Quantifying the Contribution of Aleatory and Epistemic Uncertainties in the Estimation of Probability of Failure of Primary Piping

M.I Jyrkama and M.D. Pandey
University of Waterloo, Ontario, Canada
(mjyrkama@uwaterloo.ca)

Abstract

The impact of aleatory (random) and epistemic (lack of knowledge) uncertainty on the probability of failure is typically quantified through a two-staged nested simulation approach. Addressing the regulatory concern over the confidence of the results has thus far had limited discussion in the literature. This paper shows how the two-staged approach can be used for modeling parameter uncertainty and analyzing sensitivity, which are unrelated to the confidence in the probability of failure. The results demonstrate how the uncertainty in the overall probability of failure depends only on the sampling error from simulation, not the input variable uncertainty or their separation.

1. Introduction

The reliability of primary nuclear piping is challenged by active degradation mechanisms, such as primary water stress corrosion cracking (PWSCC). The probability of failure estimate, computed using detailed fracture mechanics models [1-3], depends on the interaction of many uncertain or random variables, which are typically characterized as either *aleatory* (intrinsically variable) or *epistemic* (uncertain due to lack of knowledge). Since the estimated probability of rupture is extremely low, there is always a concern regarding the confidence of the results.

To quantify the contribution of each uncertainty on the final result, the uncertainties are typically separated in a two-stage nested simulation approach, where the epistemic parameters are sampled in the outer loop, while the aleatory variables are simulated as part of the inner loop [1, 3-5]. The overall uncertainty in the estimated probability of failure should then be governed solely by the respective number of simulations within each loop.

The concept of uncertainty separation has been a source of interest in many studies [4-11] with the primary goal of characterizing and displaying their contribution in the final outcomes. Depending on the authors, *aleatory* uncertainty has often been referred to as simply variability, or random (or stochastic) heterogeneity in a population that cannot be reduced, while *epistemic* uncertainty has been used to refer to any kind of lack of information (i.e., ignorance) with respect to model parameters, variables, structure or form, which may be reduced by further measurement or study.

The main objective of this study is to demonstrate, using simple examples, how the process of uncertainty separation may lead to misinterpretation of the results regarding the confidence (e.g., lower and upper bounds) in the estimated probability of failure. Much of the confusion is related not only to how the uncertainty in the variables and parameters is defined, but also the way in which the uncertainty is propagated to the final results.

The two-stage nested simulation approach is generally applicable in the presence of second-order uncertainties, that is, when the *parameters* of the probability distributions used to describe the random (aleatory) *variables* are themselves considered to be random variables (due to epistemic uncertainty). The results of the study show how separating variables in a first-order random variable problem renders the uncertainty in the probability of failure estimate to be *conditional* on the separated variables. Rather than uncertainty bounds, this approach leads to the estimation of *sensitivity bounds*, which may be useful for model development and interpretation of the model results.

Using simple examples, the results of the study further illustrate how the uncertainty in the estimated probability of failure in a first-order problem is due only to the sampling error from simulation, and is independent of the input variable uncertainty and their separation.

2. Problem Statement

Consider the following simple model for the time to leak, e.g., due to stress corrosion cracking, as

$$T_L = T_I + \frac{W}{R} \tag{1}$$

where T_L is the time to leak, T_I is the time to crack initiation, W is the wall thickness of the pipe, and R is the crack growth rate. The term W/R represents the time it takes for an initiated crack to grow through the pipe wall, resulting in a leak. The key question from a regulatory or fitness-for-service perspective is to determine

When is the pipe going to leak (and ultimately rupture)?

Assuming all the variables are known precisely (i.e., deterministic model), the time to leak can be assessed directly without any uncertainty (except for "model" uncertainty, which is also very important, but will not be considered in great detail in this paper). In reality, many of the parameters contributing to the leakage and failure of a pipe are unknown, and hence described by random variables. Because of the complexity of the problem, the solution is also generally obtained through numerical simulations with their own challenges and limitations. This then raises the secondary, but no less important question of

What is the uncertainty or confidence in the estimated time (or probability) of leak?

The answers to these questions are explored in the context of the simple model in Eqn. (1).

3. Basic Random Variable Model

Let us assume the pipe wall thickness, W, is constant and equal to 40 mm. Because of the uncertain nature of the cracking process, assume the time to initiation, T_I, follows the Weibull distribution with shape parameter equal to 3 and scale parameter equal to 480 months (i.e., 40 years), and the growth rate follows the Normal distribution with a mean of 5 mm/month and standard deviation of 1 mm/month. This means that the time to initiation is highly variable over time, while the crack growth is relatively fast (i.e., reaching through-wall in approximately 8 months on average in this case). These distributions are plotted in Figure 1.

The distribution for the time to grow through-wall, W/R is the reciprocal of the growth rate distribution and given as

$$g(t) = \frac{W}{t^2} f_R\left(\frac{W}{t}\right) \tag{2}$$

where $f_R(\ )$ is the distribution of the growth rate (assumed Normal in this case).

Under these assumptions, the time to leak, T_L, is also a random variable, and expressed as the sum of the time to initiation and time to grow through-wall distributions.

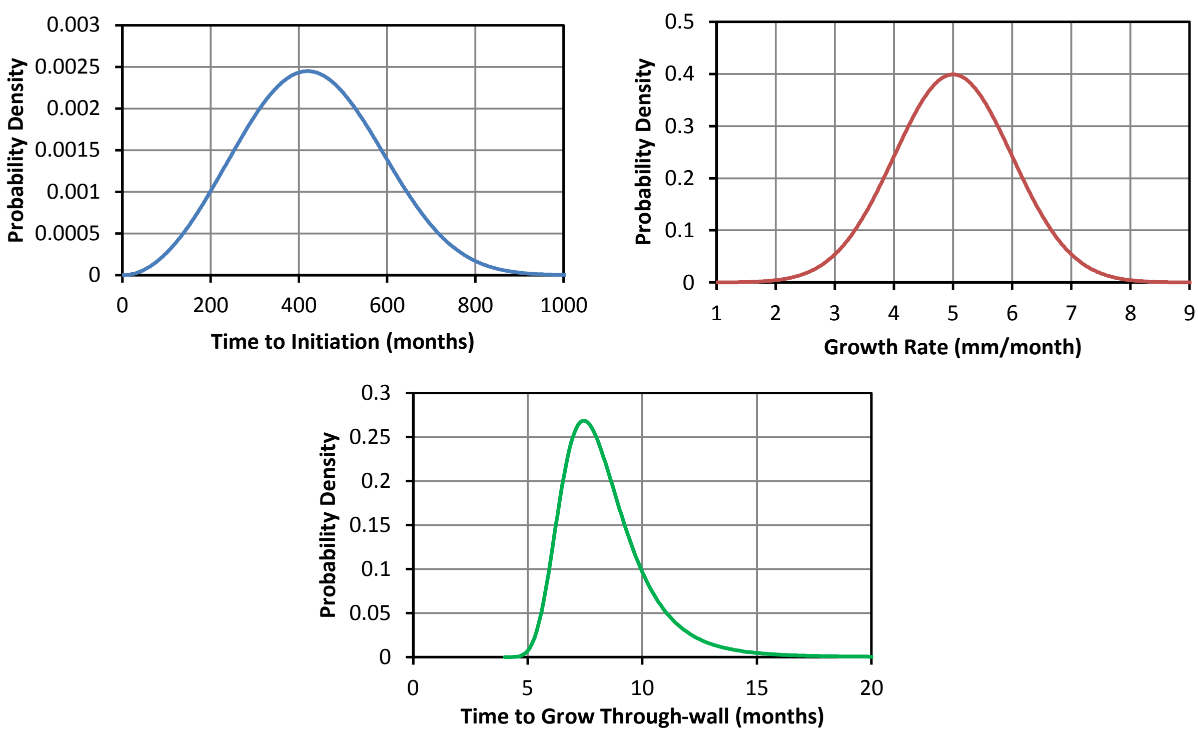

Figure 1 Distributions of time to initiation, T_I, growth rate, R, and the time to grow through-wall, W/R.

3.1 Uncertainty in Time to Leak

Even for this relatively simple problem, no analytical expression can be derived for the distribution of time to leak. Figure 2 shows the distribution of time to leak estimated from 10^7 Monte Carlo simulation trials. Disregarding the issue of simulation uncertainty for the time being, Figure 2 shows the range of variability, or (aleatory) uncertainty, in the time to leak.

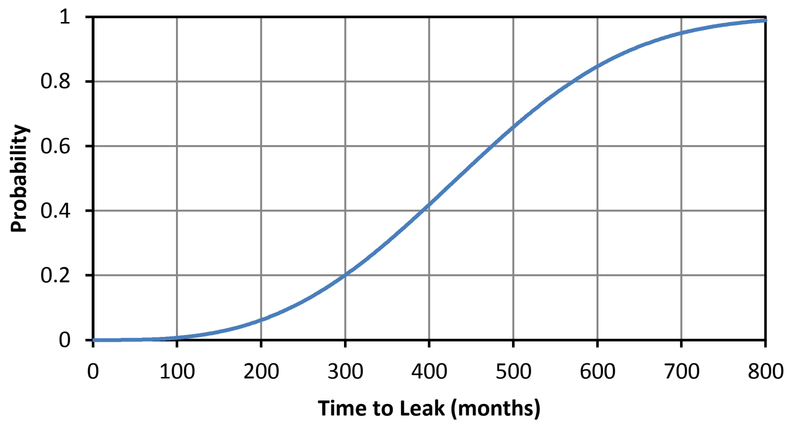

Figure 2 Cumulative distribution of time to leak, T_L, estimated using Monte Carlo simulation.

3.1.1 Prediction interval

The inherent or aleatory uncertainty in the time to leak can be characterized numerically using the concept of a *prediction interval*. For example, the 95 % prediction interval for time to leak can be obtained directly from Figure 2, with the lower limit (i.e., 0.025 percentile or quantile value) equal to 150 months, and the upper limit (0.975 percentile value) equal to 750 months. This means that for a given 40 mm thick pipe, the time to leak is predicted to be between 150 and 750 months, with 95 % probability. It is also possible to estimate the average or expected time to leak, which is equal to 437 months.

3.2 Uncertainty in Probability of Leak

Given the uncertainty in time to leak, it is not possible to answer the first question of "*when is the pipe going to leak?*" (i.e., time to leak is a random variable). Therefore, the focus is on estimating the *probability of leak* at a particular time, i.e., $P(T_L \leq t)$. This is also equivalent to the result in Figure 2, which can be interpreted as the change in the probability of leak over time. For example, the probability of leak at 720 months (i.e., 60 years) is equal to 0.96, meaning that there is a high probability that a leak will take place before 60 years.

The important comment here is that, for any problem involving a first-order function of random variables with known distributions and their parameters, only the estimated output variable, e.g., the time to leak, is by definition (i.e., inherently) random or uncertain, not the actual probability itself. The probability associated with the output variable is essentially a fixed number, which is only subject to uncertainty arising from the *estimation* (e.g., when using simulation methods). Again, this assumes that all random variables and their distributions are known precisely, and is only applicable in the context of the model (i.e., model uncertainty is ignored).

3.2.1 Uncertainty due to simulation

The epistemic uncertainty or sampling error due to simulation is straightforward to quantify using standard methods [12]. The *confidence interval* in the estimated probability is obtained from the Beta distribution (or the Normal distribution) and is plotted for the example problem in Figure 3 using only 1,000 Monte Carlo trials. As shown in Figure 3, the 95 % confidence interval is fairly narrow, even for only 1,000 simulations. Again, the issue of uncertainty or confidence arises purely from the simulation process, not the basic random variable model itself.

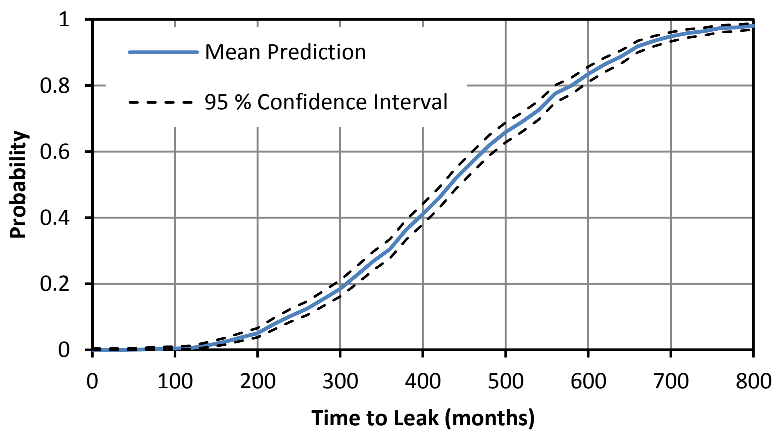

Figure 3 The 95 % confidence interval for the probability of leak using 1,000 simulation trials.

4. Second-Order Random Variable Model

In the preceding problem, the time to initiation, T_I, was assumed to follow the Weibull distribution with shape parameter equal to 3 and scale parameter equal to 480 months (i.e., 40 years). This distribution was plotted in Figure 1, and describes the inherent (or aleatory) uncertainty associated with the time to initiation. In this context, the range of variability is defined precisely by the chosen distribution parameters of 3 and 480. The obvious question then is, how good are the chosen parameters in representing the true underlying uncertainty, or in other words, what is the *uncertainty* associated with the chosen parameters?

A second-order random variable problem arises when the *parameters* of the basic random variables are considered to be uncertain or random themselves. This means that the parameters are then also described using probability distributions (with their own "hyper" parameters), similar to the Bayesian approach.

As an example, assume the Normal distribution is used to describe the *shape parameter* in the Weibull distribution of time to initiation, with its own mean and standard deviation. Rather than a single distribution, this would then result in a "family" of distributions for time to initiation, as illustrated in Figure 4. Because the parameters are commonly estimated from statistical data analysis, the parameter uncertainty is typically referred to as epistemic (i.e., it depends on the sample size, and hence may be reduced with additional data).

The obvious outcome of this second layer of uncertainty is that the actual probability of leak will now be *inherently* uncertain or random as well. In addition to the uncertainty from estimation (e.g., sampling error from simulation), the uncertainty or confidence in the probability of leak will now further be influenced by the degree of uncertainty associated with the distribution parameters. The resulting impact of this additional level of uncertainty can be quantified using a two-staged simulation approach.

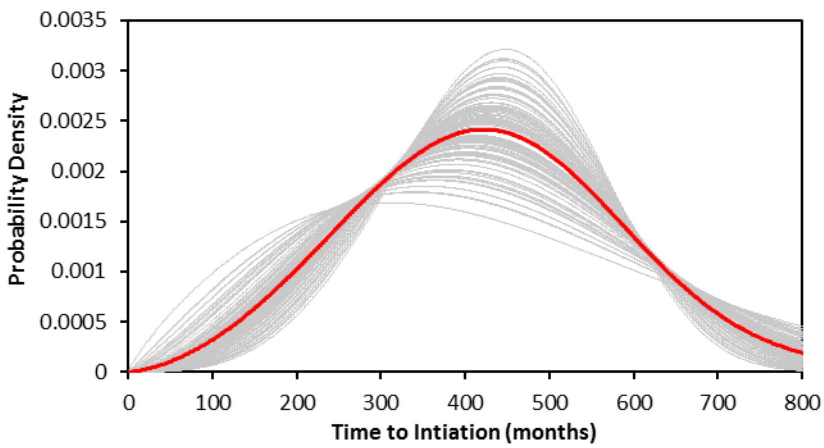

Figure 4 Family of distributions for the Weibull distributed time to initiation, T_I, assuming the shape parameter follows the Normal distribution with mean equal to 3 and standard deviation equal to 0.5 (each grey line represents a single distribution of 100 realizations of time to initiation with a random shape parameter, while the red line represents the mean or average of the 100 realizations).

4.1 Uncertainty separation using a two-staged simulation approach

The standard two-staged nested simulation approach involves first sampling the random (epistemic) *parameters* in an outer loop, followed by simulation of the (aleatory) *variables* in an inner loop for a given set of random parameters [4, 5]. This method leads to the generation of a family of distributions for the time to leak, T_L, as shown in Figure 5. Each grey line in Figure 5 represents a single realization of the time to leak distribution, reflecting the characterized (epistemic) uncertainty in the underlying distribution parameters (i.e., only the shape parameter of the Weibull time to initiation distribution in this case).

Because of the simulation approach, the uncertainty or confidence in the estimated probability of leak will depend additionally on the number of simulations. Increasing the number of simulations in the inner and outer loops reduces the overall uncertainty, as shown in Figure 5b. Naturally, some uncertainty in the probability of leak will <u>always</u> remain, regardless of the number of simulations, due to the (epistemic) uncertainty in the distribution parameters.

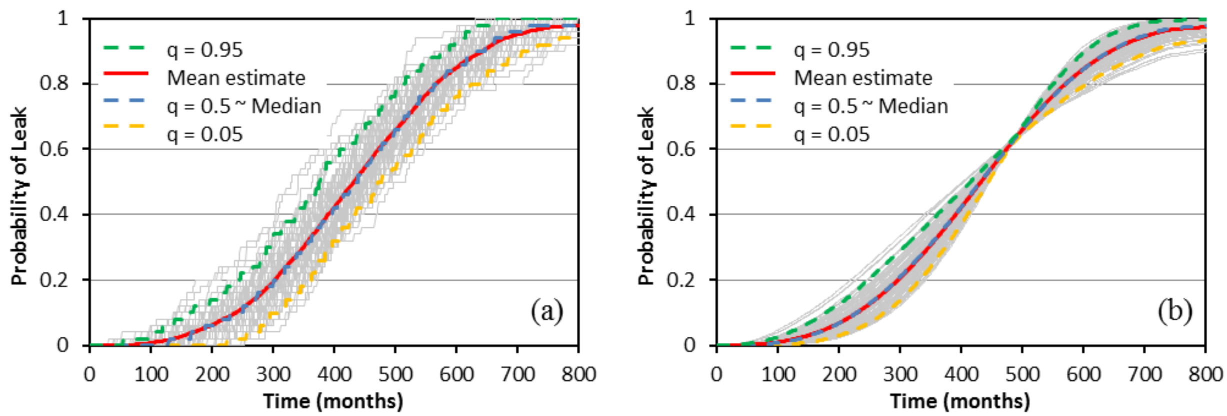

Figure 5 Estimated probability of leak over time, including the mean, median and 5 % and 95 % confidence bounds, assuming the random Weibull shape parameter for the time to initiation, and given 100 outer (epistemic) trials and (a) 50, and (b) 10^4 inner (aleatory) simulations.

The main benefit and reason for adopting the two-staged simulation approach is the relatively simple estimation and transparent presentation of the confidence bounds for the probability estimate. The estimated confidence bounds account for the sampling error from both the inner and outer loops explicitly, without the need for any complex analytical computations. In fact, no simple formulae exist for computing the confidence intervals in a general second-order random variable problem.

In summary, the two-staged nested simulation approach allows the impact of both aleatory and epistemic uncertainties to be displayed separately on the final results. The aleatory uncertainty or variability is reflected in the shape of the time to leak distribution (i.e., the "s" shape of the curves in Figure 5), while the epistemic uncertainty is characterized by the estimated confidence bounds.

5. Uncertainty Separation in a Basic Random Variable Model

The concept of the two-staged simulation approach has also been adopted in the context of the basic random variable model. For example, in the xLPR project [1, 13, 14], many of the model variables involved in the probabilistic fracture mechanics calculations can be considered either epistemic or aleatory (or also constant), which determines the sampling scheme for these variables. All epistemic variables are sampled in the outer loop, while aleatory variables are part of the inner loop [1, 14].

This approach allows the characterization of uncertainty in the probability estimate in terms of the outer epistemic variables, which is useful for sensitivity analysis. However, because the uncertainty bounds are *conditional* on the outer variables, they do not represent the uncertainty in the probability of leak (or rupture) itself. As discussed above, there is no uncertainty (beyond model and simulation uncertainty) in the actual probability in a basic first-order random variable problem. This issue of uncertainty separation and its implications are further explored in the context of the simple problem presented in Equation (1).

5.1 Sensitivity bounds

Consider the same basic random variable problem as presented previously in Section 2, where the time to initiation, T_I, follows the Weibull distribution with *known* shape parameter 3 and scale parameter equal 480 months, and the growth rate follows the Normal distribution with a mean of 5 mm/month and standard deviation of 1 mm/month.

Assume now that the time to initiation is (arbitrarily) sampled as part of the outer (epistemic) loop in a two-staged nested Monte Carlo simulation approach. This means that for each random (but constant) single value of time to initiation, t_i, from the outer loop, the time to leak is estimated in the inner loop as

$$T_L = t_i + \frac{W}{R} \tag{3}$$

which is now a function of only a single random variable (i.e., the growth rate, R).

Given the random sampling of R in the inner (aleatory) loop, the final result is again a family of curves for the estimated probability of leak as shown in Figure 6. However, in this case, the uncertainty bounds are conditional on the random time to initiation, i.e., $P(T_L \leq t \mid T_I)$. Each of the nearly vertical grey lines in Figure 6 represents the distribution of time to leak for each random realization of time to initiation, t_i. The lines are nearly vertical because of the fast crack growth rate (i.e., approximately 8 months to grow through-wall on average) relative to the time to initiation.

The variation in the grey lines represents the large uncertainty in the time to initiation as described by the Weibull distribution. Clearly, the estimated upper and lower bounds are also directly dependent (i.e., conditional) on the time to initiation distribution, and therefore, should be referred to as *sensitivity bounds*.

Figure 7 shows the results for the opposite case, where the growth rate, R, is sampled as part of the outer epistemic loop, while the time to initiation, T_I, is now part of the inner aleatory loop.

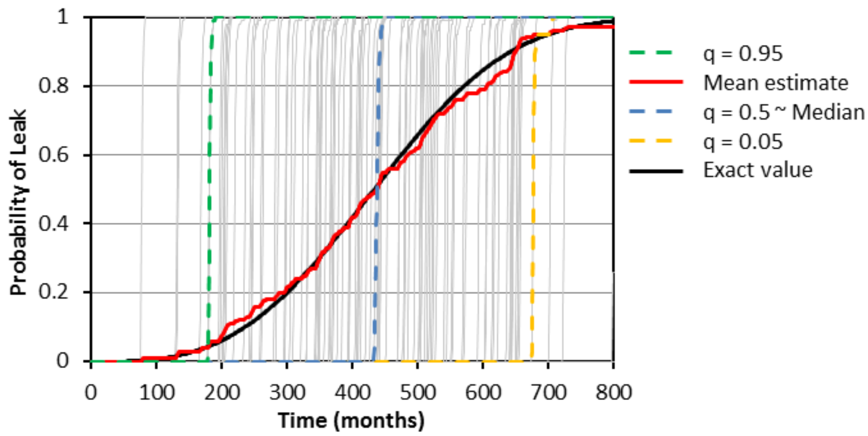

Figure 6 Probability of leak over time including the conditional mean, median and 5 % and 95 % sensitivity bounds, assuming the time to initiation, T_I, is sampled in the outer (epistemic) loop with 100 trials, while the growth rate, R, is simulated in the inner (aleatory) loop with 100 trials.

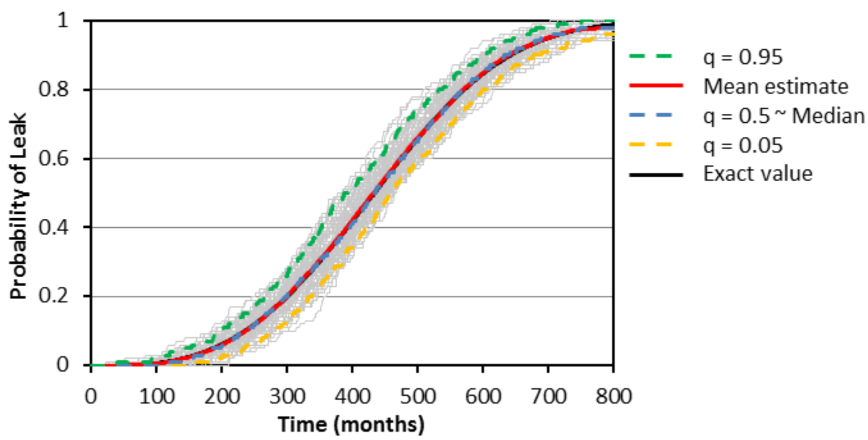

Figure 7 Probability of leak over time including the conditional mean, median and 5 % and 95 % sensitivity bounds, assuming the growth rate, R, is sampled in the outer (epistemic) loop with 100 trials, while the time to initiation, T_I, is simulated in the inner (aleatory) loop with 100 trials.

As shown by Figure 6 and Figure 7, the sensitivity bounds clearly indicate that the probability of leak (or the distribution of time to leak) is more sensitive to the uncertainty in the time to initiation, T_I, as compared to the uncertainty in the crack growth rate, R.

A key point again is that the *uncertainty* in the probability of leak is not dependent on the *uncertainty* in the input variables. It only depends on the *number of simulation trials*, as illustrated by the mean estimates (red lines) in Figure 6 and Figure 7. Increasing the number of trials would result in a better estimate for the probability of leak. The mean estimates are the same in both cases (except for the sampling error) regardless of the order of separation or looping structure, because the conditional expectations are the same. Using the rule of total probability, the conditional expectation can be expressed as

$$E[P(T_L \leq t \mid T_I = t_i)] = P(T_L \leq t) = E[P(T_L \leq t \mid R = r_i)] \tag{4}$$

which is simply equal to the unconditional distribution of time to leak, T_L.

In summary, the sensitivity bounds describe how the *distribution* of time to leak, T_L, is impacted by the uncertainty or variation in the outer (epistemic) variables. This is useful for sensitivity analysis, because it not only identifies the most critical variables, but also quantifies their impact on the estimated results. For example, reducing the uncertainty in the distribution of time to initiation, T_I, as opposed to the crack growth rate, R, would have a much larger impact on the uncertainty in the distribution of time to leak, and hence the estimate of probability of leak (i.e., the answer to the first question). However, the *uncertainty* in the probability of leak would not be affected, because it would only depend on the simulation sample size (i.e., the answer to the second question).

6. Summary and Conclusions

The estimation of probability of failure of primary piping depends on many uncertain or random variables. These variables may be characterized either as aleatory (intrinsically random) or epistemic (uncertain due to lack of knowledge). From a regulatory or fitness-for-service perspective, it is important to estimate not only the probability of failure, but also the uncertainty or confidence in the final results.

Using simple examples, this study investigated the process of uncertainty separation using a two-staged nested simulation approach and its impact on the estimated results. The two-staged approach is generally applicable to second-order random variable problems, where the parameters of the probability distributions used to describe the random variables are considered themselves to be random.

In contrast, using the two-staged approach in a basic first-order random variable problem renders the uncertainty in the probability of failure estimate to be conditional on the separated variables, and therefore leads to the estimation of sensitivity bounds, rather than the actual bounds on the probability itself. The uncertainty or confidence in the estimated probability of failure in a first-order problem is due only to sampling error from simulation, and does not depend on the input variable uncertainty or their separation.

7. References

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