Generation of Earthquake Ground Motions Preserving Non-Stationary Characteristics for Nuclear Power Plants

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Abstract

A new approach for generating a set of tri-directional earthquake time-histories compatible with the target seismic design spectra for nuclear energy facilities based on multiple sets of actual earthquake records is presented. The tri-directional spectrum-compatible earthquake time-histories are generated by using the Hilbert-Huang transform (HHT) and solving optimization problems. The generation procedure can preserve the non-stationary characteristics of the seed actual earthquake records.

1. Introduction

Seismic Response History Analysis (SRHA) has been widely used for seismic design and qualification of structures, systems, and components in nuclear energy facilities ([1], [2], [3]). To realistically characterize the seismic hazard environment, a set of representative tri-directional earthquake time-histories, which are statistically independent of each other, is required for this analysis method.

Several researchers have studied the characteristics of tri-directional earthquake ground motions. Chen [4] defined the correlation coefficient of two earthquake time-histories as an index of statistical independence of these two time-histories. Hadjian [5] conducted a similar investigation and concluded that the absolute correlation coefficient of two orthogonal ground motions was 0.32.

Levy and Wilkinson [6] have attempted to generate two orthogonal spectrum-compatible artificial time-histories, which are statistically independent. The first spectrum-compatible earthquake time-history was represented by Fourier series. The median interpolations of the Fourier amplitudes and frequencies of the first time-history were then used to construct another earthquake time-history. These two time-histories, whose correlation coefficient was low, were thought to be statistically independent. However, this approach is unable to preset the value of the correlation coefficient of two earthquake time-histories, and hence unable to guarantee two generated time-histories to be statistically independent.

In this paper, a new generation method [7] is presented for generating a set of tri-directional orthogonal earthquake time-histories compatible with the target seismic design spectra for nuclear energy facilities. Three orthogonal components of generated earthquake ground motion are statistically independent from the point of view of practical engineering. Two horizontal components of the generated ground motion have the same frequency-time-energy distribution. The generated time-histories can preserve the non-stationary characteristics of the seed actual earthquake records and can also meet the strict requirements in the codes listed in Section 3.

2. Hilbert-Huang transform (HHT)

To obtain a set of tri-directional spectrum-compatible earthquake time-histories for seismic response history analysis, the Hilbert-Huang transform (HHT), developed by Huang *et al.* [8], is applied. The HHT can represent non-stationary and nonlinear data, such as earthquake records, by decomposing the data into several components and transforming the data from time-domain to frequency-domain.

Compared to Fourier transform and wavelet transform, the HHT can meet the necessary conditions for the basis to represent a non-stationary and nonlinear time series: complete, local, and adaptive. The first condition guarantees the degree of precision of the expansion. The requirement for locality is the most crucial for non-stationarity, which means all events have to be identified by the time of their occurrence. Consequently, it is required that both the amplitude and the frequency be functions of time. The requirement for adaptivity is also crucial for both non-stationary and nonlinear data. Since it is impossible to expect a predetermined basis to fit all the phenomena in the data, an easy way to generate the necessary adaptive basis is to derive the basis from the data themselves. This is the substantial advantage of the HHT compared to other transform techniques.

The HHT is the result of the Empirical Mode Decomposition (EMD) and the Hilbert Spectral Analysis (HSA). By applying the EMD method, an earthquake record can be decomposed into n so-called Intrinsic Mode Functions (IMF) $c_j(t)$ (j=1, 2, …, n). The original earthquake record $X(t)$ is then the sum of the IMF components $c_j(t)$ plus the final residue $r_n(t)$

$$X(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) . \quad (1)$$

By applying the HSA to the n IMF components of $X(t)$ in equation 1, the earthquake record $X(t)$ can be written as

$$X(t) = \mathrm{Re} \sum_{j=1}^{n} a_j(t)\, \mathrm{e}^{\mathrm{i} \int \omega_j(t) \mathrm{d}t} , \quad (2)$$

where Re stands for the real part, $a_j(t)$ and $\omega_j(t)$ are the time-dependent amplitude and instantaneous frequency associated with the jth IMF component, respectively. The residue $r_n(t)$ is not included because of its monotonic property.

3. Code requirements for generating design ground motions

According to the codes for seismic design and analysis of nuclear energy facilities ([1], [2], [3]), the seismic design spectra for safety-related nuclear structures shall be defined for ground motions in two orthogonal horizontal and one vertical directions. Except for unusual circumstances of geology or location of seismic sources, the design spectra in two horizontal directions shall be assumed equal. The vertical component of the seismic design spectra can be obtained by scaling the corresponding coordinates of the horizontal design spectra using specified factors throughout the frequency range.

Tri-directional earthquake time-histories are needed for seismic design and analysis of nuclear energy facilities unless the uncoupled response of the structure is expected. Time-history for each direction of motion shall be compatible with the corresponding target seismic design spectrum. Three orthogonal time-histories of the design ground motion shall be statistically independent of each other.

For generating tri-directional spectrum-compatible earthquake time-histories, the quantitative acceptance criteria of earthquake time-histories in the codes are summarized:

1. Spectral accelerations at 5 % damping shall be computed at 100 points per frequency decade uniformly spaced over the log frequency scale.

2. The Peak Ground Acceleration (PGA) shall equal or exceed the PGA of the seismic design spectrum.

3. The 5 % damped response spectrum of the time-history shall not fall more than 10\% below the target spectrum at any one frequency and not exceed the target spectrum more than 30 % in the frequency range between 0.2 Hz and 25 Hz.

4. No more than 9 adjacent frequency points are allowed to fall below the target spectrum.

5. The average of the ratios of response spectrum of the time-history to the target spectrum frequency by frequency shall be equal to or greater than 1.

6. To be considered statistically independent, the correlation coefficients between three orthogonal components of one set of ground motions shall not exceed 0.3, i.e., $c = 0.3$ in equations 6 and 8.

4. Generation of tri-directional spectrum-compatible earthquake time-histories

In this paper, a conventional design spectrum (CDS) is selected as the target seismic design spectra for nuclear energy facilities. A CDS is the statistical result of processing actual earthquake records obtained from worldwide ground motion database. Since the CDS represent the design ground motions by combining the seismic hazard contributions from multiple earthquakes, the selection of seed actual earthquake records can be based on the identification of dominant magnitude-distance pairs that impact the site of interest. It is noted that the proposed generation method is suitable to any design spectra once the seed earthquake time-histories are defined.

4.1 Selection of seed earthquake time-histories

The first step in generating tri-directional spectrum-compatible earthquake time-histories is to select several sets of tri-directional orthogonal actual earthquake records. Small near-field earthquakes and large far-field earthquakes are used to characterize the high frequency portion and the low frequency portion of the target design spectrum, respectively. The selection of representative actual earthquake records refers to the results of Seismic Hazard Deaggregation (SHD) [9]. The SHD can provide the relative hazard contributions of the earthquake sources in terms of distance and magnitude. The selected actual earthquake records, induced from earthquakes with such distances and magnitudes, have relatively the most seismic hazard contributions for the site of interest.

4.2 Generation of the first horizontal spectrum-compatible earthquake time-history

Each of the selected horizontal actual earthquake records is decomposed into several IMF components via EMD. The l generated IMF components from the selected horizontal actual earthquake records are treated as the basis to represent a non-stationary earthquake time-history. Compared to Fourier transform and wavelet transform, the basis to represent a time-series are derived from the actual earthquake records themselves by EMD. Hence, these IMF components preserve the information of the nature of the selected earthquakes, which is consistent with the seismic hazard circumstances at the site of interest.

The time-dependent amplitude and the instantaneous frequency of each IMF component are then generated through HSA. The HHT amplitudes $a_{1,i}(t)$ and the instantaneous frequencies $\omega_{1,i}(t)$ of the horizontal actual earthquake records are thus scaled to obtain the first horizontal earthquake time-history

$$\mathrm{TH}_{H1}(\mathbf{x},t) = \mathrm{Re}\sum_{i=1}^{l}[x_i a_{1,i}(t)]\,\mathrm{e}^{\mathrm{i}\, x_{l+i}\int \omega_{1,i}(t)\mathrm{d}t}\ , \tag{3}$$

where $x_1, x_2, \ldots, x_l$ are the amplitude scaling parameters, $x_{l+1}, x_{l+2}, \ldots, x_{2l}$ are the frequency scaling parameters, and $\mathbf{x}=\{x_1, x_2, \ldots, x_{2l}\}^T$ is the column vector of the scaling parameters.

The purpose of generating spectrum-compatible earthquake time-history is to make the time-history rich in all frequencies. By scaling the amplitudes of the IMF components, the frequency contents over corresponding frequency ranges can be changed to enrich the time-history in all frequencies as required. Since each IMF component contains a range of frequencies, adjacent IMF components of each actual earthquake record may overlap or separate in the frequency domain. Slightly adjusting the instantaneous frequencies of the IMF components can ensure that there are no significant gaps in the frequency contents of the IMF components.

An optimization model is then constructed:

$$\text{Minimize:} \quad V = \sum_{p=1}^{q} f_p(\mathbf{x}) \,, \qquad f_p(\mathbf{x}) = \frac{\left| S_M\left[\mathrm{TH}_{H1}(\mathbf{x},t), T_p\right] - s_H(T_p) \right|}{s_H(T_p)}, \tag{4}$$
$$\text{Subject to constraints:} \quad x_r > 0, \quad r = 1,\ 2,\ \ldots,\ 2l,$$

where $S_M[\mathrm{TH}_{H1}(\mathbf{x},t),\ T_p]$ is the spectral acceleration of the first generated horizontal earthquake time-history TH_{H1} $(\mathbf{x},t)$ at a specific period T_p, $s_H(T_p)$ is the spectral acceleration of the target design spectrum for horizontal component at T_p, and V is called the objective function.

By solving the optimization problem 4 with a suitable optimization algorithm, the first horizontal earthquake time-history, whose response spectrum closely matches the target design spectrum, can be generated. The first horizontal spectrum-compatible earthquake time-history can then be obtained by scaling the generated time-history linearly to meet the acceptance criteria in the codes. It contains the desired characteristics of ground motions from actual earthquakes that contribute relatively the most seismic hazard to the site of interest. The generated spectrum-compatible earthquake time-history also preserve the non-stationary characteristics of the seed earthquake records through the HHT.

4.3 Generation of the second horizontal spectrum-compatible earthquake time-history

Having obtained the first horizontal spectrum-compatible earthquake time-history, the second horizontal spectrum-compatible earthquake time-history can be generated as follows. The first generated horizontal earthquake time-history $\mathrm{TH}_{H1}(t)$ is treated as a new seed earthquake time-history and decomposed into m IMF components as the basis to represent the second horizontal time-history via EMD.

The time-dependent amplitudes $a_{2,j}(t)$ and the instantaneous frequencies $\omega_{2,j}(t)$ of the first horizontal time-history are then generated through HSA. The phase of each IMF component of the first horizontal time-history are thus shifted to obtain the second horizontal earthquake time-history

$$\mathrm{TH}_{H2}(\mathbf{x},t) = \mathrm{Re}\sum_{j=1}^{m} a_{2,j}(t)\mathrm{e}^{\mathrm{i}\left[\int \omega_{2,j}(t)\mathrm{d}t + x_j\right]}, \tag{5}$$

where $x_1, x_2,\ \ldots,\ x_m$ are the phase shifting parameters in the range from 0 to 2π. By merely shifting the phase of each IMF component, the second generated horizontal time-history has the same frequency-time-energy distribution as the first generated horizontal earthquake time-history $\mathrm{TH}_{H1}(t)$ [10].

A constrained optimization model is then used:

$$\text{Minimize:} \quad V = \sum_{p=1}^{q} f_p(\mathbf{x}) \,, \qquad f_p(\mathbf{x}) = \frac{\left| S_M\left[\mathrm{TH}_{H2}(\mathbf{x},t), T_p\right] - s_H(T_p) \right|}{s_H(T_p)}, \tag{6}$$

$$\text{Subject to constraints:} \quad \left|\rho[\mathrm{TH}_{H1}(t), \mathrm{TH}_{H2}(t)]\right| \le c,$$

where c is a small prescribed value to ensure that these two time-histories are statistically independent in engineering sense. The correlation coefficient between the first and second generated horizontal time-histories TH_{H1}(t) and TH_{H2}(t) is defined as

$$\rho[\mathrm{TH}_{H1}(t), \mathrm{TH}_{H2}(t)] = \frac{\mathrm{E}[\{\mathrm{TH}_{H1}(t) - \mu_{H1}\}\{\mathrm{TH}_{H2}(t) - \mu_{H2}\}]}{\sigma_{H1} \cdot \sigma_{H2}},$$

where E[·] is the mathematical expectation, μ_{H1} and μ_{H2} are the mean values, and σ_{H1} and σ_{H2} are the standard deviations of $\mathrm{TH}_{H1}(t)$ and $\mathrm{TH}_{H2}(t)$, respectively [4].

By minimizing the objective function V subjected to the constraints in optimization model 6, the second horizontal earthquake time-history, which is statistically independent of the first horizontal earthquake time-history and closely matches the target design spectrum, can be generated.

Having the second horizontal earthquake time-history, which closely matches the target design spectrum, the second horizontal spectrum-compatible earthquake time-history can then be obtained by scaling the second generated time-history linearly, with the same scaling factor as used for the first generated time-history, to meet the acceptance criteria in the codes. It should be noted that linearly scaling the time-histories does not change the correlation coefficient between them.

4.4 Generation of vertical spectrum-compatible earthquake time-history

The last step is to generate vertical spectrum-compatible earthquake time-history. Each of the selected vertical actual earthquake records is decomposed into a number of IMF components via EMD. The n generated IMF components from the selected vertical actual earthquake records are treated as the basis to represent a non-stationary vertical earthquake time-history.

The time-dependent amplitude and the instantaneous frequency of each IMF component are then generated through HSA. The HHT amplitudes $a_{V,k}(t)$ and the instantaneous frequencies $\omega_{V,k}(t)$ of the vertical actual earthquake records are thus scaled to obtain the vertical earthquake time-history

$$\mathrm{TH}_V(\mathbf{x},t) = \mathrm{Re}\sum_{k=1}^{n} [x_k a_{V,k}(t)]\, \mathrm{e}^{\mathrm{i}\, x_{n+k} \int \omega_{V,k}(t)\mathrm{d}t} \quad , \tag{7}$$

where x_1, x_2, …, x_n are the amplitude scaling parameters, x_{n+1}, x_{n+2}, …, x_{2n} are the frequency scaling parameters, and $\mathbf{x}=\{ x_1, x_2, \ldots, x_{2n} \}^T$ is the column vector of the scaling parameters.

A constrained optimization model is then used:

$$\text{Minimize:} \quad V = \sum_{p=1}^{q} f_p(\mathbf{x}) \; , \qquad f_p(\mathbf{x}) = \frac{\left| S_M \left[\mathrm{TH}_V(\mathbf{x}, t), T_p \right] - s_V(T_p) \right|}{s_V(T_p)}, \tag{8}$$
$$\text{Subject to constraints:} \quad \left| \rho[\mathrm{TH}_V(t), \mathrm{TH}_{H1}(t)] \right| \le c, \quad \left| \rho[\mathrm{TH}_V(t), \mathrm{TH}_{H2}(t)] \right| \le c,$$

where $s_V(T_p)$ is the spectral acceleration of the target design spectrum for vertical component at period T_p.

By solving the optimization problem 8, the earthquake time-history in the vertical direction, whose response spectrum closely matches the target design spectrum for vertical component, can be generated. The vertical spectrum-compatible earthquake time-history can then be obtained by scaling the generated time-history linearly to meet the acceptance criteria in the codes. The generated spectrum-compatible earthquake time-history in the vertical direction is thus statistically independent of two generated spectrum-compatible earthquake time-histories in the horizontal directions, respectively.

5. Numerical example

Following the procedure described in Section 3, a set of tri-directional orthogonal spectrum-compatible earthquake time-histories can be generated. Three components of the generated earthquake ground motion are statistically independent in engineering concern. Two horizontal components of the generated ground motion have the same frequency-time-energy distribution. For illustration purpose, one numerical example is presented in this section.

5.1 Optimization algorithm

There are a great number of optimization techniques, such as the Quasi-Newton method and the Nelder-Mead method [11], which can be used to achieve the minimization of the objective function V. For the optimization problems in equations 4, 6, and 8 in the proposed generation method, the constrained nonlinear multi-variable line-search algorithm [11] is applied.

5.2 Target horizontal and vertical seismic design spectra

A conventional design spectrum (CDS) for nuclear energy facilities at Quebec City are used as the horizontal target design spectrum. The standard spectral shape of the CDS is from Standard CSA N289.3-10 [1]. The peak ground acceleration (PGA) for Quebec City is obtained by the Geological Survey of Canada (GSC), which has the probability of exceedance of 0.0004 per annum (2% in 50 years). The CDS for Quebec City is then obtained by scaling the standard spectral shape using the PGA for Quebec City.

The selected target design spectrum is for rock site conditions (average shear wave velocity to a depth of 30 m greater than 750 m/sec) and for 5% critical damping. According to the seismic design codes ([1], [2], [3]), the vertical target design spectrum for Quebec City is obtained by scaling the corresponding coordinates of the horizontal design spectrum by two-thirds throughout the entire frequency range.

5.3 Selection of seed earthquake time-histories

The seed actual earthquake records are selected based on the results of the seismic hazard deaggregation (SHD) [9]. The results of the SHD and the selected earthquake ground motions are listed in Tables 1 and 2, respectively.

Location	Motion period	Seismicity	Distance	Magnitude
Quebec City	Short	0.56 g	41 km	6.4
	Long	0.14 g	100 km	7.1

Table 1 Results of seismic hazard deaggregation

Three earthquakes are selected for the target CDS at Quebec City. Compared to the target CDS, the high frequency portion of the response spectrum of an actual earthquake time-history is highly non-smooth. To ensure the resulting response spectrum closely match the target CDS, two small near-field earthquakes are used to characterize the high frequency plateau portion of the target CDS, and one large far-field earthquake is used to describe the low frequency portion of the target CDS.

The actual earthquake accelerograms used in this paper are searched from strong motion databases of the Pacific Earthquake Engineering Research Center (PEER). Each set of ground motions has two orthogonal horizontal and one vertical acceleration time-histories. All the selected actual earthquake time-histories are recorded by free-field instruments or instrument shelters at rock site (average shear wave velocity to a depth of 30 m greater than 750 m/sec). The magnitude-distance pairs of the selected earthquake records are approximately consistent with the results of the SHD and thus generally contribute the most relative seismic hazard to the sites of interest.

Location	EQ[1]	Type[2]	D[3] (km)	M[4]	Hor-1[5],-2[5]	Ver[5]
Quebec	WN	SN	21.2	6.0	A-MTW-000,-090	A-MTW-UP
	NPS	SN	25.8	6.0	SIL-000,-090	SIL-UP
	LP	LF	79.7	6.9	RIN-000,-090	RIN-UP

1. Earthquake names: WN, Whittier Narrows, 1987/10/01; NPS, N. Palm Springs, 1986/07/08; LP, Loma Prieta, 1989/10/18 00:05. 2. Earthquake type: SN, Small near-field; LF, Large far-field. 3. The closest distance to fault rupture. 4. Moment magnitude. 5. Names of two horizontal and one vertical components of earthquake records, which can be searched from PEER database.

Table 2 Selected actual earthquake records

5.4 Tri-directional spectrum-compatible earthquake time-histories

The response spectra of the generated horizontal and vertical earthquake time-histories and the selected seed earthquake time-histories are shown in Figure 1. The ratios of response spectra of generated spectrum-compatible earthquake time-histories to the target design spectra frequency by frequency, as shown in Figure 2, are within the range from 0.9 to 1.3.

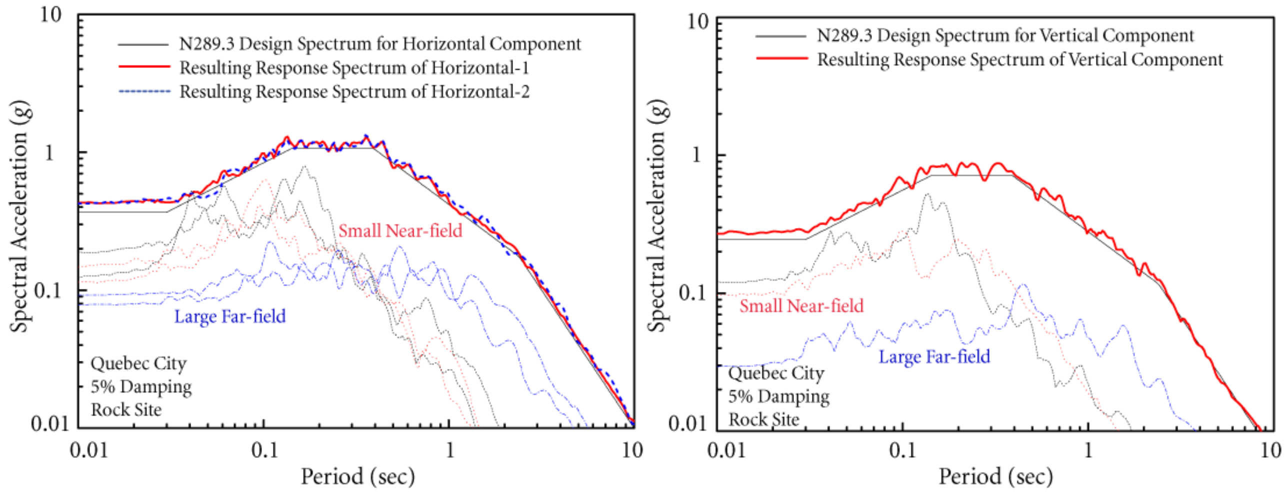

Figure 1 Resulting and target response spectra

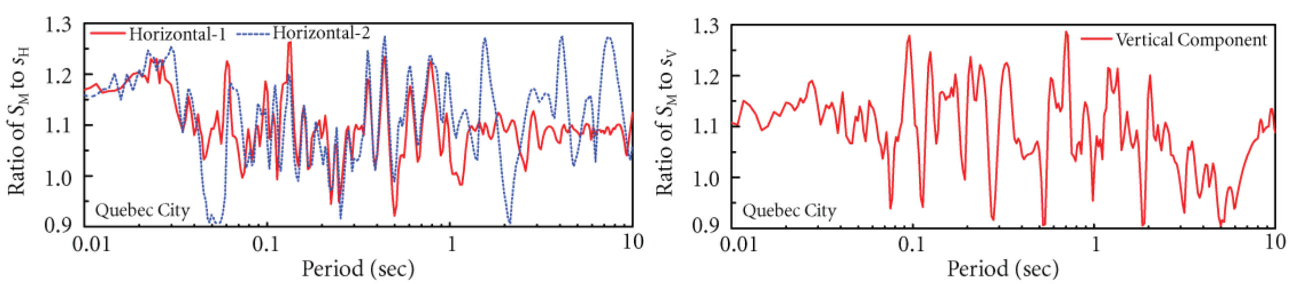

Figure 2 Errors between resulting and target response spectra.

As shown in Table 3, the PGA of each generated earthquake time-history is greater than that of the corresponding target seismic design spectrum. The average of the ratios of each resulting response spectrum to the target design spectrum frequency by frequency is greater than 1. The correlation coefficients between the components of tri-directional ground motions are less than 0.3. The generated tri-directional spectrum-compatible earthquake time-histories thus meet all the acceptance criteria in the codes ([1], [2], [3]).

Location	PGA-DS[1] (g)	PGA-TH[2] (g)	AR[3]	ρ[4]
Quebec City	$H1^5$=0.367	$H1$=0.429	$H1$=1.09	$\rho[H1,H2]$=0.058
	$H2^5$=0.367	$H2$=0.425	$H2$=1.36	$\rho[H1,V]$=0.026
	V^5=0.245	V=0.271	V=1.32	$\rho[H2,V]$=0.025

1. Peak ground acceleration of target design spectrum. 2. Peak ground acceleration of generated earthquake time-history. 3. Average of ratios of resulting response spectrum to target design spectrum. 4. Correlation coefficient between two ground motion components. 5. $H1$, $H2$, and V are two horizontal components and one vertical components of a set of tri-directional ground motions, respectively.

Table 3 Information of generated earthquake time-histories

The generated horizontal and vertical spectrum-compatible earthquake time-histories and their corresponding seed earthquake time-histories are shown in Figure 3. Although the two generated horizontal components appear to be similar, they are statistically independent of each other. The generated spectrum-compatible earthquake time-histories generally preserve the temporal characteristics of the seed actual earthquake time-histories. It is noted that the generated earthquake time-histories may need proper baseline correction to eliminate the drifts in the velocity and displacement time-histories.

6. Conclusion

A new method, which can generate a set of tri-directional earthquake time-histories compatible with the target seismic design spectra for nuclear energy facilities based on multiple sets of actual earthquake records and optimization techniques, is presented. Three orthogonal components of generated earthquake ground motion are statistically independent in engineering concern. Two horizontal components have the same frequency-time-energy distribution. The generation procedure can preserve the non-stationary characteristics of the seed actual earthquake records and can also meet the strict requirements in the codes.

The success of the proposed generation procedure in any practical situation will depend much on the availability and the selection of strong ground motion records. A proper selection of seed ground motion records can ensure the resulting response spectrum closely match the target spectrum. Although the results of seismic hazard deaggregation in terms of magnitude-distance pairs provide much useful information for earthquake record selection, strong ground motions, which are recorded around the sites of interest and reflect the most realistic seismic hazard of the sites, are still desired. This difficulty will be alleviated with the increase of the number of the available earthquake records over time.

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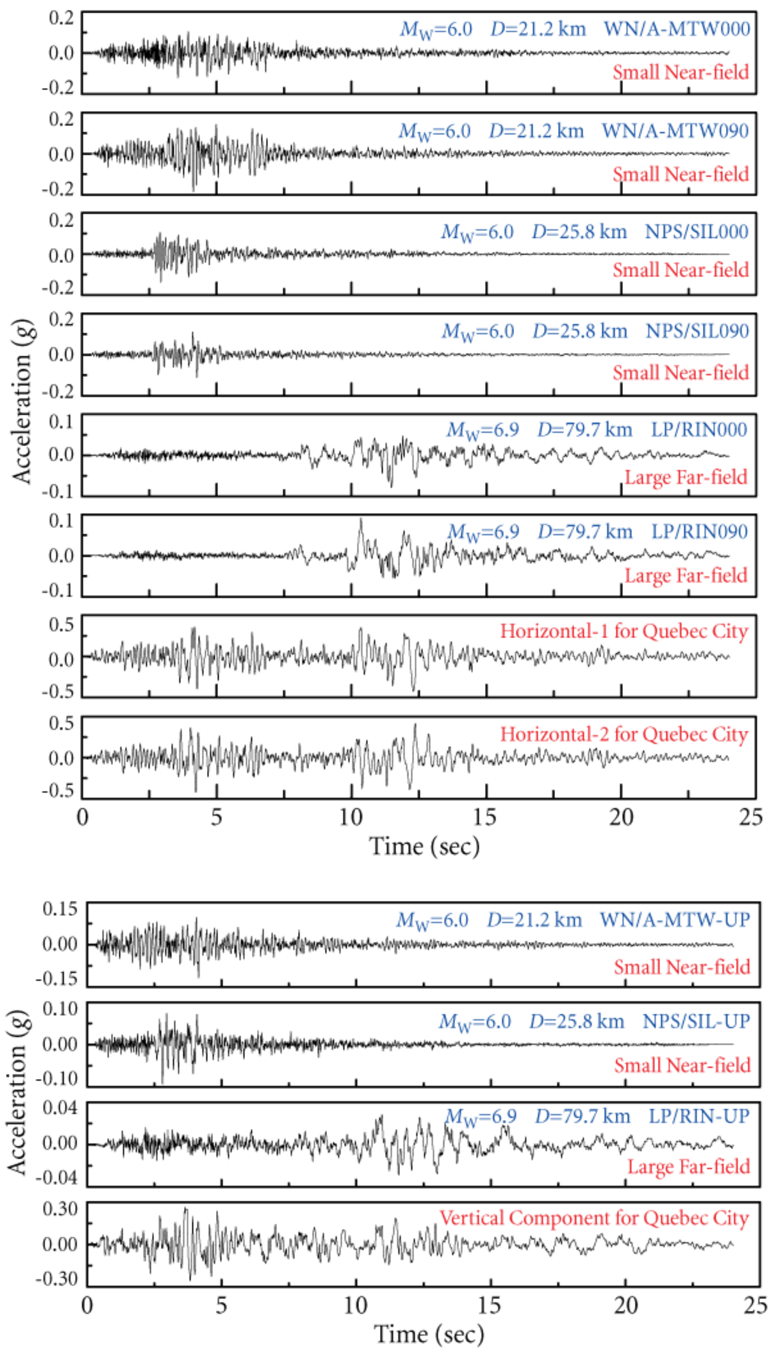

Figure 3 Seed and generated earthquake time-histories.

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