Statistical Modelling of Pressure Tube Diametral Creep

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Abstract

This paper presents the development of a weighted least squares (WLS) regression model for predicting pressure tube diametral creep over time, using in-service inspection data from 328 different pressure tubes from Bruce B and Darlington stations. The proposed model is linear in time as a function of flux and temperature, with a temperature dependent variance function. The model predicts the shape of the observed diametral creep profiles very well, and is useful not only for prediction, but also for assessing tube-to-tube variability and manufacturing properties among the inspected tubes.

1. Introduction

The cold-worked Zr-2.5 wt% Nb pressure tubes constitute the pressure boundary within the core of CANDU® reactors. They are approximately 6 metres (m) long, 104 mm in diameter, and have a wall thickness of approximately 4.2 mm. Multi-unit stations, such as Bruce B and Darlington, have a total of 480 pressure tubes in each reactor unit, with each pressure tube containing 13 individual fuel bundles. During reactor operation, the pressure tubes are subjected to severe environmental and operational stresses, including high temperatures, pressures, flow rates, and neutron irradiation. These stresses induce changes not only in tube dimensions, but also in material properties through irradiation damage and microstructural changes.

Diametral expansion, or creep, refers to the increase in pressure tube circumference over time. It occurs mostly as a result of irradiation creep with a small thermal component [1]. The key structural concern is stress rupture and creep ductility, while the main safety concern is related to the impact of flow bypass on fuel cooling. The increasing pressure tube diameter may result in flow bypass around the fuel bundles lowering the efficiency of heat transfer, thereby increasing the possibility of fuel dryout and thus limiting the operating power of the reactor. It is evident that understanding and predicting the diametral creep in pressure tubes is vital for the lifetime management of CANDU reactors.

The objective of this study is to develop a simplified statistical model for diametral creep using in-service inspection data from Bruce B and Darlington stations. The proposed weighted least squares (WLS) regression model is linear in time and accurately captures the effects of operating conditions on the diametral creep process. In addition to lifetime prediction, the fitted model can be used to explore issues such as intrinsic tube-to-tube material variability and manufacturing properties, and their impact on the results.

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2. Data Analysis

The data for this study consist of pressure tube (PT) surveillance data from Bruce B and Darlington reactors. Gauging records from a total of 328 in-service pressure tubes were available from the Bruce B and Darlington units. Some tubes have been inspected multiple times, with the inspection times ranging from 36,000 effective full power hours (EFPH) to 188,000 EFPH in some cases.

The data for each pressure tube consist of operating conditions, such as pressure, temperature and flux, estimated at mid-bundle locations, and also the corresponding diametral creep measurements from the CIGAR (Channel Inspection and Gauging Apparatus for Reactors) inspection system [2]. Because of uncertainties at the end of each fuel channel, the analysis and modelling in this study will consider data between bundles 2 and 12 only. The channel ends have much lower rates of diametral creep, and therefore are not as critical to overall pressure tube life.

Figure 1 shows typical CIGAR measured diameter profiles for a selected pressure tube. The plot includes the measured maximum, minimum, and mean pressure tube diameters from two different outages.

As shown in Figure 1 the pressure tube diameter increases along the axial distance from the inlet end toward the outlet, as well as a function of operating time (i.e., between the two outages). The regular "dips" in the data, approximately 500 mm apart, indicate the end of each fuel bundle, where the slightly lower irradiation flux results in lower rate of diametral expansion. The minimum and maximum diameter data also reveal the location of the fuel channel spacers as indicated by the four small "bumps" in the profiles. Scrape samples (used for deuterium ingress monitoring) are also visible in the data, as evidenced by the rectangular peaks on the maximum diameters.

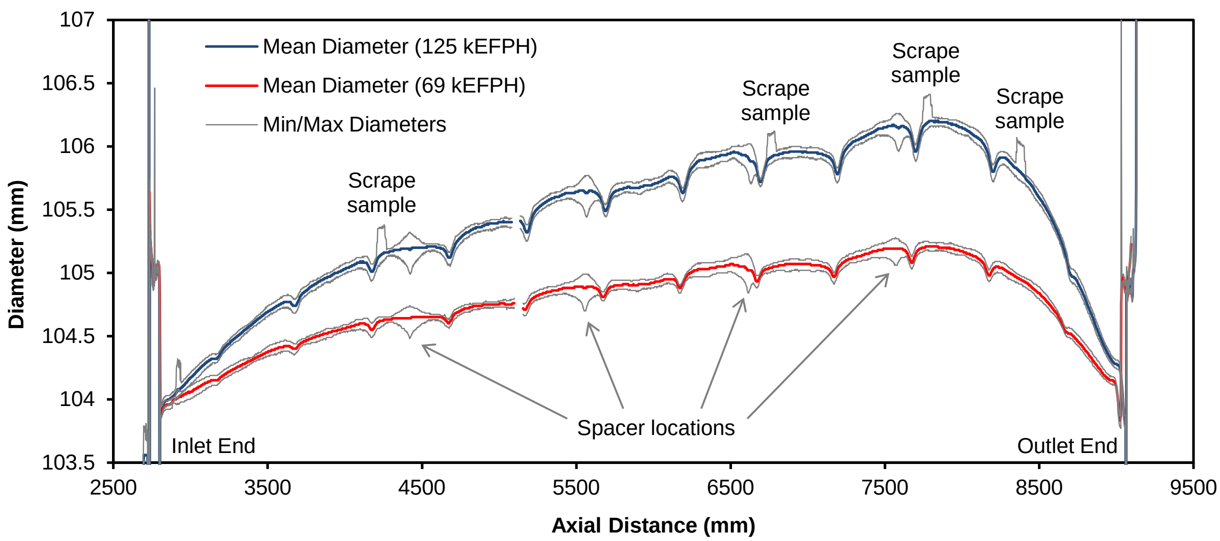


Figure 1 Typical CIGAR gauging diameter data for a selected pressure tube inspected 7 calendar years apart.

3. Model Development

Pressure tube diametral creep occurs mainly as a result of irradiation, and following a brief initial transient has been shown to increase linearly over time [1]. This means that the lifetime (i.e., the time to reach a critical value of diametral creep) of inspected pressure tubes can be predicted with very high confidence. However, because of time and cost limitations, only a small number of tubes can reasonably be inspected in each reactor during the operating life, therefore, there is a need to develop models for predicting the lifetime of the remaining population of uninspected tubes.

In this study, the pressure tube diametral creep is modelled as a function of operating conditions, which are derived from thermalhydraulic codes for all the channels in the reactor core. Figure 2 illustrates the temperature, pressure, and fast neutron flux profiles for a typical pressure tube.

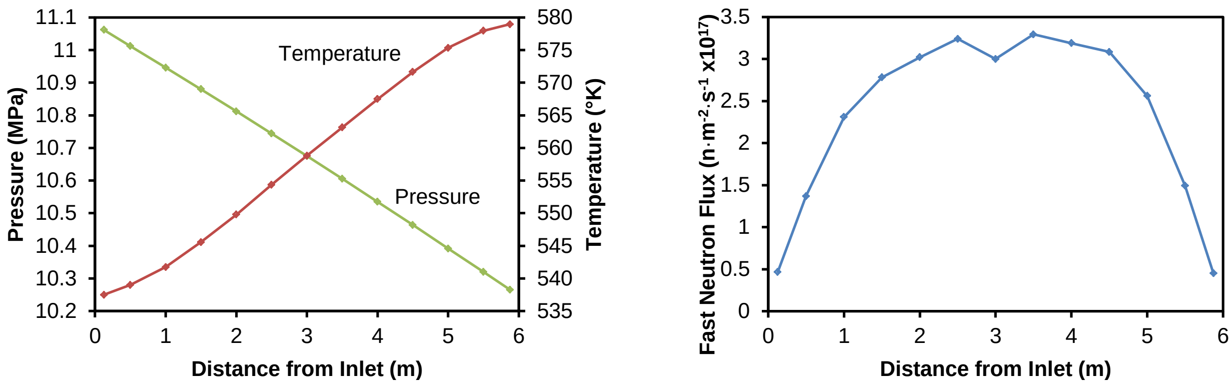


Figure 2 Typical pressure tube temperature, pressure, and fast (> 1 MeV) neutron flux profiles.

As shown in Figure 2, pressure and temperature are nearly complementary (i.e., reciprocal) variables, therefore, the proposed statistical model is formulated in terms of flux and temperature only[1]. While a simple multiplicative relationship of flux, temperature and time yields good results (regression $R^2 \sim 0.9$), a more generic linear model for diametral creep is

$$\varepsilon_{d_i}^{creep} = \beta_0 + \beta_1 \phi_i^a t_i + \beta_2 (T_i - T_0)^b t_i + \beta_3 \phi_i^a (T_i - T_0)^b t_i + \delta_i \quad (1)$$

where ϕ is the fast neutron flux ($\times 10^{17}$ $n \cdot m^{-2} \cdot s^{-1}$), T is the temperature (K), T_0 is a constant reference temperature (K), t is the operating time (kEFPH), β_k are the regression coefficients, δ_i is the error term associated with the i^{th} observation, and a and b are constants to be determined.

As shown above, Equation 1 is a linear function in time that includes the additive and multiplicative effects of flux and temperature. The initial transient is also included in the model as the (fitted) constant intercept term β_0. Because the channel temperatures range from about 528 K to 583 K among the Bruce B and Darlington reactors, the reference temperature T_0 is included in the model as a scaling factor to better account and model the influence of temperature on the results.

[1]The temperature dependence will also subsume some of the strain dependence on microstructure since these tubes are oriented with their back ends at the outlet and the microstructure is known to vary from the front-end to the back-end.

3.1 Model Fitting

It has been established [3] that manufacturing procedures do affect the microstructure and therefore the pressure tube performance with respect to deformation. Due to differing manufacturing processes, three sub-populations of pressure tubes exist within the Bruce B and Darlington stations. These are denoted as series G, H and HM and are each fitted separately to the model. Again as a reminder, only data between bundles 2 and 12 will be considered in the analysis. The reference temperature, T_0, is assumed to be 510 K and 520 K for the Bruce B and Darlington stations, respectively.

3.1.1 Ordinary Least Squares (OLS) Model

The proposed linear model for pressure tube diametral creep in Equation 1 is fitted using ordinary least squares (OLS). Assuming the constant exponent terms *a* and *b* are both equal to 1, the goodness of fit for the regression is quite high for all the tube types. As an example, Figure 3 shows a plot of the fitted vs. observed values of diametral strain for all H-series tubes (a total of 90 different pressure tubes with a total of 1342 data points), with an overall coefficient of determination, R^2, equal to 0.9458.

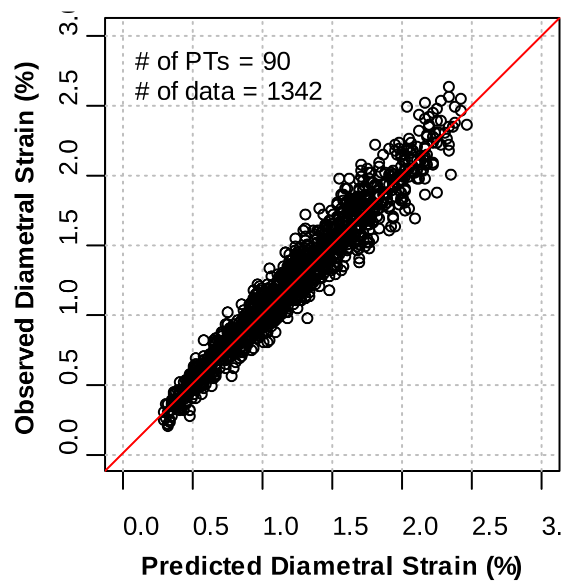


Figure 3 Fitted vs. observed diametral creep for H-series tubes using the OLS model.

In classical OLS regression, the model error or residual, is considered as an independent and identically distributed (iid) random variable following the Normal distribution with zero mean and a constant variance. Figure 4 shows the residual plot and the Normal probability plot for the OLS model of the H-series tube data. As shown in Figure 4, the model residuals exhibit heteroscedasticity, indicating that the error variance is not constant. That is, the model error (or residual) increases with the fitted value (i.e., diametral creep), as shown in Figure 4a by the “fanning out” pattern, thereby violating the assumptions behind the OLS approach. The same pattern is also evident for the HM- and G-series tubes when using the OLS model (results not shown here).

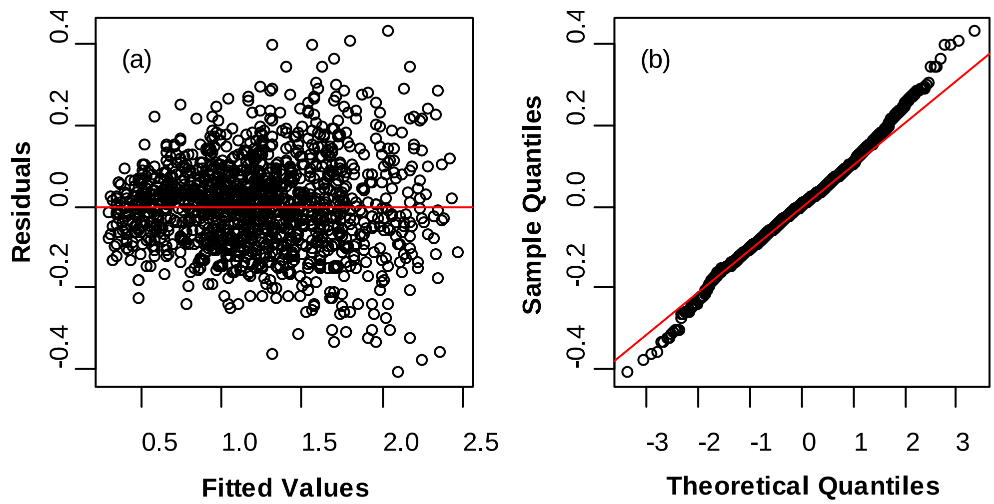


Figure 4 The (a) residual plot and (b) Normal probability plot of the H-series tube OLS residuals.

3.1.2 Weighted Least Squares (WLS)

In weighted least squares (WLS), the error variance is assumed to be non-constant and modelled explicitly by a specified function. The classic fanning out pattern observed in Figure 4a suggests a variance weighting scheme proportional to the square of the predictors. Therefore, because diametral creep tends to increase with temperature, the variance function is chosen as

$$\sigma_i^2 = \sigma_0^2 (T_i - T_0)^2 \tag{2}$$

where σ_0 is the standard error of regression. Figure 5 shows the weighted residuals and the Normal probability plot for the H-series tube data using the WLS model. As shown in Figure 5, the WLS fitted residuals look more evenly scattered and fit the Normal distribution well.

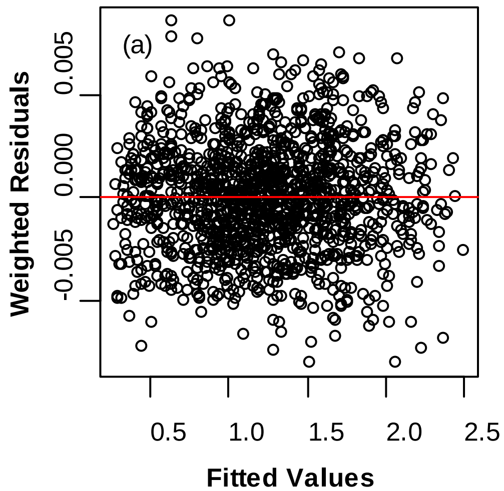
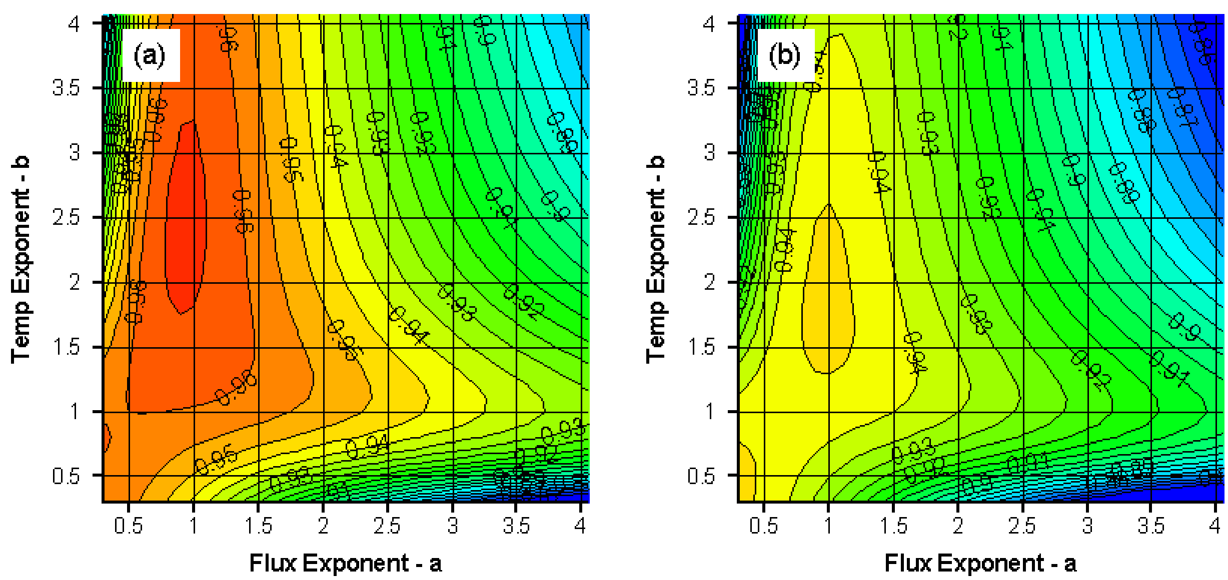


Figure 5 The (a) weighted residuals and (b) Normal probability plot of the weighted H-series tube WLS residuals.

3.1.3 Optimal Exponents *a* and *b*

The constant exponents *a* and *b* were introduced in Equation 1 to make the proposed model generic. Figure 6 illustrates the impact of these exponents on the model results. The plots in Figure 6 show the coefficient of determination, R^2, as a function of the two exponents for the WLS models of the three major tube sub-populations.

As shown in Figure 6, the optimal model fit (in terms of R^2) is obtained by increasing the value of the temperature exponent *b* to around 2, while keeping the value of the flux exponent *a* equal to 1.



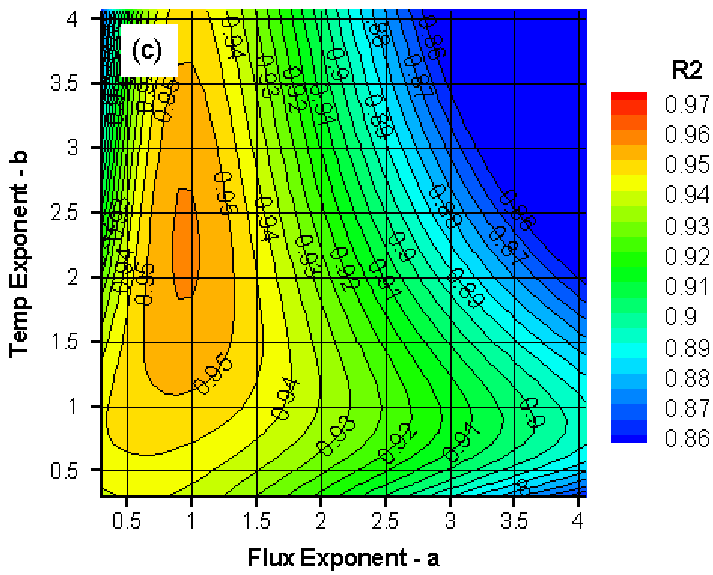


Figure 6 WLS fitted R^2 as a function of constants *a* and *b* for the (a) H-series tubes, (b) HM-series tubes, and (c) G-series tubes.

3.2 Proposed Model

The final proposed model for pressure tube diametral creep is linear in time with the following relationship

$$\varepsilon_{d_i}^{creep} = \beta_0 + \beta_1\phi_i t_i + \beta_2(T_i - T_0)^2 t_i + \beta_3\phi_i(T_i - T_0)^2 t_i + \delta_i \tag{3}$$

The model is fitted using weighted least squares (WLS) with the following variance function

$$\sigma_i^2 = \sigma_0^2(T_i - T_0)^2 \tag{4}$$

The estimated model parameters for the three manufacturing series are shown in Table 1. As seen in Table 1, the goodness of fit is very good in all cases as indicated by the high values of R^2. The estimated regression coefficients are also very similar, indicating that the process of diametral creep is similar between the different reactors and tube types. The estimates for the intercept term, β_0, are also very consistent with the magnitude of the initial transient (i.e., around 0.1 % strain).

Table 1 Parameter estimates for the three pressure tube series using the proposed WLS model.

Tube Series	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\sigma}_0$	R^2
G	0.1121	2.66E-03	9.70E-07	4.85E-07	3.15E-03	0.9554
H	0.0755	2.52E-03	1.99E-06	4.32E-07	2.66E-03	0.9654
HM	0.1219	3.04E-03	2.09E-06	5.20E-07	4.11E-03	0.9460

4. Model Results

The first objective is to compare the model predictions of the inspected pressure tubes for further verification of the modelling approach. Figure 7 shows the observed and predicted diametral creep profiles for selected H-series tubes. As shown in Figure 7, the creep profiles for the first two channels are predicted extremely well, while the other two selected channels are under- and over-predicted by the model, respectively. Despite these differences, however, the predicted profiles are highly consistent with the observations along the axial length, indicating that the model is predicting the general creep behavior very well. Rather than operating conditions (i.e., flux and temperature), the nearly constant shift in the profiles must be due to other factors, such as the intrinsic metallurgical or manufacturing properties of the individual tubes.

The developed model can also be used to make predictions for the uninspected pressure tube populations. Consider, for example, a pressure tube with a fast neutron flux of 3.0 x 10^{17} $n \cdot m^{-2} \cdot s^{-1}$ and temperature equal to 572 K at bundle 10, which is typically the location of maximum diametral creep. Assuming an H-series pressure tube and a reference temperature of 520 K, the mean diametral strain at, for example, 210 kEFPH is estimated using Equation 3 and the parameter estimates for H-series tubes from Table 1 as

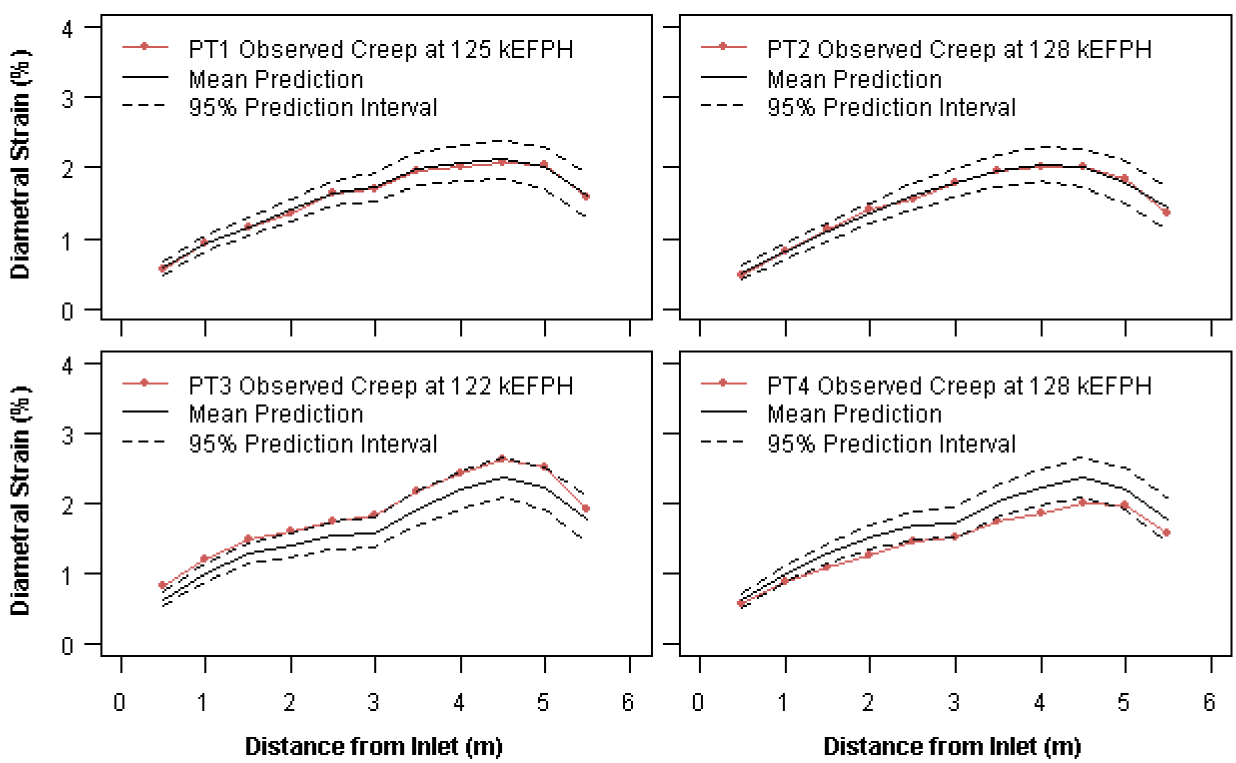


Figure 7 Observed and predicted diametral creep profiles for selected H-series tubes.

$$\begin{aligned}\varepsilon_d^{creep} &= \beta_0 + \beta_1\phi t + \beta_2 (T-T_0)^2 t + \beta_3 \phi (T-T_0)^2 t \\ &= 0.0755 + (2.52\text{E-}03)(3.0)(210) + (1.99\text{E-}06)(572-520)^2(210) \\ &\quad + (4.32\text{E-}07)(3.0)(572-520)^2(210) \\ &= 3.53\,\%\end{aligned} \tag{5}$$

The uncertainty or standard deviation associated with the mean prediction is given by the variance function in Equation 4 as

$$\begin{aligned}\sigma &= \sqrt{\sigma_0^2 (T-T_0)^2} \\ &= \sqrt{(2.66\text{E-}03)^2(572-520)^2} \\ &= 0.14\,\%\end{aligned} \tag{6}$$

Because of the large number of data used in the model fitting, the prediction interval can be obtained from the Normal distribution (rather than the t-distribution) with mean equal to 3.53 % and standard deviation equal to 0.14 %. For the 95 % prediction interval, the prediction limits are obtained using the Normal percentile function, for example by using the NORMINV() function in Excel as

$$\begin{aligned}\varepsilon_{0.025}^{creep} &= NORMINV(0.025, 3.53, 0.14) = 3.26\,\% \\ \varepsilon_{0.975}^{creep} &= NORMINV(0.975, 3.53, 0.14) = 3.80\,\%\end{aligned} \tag{7}$$

Therefore, there is a 95 % probability that the diametral creep at bundle 10 of this uninspected H-series pressure tube will be between 3.26 % and 3.80 % at 210 kEFPH. Figure 8 shows the predicted creep profiles (i.e., mean and 95 % prediction interval) for this channel both axially at 210 kEFPH, as well as at bundle 10 over time.

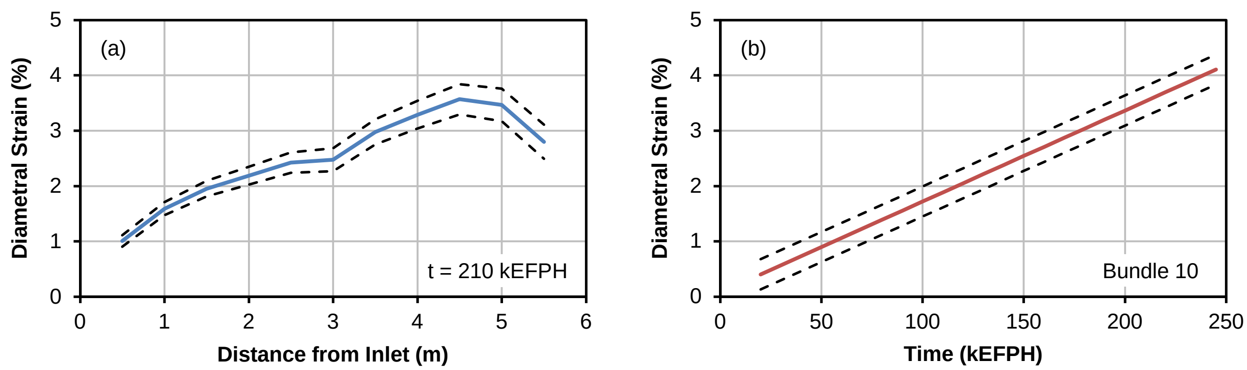


Figure 8 Predicted mean diametral creep and 95 % prediction interval (dashed lines) for the H-series pressure tube vs. (a) axial location at 210 kEFPH, and (b) time at bundle 10.

4.1 Analysis of Residuals

In addition to future prediction, the fitted WLS regression model can also be used to assess tube-to-tube variability within the inspected pressure tube sub-populations. The objective is to identify factors other than operating conditions, such as microstructure, manufacturing, etc., that may have an impact on the diametral creep process.

Figure 9 shows the tube specific diametral strain relative residuals versus axial distance for selected tube types and reactor units. That is, each line in Figure 9 represents the relative residuals (observed/predicted) for a particular tube. A positive residual indicates that the observed value is relatively higher than the population mean.

As shown in Figure 9, the population variability is generally within approximately +/- 20 % or less of the mean in all cases. The HM-series tubes tend to have slightly higher overall variability, while the observed creep rates of G-series tubes in Unit W appear to be lower than expected. It is clearly evident from Figure 9, however, that the tube specific residuals are highly consistent and nearly parallel with respect to axial distance. This indicates that the WLS regression model is predicting the shape of the diametral creep profiles very well (this was also illustrated earlier in Figure 7).

The model does appear to under-predict the mid-channel diametral creep slightly for channels with complex flux profiles (i.e., flux dips in the middle of the channel at bundle 7), as shown by the localized increase in the residuals approximately 3 m from the inlet end in some cases. However, the residuals are still highly consistent, indicating that the variability must be due to factors other than the operating conditions. This allows the model to readily be used for detailed comparisons of material factors and manufacturing variability within the sub-populations. Furthermore, using the same form of the model, all measured sub-populations of pressure tubes (in addition to those presented here) could be combined and fit with a global set of parameters to assess the relative deformation across the entire pressure tube population.

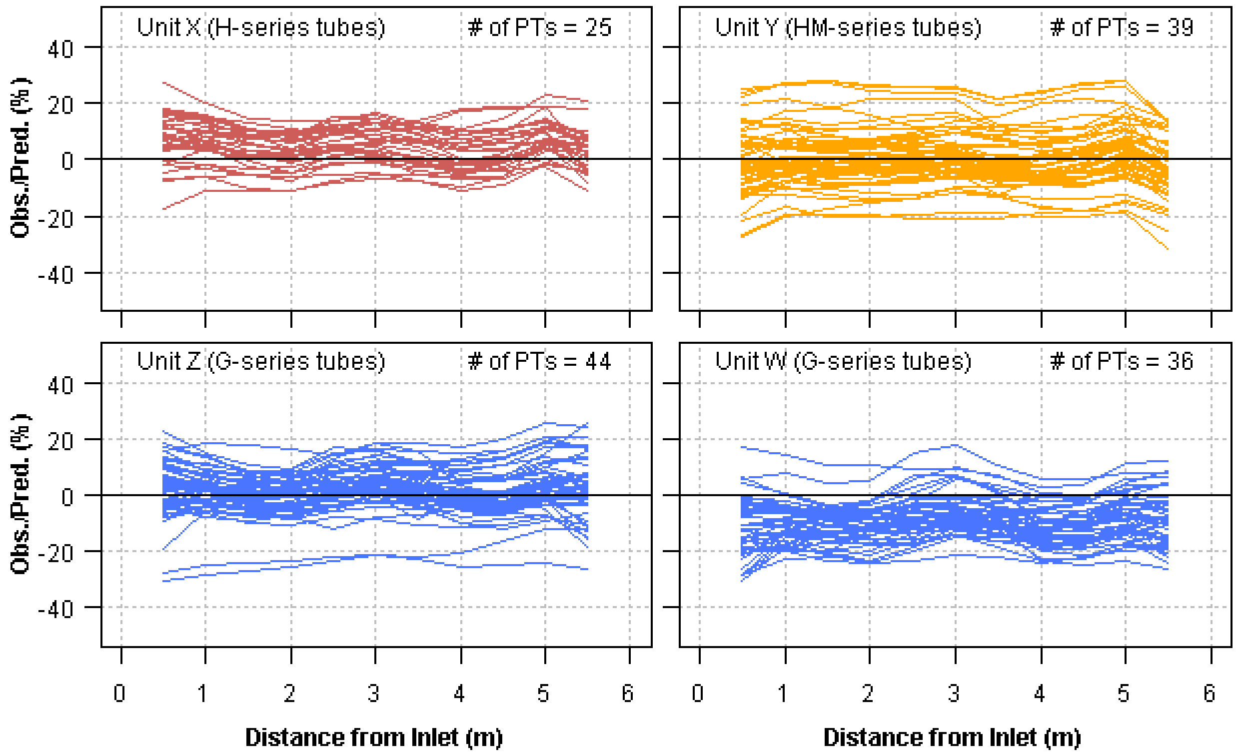


Figure 9 Tube specific residuals for selected tube series and reactor units.

5. Summary and Conclusions

Understanding and predicting the diametral creep in pressure tubes is vital for the lifetime management of CANDU reactors. In this study, in-service inspection data from 328 different pressure tubes at Bruce B and Darlington stations was used to develop a weighted least squares (WLS) regression model for predicting diametral creep in pressure tubes as a function of operating conditions. The proposed model is linear in time and accounts for the additive and multiplicative effects of both flux and temperature, with a temperature dependent variance function.

The results of the study showed that the proposed model can predict the shape of the diametral creep profiles very well. The model can therefore readily be used not only for predicting the creep deformation in uninspected pressure tubes, but also for investigating the impact of material variability and manufacturing properties in the inspected pressure tube populations.

6. Acknowledgements

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7. References

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