Computationally Efficient Seismic Risk Analysis for Nuclear Energy Facilities

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Abstract

A simplified Vector-valued Seismic Risk Analysis (VSRA) approach is developed for nuclear energy facilities. The seismic hazard model from Vector-valued Probabilistic Seismic Hazard Analysis is simplified based on Vector-valued Seismic Hazard Deaggregation (VSHD). Numerical integrations in the standard formulation of vector-valued PSHA are then avoided. Ground motion parameters used for further structural analysis can be obtained directly from the result of VSHD. Numerical results show that the proposed approach greatly improves the efficiency of seismic risk analysis without losing computational accuracy and resorting to extensive PSHA from seismologists.

1. Introduction

Seismic risk has been defined as the potential damage or loss that may occur in hazardous earthquake events and the associated probabilities of occurrence (or exceedance) over a specified period of time [1]. It follows that the seismic risk can be described by relationships of the form

The accuracy and efficiency of seismic risk analysis inherently depends on the methodologies or analytical tools employed in hazard analysis and structural analysis.

Probabilistic Seismic Hazard Analysis (PSHA) [1] is usually performed to estimate the seismic hazard for a site of interest by integrating all possible earthquake occurrences in terms of magnitude and source-site distance. A recently developed tool called vector-valued PSHA [2] determines a joint hazard of multiple ground motion parameters occurring simultaneously at a site, and is believed to be more accurate. However, due to the extensive computational efforts, the vector-valued PSHA has not yet been commonly applied [3]. As a result, seismic risk of engineering structures cannot be accurately determined with computational ease.

In this paper, a simplified Vector-valued Seismic Risk Analysis (VSRA) approach is proposed based on Seismic Hazard Deaggregation (SHD) [4], which determines a single controlling earthquake in terms of magnitude m_c , source-site distance r_c , and the occurrence rate v_c in an average sense. Resorting to a certain ground motion prediction equation, the controlling earthquake can be applied in VSRA approximately without performing the extensive PSHA.

2. Vector-valued Seismic Risk Analysis

The vector-valued PSHA determines the joint hazard of multiple ground motion parameters occurring simultaneously at a site. In the general k-dimensional case, the joint mean rate of exceedance of spectral accelerations $S_a(T_1), S_a(T_2), ..., S_a(T_k)$ at periods $T_1, T_1, ..., T_k$ is given by

$$\lambda_{s_1...s_k} = \sum_{i=1}^{N_s} v_i \left\{ \iint_{r,m} P\{S_a(T_1) > s_1, \dots, S_a(T_k) > s_k | m, r\} f_{M,R}(m,r) \, \mathrm{d}m \, \mathrm{d}r \right\}_i.$$
(2)

The first term in the integrand is the joint distribution of spectral accelerations given a scenario earthquake in terms of magnitude and distance, which has been empirically tested to follow multivariate lognormal distribution [5].

Fragility analysis determines the probability that a structure exceeds a specific level of damage state or probability of failure conditional on seismic hazard

Fragility =
$$P\{ \text{Damage} > d | \text{Hazard} \}.$$
 (3)

Damage state of a structure or component is calibrated by typical demand parameters such as inter-story drift, maximum floor acceleration. In this study the maximum inter-story drift δ_{\max} is used. Under the lognormality assumption of δ_{\max} , one has

$$\delta_{\max} = a \cdot S_a^{b_1}(T_1) \dots S_a^{b_k}(T_k), \quad \text{or} \quad \ln \delta_{\max} = \ln a + b_1 \cdot \ln S_a(T_1) + \dots + b_k \cdot \ln S_a(T_k).$$
(4)

Fragility model can be constructed analytically, based on damage distributions from dynamic analysis [6] and regression analysis, as

$$P\{ \delta_{\max} > z \mid S_a(T_1) = s_1, \dots, S_a(T_k) = s_k\} = 1 - \Phi\left(\frac{\ln z - \mu_{\ln \delta_{\max} \mid S_a(T_1) = s_1, \dots, S_a(T_k) = s_k}}{\sigma_{\ln \delta_{\max} \mid S_a(T_1) = s_1, \dots, S_a(T_k) = s_k}}\right),$$
(5)

where $\mu_{\ln \delta_{\max} | S_{a}(T_{1}) = s_{1},...,S_{a}(T_{k}) = s_{k}}$ and $\sigma_{\ln \delta_{\max} | S_{a}(T_{1}) = s_{1},...,S_{a}(T_{k}) = s_{k}}$ are the conditional mean and standard deviations.

By combining the seismic hazard analysis and fragility analysis, the seismic risk, *i.e.*, mean rate of exceedance (MRE) of δ_{max} exceeding a threshold *z* can be evaluated as

$$\lambda_{\delta_{\max}}(z) = \int_{0}^{\infty} \int_{0}^{\infty} P\{\delta_{\max} > z \mid S_{a}(T_{1}) = s_{1}, \dots, S_{a}(T_{k}) = s_{k}\} \cdot \frac{\partial^{k} \lambda_{s_{1} \dots s_{k}}}{\partial s_{1} \dots \partial s_{k}} \cdot ds_{1} \dots ds_{k}$$

$$= \int_{0}^{\infty} \int_{0}^{\infty} P\{\delta_{\max} > z \mid S_{a}(T_{1}) = s_{1}, \dots, S_{a}(T_{k}) = s_{k}\} \cdot \sum_{i=1}^{N_{i}} v_{i} \left\{ \iint_{r,m} P\{S_{a}(T_{1}) = s_{1}, \dots, S_{a}(T_{k}) = s_{k} \mid m, r\} f_{M,R}(m,r) dm dr \right\}_{i} ds_{1} \dots ds_{k},$$
(6)

where $\frac{\partial^{k} \lambda_{s_{1}...s_{k}}}{\partial s_{1}...\partial s_{k}}$ is the joint mean rate density of the simultaneous occurrence of multiple spectral accelerations, and the term $P\{\delta_{\max} > z | S_{a}(T_{1}) = s_{1},...,S_{a}(T_{k}) = s_{k}\}$ in the integrand represents the probability that δ_{\max} exceeds a specified value z, given $S_{a}(T_{1}) = s_{1},...,S_{a}(T_{k}) = s_{k}$.

As seen from equation (13), the VSRA involves a multi-fold integration in term of m and r, and a vector of spectral ordinates, which needs intensive computational efforts.

In seismic analysis and design, seismic hazard deaggregation (SHD) [4] is often performed to determine a single controlling earthquake in terms of magnitude, distance associated with its occurrence rate for the site of interest in an average sense. In concept, the Vector-valued Seismic Hazard Deaggregation (VSHD) considers the simultaneous occurrence of spectral accelerations at multiple periods, thus has clearer physical meaning than that of SHD. The resulting controlling earthquake (m_c , r_c and v_c) is the dominant seismic hazard contributor to induce the spectral accelerations $s_1, s_2, ..., s_k$ over the entire frequency range of interest simultaneously. The procedure of Vector-valued SHD can be found in [4].

By substituting m_c and r_c into the standard VPSHA, the seismic hazard can be approximated by

$$\lambda_{s_1...s_k} = \nu_C \cdot P\{S_a(T_1) > s_1, ..., S_a(T_k) > s_k \mid m_C, r_C\}$$
(7)

Incorporating this approximation into the standard Seismic Risk Analysis, one has

$$\lambda_{\delta_{\max}}(z) = \int_{0}^{\infty} \int_{0}^{\infty} P\{\delta_{\max} > z \mid S_a(T_1) = s_1, ..., S_a(T_k) = s_k\} \cdot \frac{\partial^k \lambda_{s_1...s_k}}{\partial s_1 \dots \partial s_k} \cdot ds_1 \dots ds_k$$

$$= \int_{0}^{\infty} \int_{0}^{\infty} P\{\delta_{\max} > z \mid S_a(T_1) = s_1, ..., S_a(T_k) = s_k\} \cdot v_C \cdot P\{S_a(T_1) = s_1, ..., S_a(T_k) = s_k \mid m_C, r_C\} ds_1 \dots ds_k,$$
(8)

in which the integration of all possible potential earthquakes is avoided and approximated by a single most-likely earthquake.

3. Numerical example

In this example, a 20-story steel moment resisting frame structure is assumed to be located at a hypothetical site with configuration of seismic sources as shown in Figure 1.



Figure 1: Hypothetical configuration of seismic sources

3.1 Analytical Results

To perform VPSHA, the commonly used truncated exponential distribution of magnitude and the uniform distribution of seismic focus are adopted. The ground-motion prediction equation by

Abrahamson and Silva (1997) [7] is used to obtain the mean and standard deviation values for the conditional probability distributions of spectral accelerations. The matrix of correlation coefficient from Baker and Jayaram (2008) [5] is used in the joint probability model.

Since the focus of this study is not fragility analysis, the existing regression results given in [2] is used here, which is based on nonlinear dynamic analysis running 14 accelerograms recorded on stiff soil in California. The statistics of the quantity δ_{max} is strongly jointly dependent on the S_a at both first ($T_1 = 4.0 \text{ sec}$) and second ($T_2 = 1.33 \text{ sec}$) vibration periods of the undamaged building, as listed in Table 1.

Scalar IM	mean	Standard deviation
$S_a(T_1)$	$\ln \delta_{\max} = -2.32 + 0.70 \ln S_a(T_1)$	$\sigma_{\ln\delta_{\max} \mathbf{S}_{a}(T_{1})}=0.37in$
Vector IM	mean	standard deviation
$[S_a(T_1), S_a(T_2)]$	$\ln \delta_{\max} = -2.49 + 0.58 \ln S_a(T_1) + 0.62 \ln S_a(T_2)$	$\sigma_{\ln\delta_{\max} \mathbf{S}_a(T_1),\mathbf{S}_a(T_2)} = 0.23in$

Table 1: Statistics of response parameter δ_{max}

The VSHD is performed for eight controlling periods (*i.e.* 0.01, 0.02, 0.05, 0.1, 0.3, 0.5, 1, 5 sec). A 3-D histogram shown in Figure 2 indicates that seismic hazard is dominated by Source 1. The resulting controlling earthquake is $m_c = 5.81$, $r_c = 29.66$, $v_c = 0.045$.



Figure 2: Vector-valued seismic hazard deaggregation

Accurate SRA and approximate VSRA are performed for scalar and 2-dimensional vector-valued case, whose results are shown in Figures 3 together for the purpose of comparison.



Figure 3: Scalar and 2-D vector-valued seismic risk curves

- 4 of 5 pages -

3.2 Comparative analysis

The risk curves based on scalar ground motion parameter are conservative by a factor as much as $15\sim20$ at probability level 1×10 -4, implying that for flexible structure, whose higher-mode response is significant, a vector-valued seismic risk analysis is more appropriate.

Accuracy of the approximation is evaluated using error analysis. According to ASCE/SEI 43-05 [8], 3 failure probability levels are of great interest as given in Table 2. Computational efficiency can be seen roughly from the time costs for risk analysis tabulated in Table 3.

 Table 2: Computational error

P_F	SRA	2-D VSRA
1×10^{-4}	8.3%	12.0%
4×10^{-5}	2.9%	9.1%
1×10^{-5}	2.1%	0.0%

Table 3: Time costs for seismic risk analysis

Computational scheme	Time cost
scalar SRA, accurate	10 min
scalar SRA, approximate	0.1 sec
2-D VSRA, accurate	8.2 day
2-D VSRA, approximate	14.5 sec

4. Conclusions

The vector-valued probabilistic seismic hazard analysis (VPSHA), along with the seismic fragility model, are applied to establish a vector-valued seismic risk analysis (VSRA). However, due to the extensive computational effort, VSRA is difficult to be applied in engineering practice.

The vector-valued seismic hazard deaggregation (VSHD) determines a single controlling earthquake in an average sense in terms of magnitude, souce-site distance, and occurrence, which contributes dominant seismic hazard to the site.

To simplify the VSRA, an approximate approach is developed based on the controlling earthquake obtained from VSHD. Numerical results show that the proposed approximate approach can give comparable results to the accurate ones, but the efficiency is greatly improved.

5. References

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- 5 of 5 pages -