Monte Carlo Fuel Temperature Reactivity Coefficient Calculations by Adjoint-Weighted Correlated Sampling Method

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Abstract

It is very time-consuming to estimate the fuel temperature reactivity coefficient (FTC) with high accuracy based on the subtraction of two reactivity values calculated at different fuel temperatures by a Monte Carlo (MC) neutron transport analysis code. We present a new MC FTC estimation method based on the perturbation techniques. The effectiveness of the new method is examined through the continuous energy MC neutronics calculations for the CANDU lattice problem. It is shown that the proposed method can predict the reactivity change due to the fuel temperature variation efficiently. From its numerical results, the reaction-wise contributions to the FTC are investigated with different resonance scattering models – the free-monatomic-gas models with constant cross section and resonance cross section.

1. Introduction

With increasing computer power, the Monte Carlo (MC) particle transport methods have been successfully applied for designs and analyses of nuclear systems by using continuous-energy cross section libraries and detailed geometry data. However, it is still challenging to estimate the change of the reactivity or the multiplication factor, *k*, due to a small perturbation of a nuclear design parameter by using the MC direct subtraction method.

Since the two conventional MC perturbation techniques such as the correlated sampling and the differential operator sampling (DOS) methods were applied to estimate the temperature coefficient of the coolant in a D_2O test reactor [1], there have been significant advances in the MC sensitivity calculations. Nagaya and Mori [2] strengthened the two conventional methods by taking into account the fission source perturbation (FSP). Recently, the MC perturbation techniques based on the adjoint flux estimated in the MC forward calculations have been developed and successfully applied for the perturbation problems [3] and the nuclear data sensitivity and uncertainty (S/U) analyses [4]. Also, it is notable that the first-order DOS method with FSP is equivalent to the adjoint weighted perturbation (AWP) method [4].

By using the MC perturbation techniques, the density reactivity coefficient of coolant or moderator can be readily estimated with great efficiency [1]. And there was an approach to estimate the collision density perturbation due to the resonance parameters variation from a temperature change for the MC transport analysis [5]. However, this work was not extended to the MC Doppler coefficient calculations.

In this paper, we present a MC fuel temperature reactivity coefficient (FTC) calculation method based on the adjoint-weighted correlated sampling method. In this method, the fuel temperature variation is regarded as changes of the cross section library sets of fuel regions and the fuel temperature applied for the free-monatomic-gas scattering models [6-9] in the MC neutron simulations. The FTCs for the CANDU lattice problem estimated by the proposed method with two different scattering models – the free-monatomic-gas models with constant cross section and resonance cross section – are compared with those calculated by the MC direct subtractions and WIMS-IST [10].

2. Adjoint-Weighted Correlated Sampling Method

The MC AWP method based on the correlated sampling technique is described in this section to estimate the k change due to a perturbation of a parameter x, Δx.

The MC power method solves the eigenvalue equation generation-by-generation given by

$$S = \frac{1}{k}\mathbf{H}S\,. \tag{1}$$

The fission source density (FSD), S, satisfies $\int S(\mathbf{P})d\mathbf{P} = 1$ where $\mathbf{P}$ denotes the state vector of a neutron in the six-dimensional phase space, ($\mathbf{r}$, E, $\mathbf{\Omega}$). $\mathbf{H}S$ in Eq. (1) implies

$$\mathbf{H}S = \int d\mathbf{P}' H(\mathbf{P}' \to \mathbf{P}) S(\mathbf{P}')\,, \tag{2}$$

where $H(\mathbf{P}' \to \mathbf{P})$ means the number of first-generation fission neutrons born per unit phase space volume about $\mathbf{P}$, due to a parent neutron born at $\mathbf{P}'$.

Using the transport kernels [11], $\mathbf{H}S$ of Eq. (2) can be explicitly expressed as [4]

$$\mathbf{H}S = \sum_{j=0}^{\infty}\int dE''\int d\mathbf{\Omega}'' C_f(\mathbf{r}; E'', \mathbf{\Omega}'' \to E, \mathbf{\Omega})\int d\mathbf{P}_0 K_{s,j}(\mathbf{P}_0 \to \mathbf{r}, E'', \mathbf{\Omega}'')\int d\mathbf{r}' T(E', \mathbf{\Omega}'; \mathbf{r}' \to \mathbf{r}_0) S(\mathbf{P}'). \tag{3}$$

$K_{s,j}$ denotes the j-th scattering kernel defined by

$$\begin{aligned} K_{s,0}(\mathbf{P}_0 \to \mathbf{P}) &= \delta(\mathbf{P}_0 - \mathbf{P}),\\ K_{s,1}(\mathbf{P}_0 \to \mathbf{P}) &= K_s(\mathbf{P}_0 \to \mathbf{P}),\\ K_{s,j}(\mathbf{P}_0 \to \mathbf{P}) &= \int d\mathbf{P}_{j-1}\cdots\int d\mathbf{P}_1 K_s(\mathbf{P}_{j-1} \to \mathbf{P})\cdots K_s(\mathbf{P}_0 \to \mathbf{P}_1);\ j = 2, 3, \cdots, \end{aligned} \tag{4}$$

and $E_0 \equiv E'$, $\mathbf{\Omega}_0 \equiv \mathbf{\Omega}'$. K_s is the scattering transport kernel defined by the product of the scattering collision kernel, C_s, and the transition kernel, T:

$$K_s(\mathbf{P}' \to \mathbf{P}) = T(E,\mathbf{\Omega};\mathbf{r}' \to \mathbf{r}) \cdot C_s(\mathbf{r}';E',\mathbf{\Omega}' \to E,\mathbf{\Omega}); \tag{5}$$

$$T(E,\mathbf{\Omega};\mathbf{r}' \to \mathbf{r}) = \frac{\Sigma_t(\mathbf{r},E)}{|\mathbf{r}-\mathbf{r}'|^2} \exp\left[-\int_0^{|\mathbf{r}-\mathbf{r}'|} \Sigma_t\left(\mathbf{r} - s\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}, E\right) ds\right] \delta\left(\mathbf{\Omega} \cdot \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} - 1\right), \tag{6}$$

$$C_s(\mathbf{r}';E',\mathbf{\Omega}' \to E,\mathbf{\Omega}) = \sum_i \sum_{r \neq fis.} \nu_r^i \frac{\Sigma_r^i(\mathbf{r}';E',\mathbf{\Omega}')}{\Sigma_t(\mathbf{r}',E')} f_r^i(E',\mathbf{\Omega}' \to E,\mathbf{\Omega}). \tag{7}$$

ν_r^i and Σ_r^i are the number of neutrons produced from, and the macroscopic cross section of, respectively, a reaction *r* of isotope *i*. $f_r^i(E',\mathbf{\Omega}' \to E,\mathbf{\Omega})dEd\mathbf{\Omega}$ is the probability that a collision of type *r* of isotope *i* by a neutron of direction $\mathbf{\Omega}'$ and energy E' will produce a neutron in direction interval $d\mathbf{\Omega}$ about $\mathbf{\Omega}$ with energy in dE about E. The rest of the notation follows standard. C_f is the fission collision kernel and can be written as

$$C_f(\mathbf{r};E',\mathbf{\Omega}' \to E,\mathbf{\Omega}) = \frac{\chi(E' \to E)}{4\pi} \cdot \frac{\nu(E')\Sigma_f(\mathbf{r},E')}{\Sigma_t(\mathbf{r},E')}. \tag{8}$$

From the perturbation theory, the first-order change in the fundamental mode eigenvalue *k* due to a change of the parameter *x* can be expressed as [4]

$$\Delta k \cong <\phi_0^\dagger, \Delta\mathbf{H}S_0>; \tag{9}$$

$$\Delta\mathbf{H} = \frac{\partial\mathbf{H}}{\partial x}\Delta x. \tag{10}$$

$\phi_0^\dagger$ and S_0 denote the fundamental mode adjoint flux and FSD, respectively. From the physical meaning of $\phi_0^\dagger$ – the iterated fission probability [12], Eq. (9) can be written by [4]

$$\Delta k \cong \frac{1}{k_0{}^n} < \mathbf{H}^n, \Delta\mathbf{H}S_0 >, \tag{11}$$

where k_0 is the fundamental mode eigenvalue. *n* denotes the convergence interval of the adjoint flux [4].

In the correlated sampling method, $\Delta\mathbf{H}S_0$ in Eq. (11) is calculated by the difference of $\mathbf{H}S_0$ in the perturbed and unperturbed systems. From Eq. (3), $\mathbf{H}S_0$ for the unperturbed and perturbed systems can be written by

$$\mathbf{H}S_0 = \sum_{j=0}^{\infty}\int dE''\int d\mathbf{\Omega}'' C_f(\mathbf{r};E'',\mathbf{\Omega}'' \to E,\mathbf{\Omega})\int d\mathbf{P}_0 K_{s,j}(\mathbf{P}_0 \to \mathbf{r},E'',\mathbf{\Omega}'')\int d\mathbf{r}' T(E',\mathbf{\Omega}';\mathbf{r}' \to \mathbf{r}_0)S_0(\mathbf{P}'), \tag{12}$$

$$\mathbf{H}^* S_0 = \sum_{j=0}^{\infty}\int dE''\int d\mathbf{\Omega}'' C_f^*(\mathbf{r};E'',\mathbf{\Omega}'' \to E,\mathbf{\Omega})\int d\mathbf{P}_0 K_{s,j}^*(\mathbf{P}_0 \to \mathbf{r},E'',\mathbf{\Omega}'')\int d\mathbf{r}' T^*(E',\mathbf{\Omega}';\mathbf{r}' \to \mathbf{r}_0)S_0(\mathbf{P}'). \tag{13}$$

The operators with superscript asterisk in Eq. (13) denote those in the perturbed system where the parameter x is changed by Δx.

From Eqs. (12) and (13), $\Delta \mathbf{H}S_0$ can be expressed as

$$\begin{aligned}\Delta\mathbf{H}S_0 &\cong \mathbf{H}^* S_0 - \mathbf{H}S_0 \\ &= \sum_{j=0}^{\infty}\int dE''\int d\mathbf{\Omega}''\int d\mathbf{P}_0\int d\mathbf{r}'\, u^j(\mathbf{P}' \to \mathbf{P}) \\ &\quad \otimes C_f(\mathbf{r};E'',\mathbf{\Omega}'' \to E,\mathbf{\Omega})K_{s,j}(\mathbf{P}_0 \to \mathbf{r},E'',\mathbf{\Omega}'')T\ (E_0,\mathbf{\Omega}_0;\mathbf{r}' \to \mathbf{r}_0)S_0(\mathbf{r}',E_0,\mathbf{\Omega}_0);\end{aligned} \tag{14}$$

$$u^j(\mathbf{P}' \to \mathbf{P}) = \frac{C_f^*(\mathbf{r};E'',\mathbf{\Omega}'' \to E,\mathbf{\Omega})}{C_f(\mathbf{r};E'',\mathbf{\Omega}'' \to E,\mathbf{\Omega})}\cdot\prod_{p=0}^{j-1}\frac{K_s^*(\mathbf{P}_p \to \mathbf{P}_{p+1})}{K_s(\mathbf{P}_p \to \mathbf{P}_{p+1})}\cdot\frac{T^*(E_0,\mathbf{\Omega}_0;\mathbf{r}' \to \mathbf{r}_0)}{T(E_0,\mathbf{\Omega}_0;\mathbf{r}' \to \mathbf{r}_0)} - 1 \tag{15}$$

As shown in Eq. (14), $\Delta\mathbf{H}S_0$ is calculated by cumulating u^j at all the neutron tracks in the MC correlation sampling method. From Eq. (11), Δk is measured by weighting $\Delta\mathbf{H}S_0$ estimated at a cycle with the number of fission neutrons born many cycles, say n, after the cycle.

3. MC FTC Estimation by Perturbation Method

3.1 Sensitivity of *k* to the Fuel Temperature Change

The fuel temperature change can be regarded as the variation the microscopic cross sections of all isotopes comprising the fuel over the thermal and resonance energy regions in response to the thermal motion changes of the nuclei. Therefore in the case to estimate the FTC by using the MC perturbation techniques, $\Delta\mathbf{H}$ in Eq. (11) can be expressed as

$$\Delta\mathbf{H} = \sum_{\substack{m\in fuel,\\ i,r\notin s,E}}\frac{\partial\mathbf{H}}{\partial x_r^{m,i}(E)}\Delta x_r^{m,i}(E) + \sum_{\substack{m\in fuel,\\ i,r\in s,E,\mathbf{\Omega}}}\frac{\partial\mathbf{H}}{\partial x_r^{m,i}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}')}\Delta x_r^{m,i}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}'). \tag{16}$$

m, i, and r are the region, isotope, and reaction type indices, respectively. s denotes the scattering reactions such as the elastic and inelastic scatterings, etc. $x_r^{m,i}(E)$ denotes the microscopic cross section of reaction r of isotope i in region m at neutron energy E while $x_r^{m,i}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}')$ the double differential microscopic cross section where $x_r^{m,i}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}')dE'd\mathbf{\Omega}'$ means the cross section that the reaction by a neutron of direction $\mathbf{\Omega}$ and energy E will produce a neutron in direction interval $d\mathbf{\Omega}'$ about $\mathbf{\Omega}'$ with energy in dE' about E'.

The first term of the right hand side (RHS) of Eq. (16), $\left(\partial\mathbf{H}/\partial x_r^{m,i}(E)\right)\Delta x_r^{m,i}(E)$, can be readily estimated by the correlated sampling method from the microscopic cross sections corresponding to the reference and the changed temperatures for a given neutron energy E. However the exactly Doppler-broadened double differential scattering cross sections, $x_r^{m,i}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}')$, are not provided by the nuclear data processing code, NJOY [13] because it is based on the asymptotic slowing-down scattering model in which the neutron energy and flight direction after a collision are determined using the two-body reaction kinematics with the assumption of no thermal motion of a target nucleus. To overcome this problem, the general purpose MC neutron transport analysis codes such as MCNP [6], MVP [9], McCARD [14], etc. are equipped with the free-monatomic-gas models to simulate the elastic scattering reactions by taking into account the velocity vector distribution of the target nucleus according to the material temperature.

The microscopic scattering cross section at neutron energy E and temperature T, x_s^{eff}, at temperature T can be written by

$$x_s^{eff}(\upsilon_n,T)=\frac{1}{\upsilon_n}\int_{V:\upsilon_r>0}\int_{-1}^{1}\upsilon_r x_s(\upsilon_r)M^T(V)dV\frac{d\mu_t}{2}. \tag{17}$$

υ_n and υ_r denote the neutron speed and the relative speed of the neutron in the target-at-rest frame, respectively. $x_s(\upsilon_r)$ is the scattering cross section at 0K. μ_t is the cosine of the angle between the neutron and target velocity vector. $M^T(V)$ is the spectrum of the target velocity, V, given by the Maxwellian Boltzmann distribution, $p(\beta,V)$ [6]:

$$p(\beta,V)=\frac{4}{\sqrt{\pi}}\beta^3V^2e^{-\beta^2V^2};\ \beta=\sqrt{\frac{AM_n}{2kT}}. \tag{18}$$

A and M_n are the atomic mass ratio of the target nucleus to the neutron and the neutron mass, respectively. k is the Boltzmann constant.

Eq. (17) implies that the probability distribution of V and μ_t, $P^T(V,\mu_t)$ is

$$P^T(V,\mu_t)=\frac{x_s(\upsilon_r)\upsilon_r p(\beta,V)}{2x_s^{eff}(\upsilon_n)\upsilon_n}. \tag{19}$$

There are two approaches to sample V and μ_t based on Eq. (20): the free-monatomic-gas model with constant cross section (constant cross section model, hereafter) and the free-monatomic-gas model with resonance cross section (exact model, hereafter).

Because the free-monatomic-gas models are generally applied for the elastic scatterings of thermal neutrons, Eq. (16) can be rewritten by

$$\Delta\mathbf{H} = \sum_{\substack{m\in fuel,\\ i,r\neq els,E}} \frac{\partial \mathbf{H}}{\partial x_r^{m,i}(E)} \Delta x_r^{m,i}(E) + \sum_{\substack{m\in fuel,\\ i,E,\mathbf{\Omega}}} \frac{\partial \mathbf{H}}{\partial x_{els}^{m,i}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}')} \Delta x_{els}^{m,i}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}'), \qquad (20)$$

where *els* denotes the elastic scattering reaction.

Therefore in order to estimate $\left(\partial\mathbf{H}/\partial x_{els}^{m,i}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}')\right)\Delta x_{els}^{m,i}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}')$ in Eq. (20), the free-monatomic-gas models should be considered in the correlated sampling method.

3.2 Correlated Sampling Algorithm for Free-Monatomic-Gas Models

In order to derive the correlated sampling algorithm for the elastic scattering reaction, suppose that the neutron of the k-th track at energy E_k and $\mathbf{\Omega}_k$ is scattered to E_{k+1} and $\mathbf{\Omega}_{k+1}$, by the elastic scattering reaction with isotope i' in region m' in the MC random walk process. Then the elastic scattering collision kernel can be expressed as

$$C_k^s \equiv C_k^{s,1}\cdot C_k^{s,2}; \qquad (21)$$

$$C_k^{s,1} = N_{i'}^{m'} x_t^{m',i'}(E_k)\big/\Sigma_t^{m'}(E_k), \qquad (22)$$

$$C_k^{s,2} = \frac{x_{els}^{m',i'}(E_{r,k})}{x_t^{m',i'}(E_{r,k})} P^T(V,\mu_t)\, f_{els}^{i'}(v_r,V\to E_{k+1},\mathbf{\Omega}_{k+1}). \qquad (23)$$

Then the ratios of the perturbed and unperturbed kernels can be written as

$$\begin{aligned}\frac{C_k^{*s,2}}{C_k^{s,2}} &= \frac{\dfrac{x_{els}^{*m',i'}(E_{r,k})}{x_t^{*m',i'}(E_{r,k})} P^{T+\Delta T}(V,\mu_t)\, \cancel{f_{els}^{i'}(v_r,V\to E_{k+1},\mathbf{\Omega}_{k+1})}}{\dfrac{x_{els}^{m',i'}(E_{r,k})}{x_t^{m',i'}(E_{r,k})} P^{T}(V,\mu_t)\, \cancel{f_{els}^{i'}(v_r,V\to E_{k+1},\mathbf{\Omega}_{k+1})}} \\ &= \frac{x_{els}^{*m',i'}(E_{r,k})\big/x_t^{*m',i'}(E_{r,k})}{x_{els}^{m',i'}(E_{r,k})\big/x_t^{m',i'}(E_{r,k})}\cdot\frac{P^{T+\Delta T}(V,\mu_t)}{P^{T}(V,\mu_t)}.\end{aligned} \qquad (24)$$

In the constant cross section model, the temperature dependent elastic scattering cross section is approximated by

$$\sigma_s^{eff}(v_n) = \sigma_{s0} g(\beta); \tag{25}$$

$$g(\beta) = \frac{1}{\beta^2}\left\{\left(\beta^2 + \frac{1}{2}\right) erf\,\beta + \frac{1}{\sqrt{\pi}}\beta\exp(-\beta^2)\right\}, \tag{26}$$

where erf is the error function.

Then the insertion of Eq. (25) into Eq. (19) gives

$$P_{CXS}^{T}(V,\mu_t) = \frac{\cancel{\sigma_{s0}}\, v_r\, p(\beta,V)}{2\cancel{\sigma_{s0}}\, g(\beta) v_n} = \frac{v_r\, p(\beta,V)}{2g(\beta)v_n}. \tag{27}$$

Using Eq. (23), $C_k^{*s,2}/C_k^{s,2}$ for the constant cross section model can be calculated by

$$\frac{C_k^{*s,2}}{C_k^{s,2}} = \frac{x_{els}^{*m',i'}(E_{r,k})/x_t^{*m',i'}(E_{r,k})}{x_{els}^{m',i'}(E_{r,k})/x_t^{m',i'}(E_{r,k})} \cdot \frac{p(\beta^*,V)/g(\beta^*)}{p(\beta,V)/g(\beta)}. \tag{28}$$

Using Eq. (19), $P^{T+\Delta T}(V,\mu_t)/P^{T}(V,\mu_t)$ in Eq. (24) can be written as

$$\frac{P^{T+\Delta T}(V,\mu_t)}{P^{T}(V,\mu_t)} = \frac{\dfrac{\sigma_s(v_r)v_r\,p(\beta^*,V)}{2x_{els}^{*m',i'}(E)v_n}}{\dfrac{\sigma_s(v_r)v_r\,p(\beta,V)}{2x_{els}^{m',i'}(E)v_n}} = \frac{p(\beta^*,V)/x_{els}^{*m',i'}(E)}{p(\beta,V)/x_{els}^{m',i'}(E)}. \tag{29}$$

Using Eq. (29), $C_k^{*s,2}/C_k^{s,2}$ for the exact model can be calculated by

$$\frac{C_k^{*s,2}}{C_k^{s,2}} = \frac{x_{els}^{*m',i'}(E_{r,k})/x_t^{*m',i'}(E_{r,k})}{x_{els}^{m',i'}(E_{r,k})/x_t^{m',i'}(E_{r,k})} \cdot \frac{p(\beta^*,V)/x_{els}^{*m',i'}(E)}{p(\beta,V)/x_{els}^{m',i'}(E)}. \tag{30}$$

4. Numerical Results

The MC burnup analysis for a typical CANDU lattice of the Wolsung nuclear reactors is conducted by the Seoul National University MC code, McCARD [14]. The MC eigenvalue calculations are performed for 1,000 active and 20 inactive cycles on 10,000 histories per cycle by using the continuous-energy cross section libraries generated from ENDF/B-VII.0.

Table 1 and Figure 1 shows the comparison of k_{inf}'s calculated by McCARD and WIMS-IST. From the table and figure, we can observe that the results from two codes agree very well each other. The RMS and maximum difference of k_{inf}'s are 126 pcm and 193 pcm, respectively.

Table 1. Comparison of k_{inf}'s calculated by McCARD and WIMS-IST for the CANDU lattice

Burnup (MWd/tU)	McCARD (k_{inf} ± SD*)	WIMS-IST	DIFF (k^{WIMS}-k^{McCARD})
0	1.12187 ± 0.00012	1.11996	-0.00191
3.45	1.11448 ± 0.00012	1.11381	-0.00067
17	1.09311 ± 0.00014	1.09429	0.00118
34	1.08379 ± 0.00013	1.08522	0.00143
68	1.07891 ± 0.00013	1.07960	0.00069
172	1.07427 ± 0.00013	1.07486	0.00059
689	1.07782 ± 0.00013	1.07879	0.00097
1723	1.07735 ± 0.00015	1.07859	0.00124
1896	1.07641 ± 0.00014	1.07724	0.00083
2068	1.07470 ± 0.00015	1.07568	0.00098
2413	1.07128 ± 0.00013	1.07198	0.00070
2758	1.06697 ± 0.00015	1.06775	0.00078
3103	1.06233 ± 0.00014	1.06309	0.00076
5171	1.02905 ± 0.00015	1.03068	0.00163
6895	1.00006 ± 0.00014	1.00229	0.00223
8619	0.97325 ± 0.00016	0.97518	0.00193

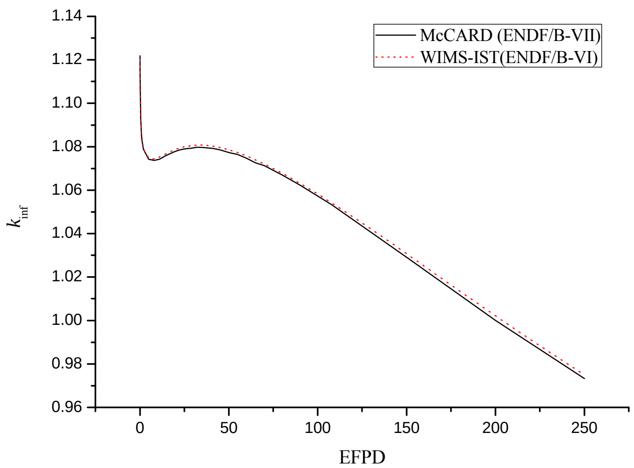

Figure 1 Comparison of k_{inf}'s by McCARD and WIMS-IST for the CANDU lattice

For the BOC state of the CANDU lattice, Δk for the fuel temperature change from 960.16K to 1060.16K is estimated by the proposed MC adjoint-weighted correlated sampling method and the direct subtractions. The MC perturbation calculations are performed for 5,000 active cycles on 200,000 histories per cycle while the MC direct subtraction calculations for 1,000 active cycles on 10,000,000 histories per cycle.

Tables 2 and 3 show the comparison of Δk's calculated by the MC perturbation method with the constant cross section model and the exact model, respectively, with the direct subtractions' from McCARD and WIMS-IST. From the table, we can see that the new method can predict Δk efficiently with high accuracy.

Table 2. Comparison of Δk's calculated by the MC adjoint-weighted correlated sampling method and direct subtractions with the constant cross section model for the CANDU lattice problem at BOC

Reaction Type		McCARD (Perturbation)	McCARD (Direct Sub.)	WIMS-IST (Direct Sub.)
U-235	(n,fis)	-0.00003 (0.46%*)		
U-238	(n,γ)	-0.00111 (0.07%)		
	(n,n)	0.00019 (8.00%)		
	(n,n')	0.00000 (0.04%)		
O-16	(n,n)	-0.00044 (1.50%)		
	(n,n')	0.00000 (0.10%)		
Total		-0.00138 (0.90%)	-0.00141 (0.92%)	-0.00131

* The value in the parentheses means the relative standard deviation.

Table 3. Comparison of Δk's calculated by the MC adjoint-weighted correlated sampling method and direct subtractions with the exact model for the CANDU lattice problem at BOC

Reaction Type		McCARD (Perturbation)	McCARD (Direct Sub.)	WIMS-IST (Direct Sub.)
U-235	(n,fis)	-0.00003 (0.46%*)		
U-238	(n,γ)	-0.00080 (0.06%)		
	(n,n)	-0.00027 (3.70%)		
	(n,n')	0.00000 (0.03%)		
O-16	(n,n)	-0.00042 (2.27%)		
	(n,n')	0.00000 (0.14%)		
Total		-0.00153 (0.91%)	-0.00148 (0.96%)	-0.00131

* The value in the parentheses means the relative standard deviation.

5. Conclusion

We have developed a MC FTC calculation method based on the perturbation theory. It is shown that the new adjoint-weighted correlated sampling method can efficiently predict Δk due to the fuel temperature variation very accurately for the CANDU lattice problem. Application results for the different burnup steps will be presented.

6. References

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