Multi-Group Fusion Reactivites for Maxwellian and Non-Maxwellian Ion Velocity Distributions

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Abstract

Ion velocity distributions in magnetically confined thermonuclear fusion plasmas under reacting conditions can deviate considerably from Maxwellian distributions. However, fusion reactivities used for power balances in fusion reactors are generally available for two interacting Maxwellian species at the same temperature. In the present work we report total fusion reactivities as well as fractional contributions to these from different ion energy groups, for a number of representative non-Maxwellian ion velocity distributions. Both isotropic as well as anisotropic distributions have been considered. The effect of unequal ion kinetic temperatures on the fusion reactivity has also been considered.

1. Introduction

Thermonuclear fusion plasmas are *far-from-equilibrium* systems. By virtue of this, applicability of fusion reactivity calculated under Maxwellian conditions is likely to be of limited value for practical thermonuclear fusion reactors (e.g., ITER). Furthermore, in a reacting system, the ion velocity distribution function cannot be expected to be purely Maxwellian even at equilibrium or near equilibrium (quasi-equilibrium) conditions. Most of the available literature about the fusion systems consider the ion speed (or velocity) distribution in such systems to be Maxwellian and the reason stated in most of the resources is that the ion speed distribution in the high ion density fusion systems rapidly approaches Maxwell-Boltzmann distribution due to a large number Coulomb collisions between the ions within a very short period of time. But local thermal accelerations and some unwanted acceleration of the reacting ions due to local disturbances in the magnetic field are always there and these events have a significant effect on the ion velocity distributions. Distortion in the fuel-ion velocity distribution also occurs during *neutral-beam-injection* plasma heating as well as the interaction of the fuel ions with high energy charged particle reaction products. These enhance the tail part of the Maxwellian spectrum. On the other hand, faster reactions at the high-energy end of the spectrum as well as a higher rate of loss of faster fuel ions in mirror machines can result in a tail-depleted Maxwellian spectrum.

There is indeed a limited availability of the reported fusion reactivity for non-Maxwellian ion velocity distribution functions. Fusion reactivity for non-Maxwellian ion velocity distributions (both isotropic and non-isotropic) is worth investigating in connection with practical fusion reactors. It is also of interest to investigate group-wise contribution to the total fusion reactivity both in the center-of-mass system and laboratory coordinate systems. This gives us an idea about the velocity (or energy) ranges of ions whose interactions result in comparatively higher fractional contribution to the fusion reactivity and how these fractional contributions get affected with the change in kinetic temperature and spectrum of the ions. Actually, this information is important because higher reactivity in a certain range of ion speeds (or energies) indicates that the population of ions belonging to that range is going to deplete fast. So, there may well exist a velocity range-wise natural (automatic) ion population restoration mechanism so that the fusion reaction rate of the system is maintained. This shows that group-wise fusion reactivities have an impact on the ion spectrum itself. Such information may become very significant in hot thermonuclear plasma diagnostics and characterization, where the group-wise as well as the total fusion reactivity add value to the fusion database in finding appropriate correlations between various observed parameters.

2. Concept of Multi-Group Fusion Reactivities

The calculation of multi-group reactivities requires division of the total energy range or speed of the reacting species into a number of groups. In this work we have chosen three equally populated energy (or speed) groups. In the laboratory frame of reference this amounts to creating three groups for each of the reacting species such that each group has one-third of the total population of that species. In the center-of-mass system, three groups are created in the relative velocity space of the interacting species. Division of the ionic species into such equally populated groups helps us to analyze the fractional contribution to total reactivity from ions in each group interacting with themselves or from ions of one group interacting with those in another group. Three groups seem to be sufficient for the purpose of highlighting the main features of fractional reactivity contributions for different ion distribution functions used in the present work. We may designate these groups as slow, intermediate and fast energy (or speed) groups.

2.1 Laboratory Frame of Reference

For the D-T reaction considered in the present work, three groups for deuterons and tritons are created by considering the cut-off speeds V_{D1}, V_{D2} and V_{T1}, V_{T2}, respectively, as follows **:**

$$\int_0^{V_{D1}} f(V_D)\,\mathrm{d}V_D = \int_{V_{D1}}^{V_{D2}} f(V_D)\,\mathrm{d}V_D = \int_{V_{D2}}^{\infty} f(V_D)\,\mathrm{d}V_D = {}^1\!/_3 \qquad (1)$$

and

$$\int_0^{V_{T1}} f(V_T)\,\mathrm{d}V_T = \int_{V_{T1}}^{V_{T2}} f(V_T)\,\mathrm{d}V_T = \int_{V_{T2}}^{\infty} f(V_T)\,\mathrm{d}V_T = {}^1\!/_3 \qquad (2)$$

where $f(V_D)$ and $f(V_T)$ are the *speed distribution functions* for deuterons and tritons, respectively. Group-wise fusion reactivities, $<\sigma v>$ for isotropic ion distributions can be calculated using the following mathematical expression [7] **:**

$$<\sigma v> = 8\pi^2 \int_{V_{Dmin}}^{V_{Dmax}} dV_D\, V_D\, f_D(\boldsymbol{V_D}) \int_{V_{Tmin}}^{V_{Tmax}} dV_T\, V_T\, f_T(\boldsymbol{V_T}) \int_{|V_D - V_T|}^{|V_D + V_T|} dv\, v^2 \sigma(v) \qquad (3)$$

where σ is the microscopic cross section for the D-T reaction [5], $f_D(\boldsymbol{V_D})$ and $f_T(\boldsymbol{V_T})$ represent the *velocity distribution functions* for deuterons and tritons. For the isotropic distribution function, these are related to the corresponding speed distributions by $f(\boldsymbol{V}) = \frac{f(V)}{4\pi V^2}$. In Eq. 3, v represents the relative speed between the reacting ion species, i.e. v $=|\boldsymbol{V_D} - \boldsymbol{V_T}|$. The speed limits in Eq. 3 are appropriately chosen for the reacting groups. For instance, if one wishes to find the reactivity contribution resulting from the interaction of deuterons in the slow group with tritons in the intermediate group, then $V_{Dmin} = 0$, $V_{Dmax} = V_{D1}$, $V_{Tmin} = V_{T1}$ and $V_{Tmax} = V_{T2}$.

It may be mentioned that an alternative computational form for calculating group-wise reactivities can be written as follows:

$$<\sigma v> = 8\pi^2 \int_0^1 d\mu \int_{V_{Dmin}}^{V_{Dmax}} dV_D\, V_D^2\, f_D(\boldsymbol{V_D}) \int_{V_{Tmin}}^{V_{Tmax}} dV_T V_T^2\, f_T(\boldsymbol{V_T})\ [\, v_1\sigma(v_1) + v_2\sigma(v_2)] \tag{4}$$

where $v_1 = (V_D^2 + V_T^2 - 2V_D V_T \mu)^{\frac{1}{2}}$ and $v_2 = (V_D^2 + V_T^2 + 2V_D V_T \mu)^{\frac{1}{2}}$. It can be easily seen that Eqs. 3 and 4 are equivalent .Two of the integrations employed in both the equations are same. However, integration over v in Eq. 3 is replaced by integration over μ in Eq. 4, where μ= cosθ and θ is the angle between $\boldsymbol{V_D}$ and $\boldsymbol{V_T}$. Both the forms have been used for calculations in this work and are found to give identical results. Furthermore, the partial group reactivities can be added and checked against the total reactivity available in the published literature for the Maxwellian distribution [1, 2].

2.2 Center-of-Mass Frame of Reference

An easier alternative to group-wise reactivities in the laboratory frame is to calculate the same in the center-of-mass system. In this case, it is the relative speed of the interacting species which is divided into three groups of equal ion populations. For this we need only to find v_1 and v_2 such that

$$\int_0^{v_1} f_{DT}(v)\, dv = \int_{v_1}^{v_2} f_{DT}(v)\, dv = \int_{v_2}^{\infty} f_{DT}(v)\, dv = {}^1/_3 \tag{5}$$

where $f_{DT}(v)$ is the relative speed distribution function between the deuterons and tritons. The group-wise reactivities in the relative speed space are then given by

$$<\sigma v> = \int_{v_{min}}^{v_{max}} v\, f_{DT}\,(v)\sigma(v) dv \tag{6}$$

where v_{min} and v_{max} are chosen appropriately.

3. Ion Distribution Functions

In the present work, we have chosen four different kinds of distribution functions (three isotropic and one anisotropic) for calculating the group-wise fusion reactivities in the relative velocity space as well as the laboratory coordinate system. These are given below:

(a) Maxwellian speed distribution

This distribution is expressed by the following mathematical expression:

$$f(V) = 4\pi V^2 \left(\frac{m}{2\pi T}\right)^{\frac{3}{2}} e^{\frac{-mV^2}{2T}} \tag{7}$$

where m is the mass, T is the temperature (in joules) and V is the ion speed.

(b) Generalized Maxwellian speed distribution

This distribution is expressed by the following mathematical expression [6]:

$$f(V) = 4\pi V^2 Y_m \left(e^{\frac{-(\alpha_m) m V^2}{2}}\right)^{\frac{p}{2}} \tag{8}$$

where $Y_{\mathbf{m}} = \left(\frac{1}{4\pi}\right)\left(\frac{p}{\Gamma\left(\frac{3}{p}\right)\left(\frac{3T}{m}\right)^{\frac{3}{2}}}\right) \left(\frac{\Gamma\left(\frac{5}{p}\right)}{\Gamma\left(\frac{3}{p}\right)}\right)^{3/2}$, $\alpha_m = \left(\frac{2}{3T}\right)\left(\frac{\Gamma\left(\frac{5}{p}\right)}{\Gamma\left(\frac{3}{p}\right)}\right)$

where Γ represents the gamma function; m, T, V are as in Eq. 7, and p is a parameter which determines the nature of the distribution. For p=2, the distribution is Maxwellian, whereas for $p < 2$ and $p > 2$ we obtain tail-enhanced and tail-depleted Maxwellian distributions, respectively.

(c) Kappa or Lorentzian speed distribution

This distribution is expressed by the following mathematical expression:

$$f(V) = 4\pi V^2 \left(\frac{1}{2\pi\left(K V_K^2\right)^{\frac{3}{2}}}\right) \left(\frac{\Gamma(K+1)}{\Gamma(K-0.5) * \Gamma\left(\frac{3}{2}\right)}\right) \left(1 + \left(\frac{V^2}{K V_K^2}\right)\right)^{-(K+1)} \tag{9}$$

where K is a *parameter which determines high energy power-law index*; m, T, V, Γ are as in Eq. 8, V_K is an equivalent thermal speed, such that $V_K = \left(\left(\frac{2K-3}{K}\right)\left(\frac{T_K}{m}\right)\right)^{1/2}$; T_K is related to *Maxwellian temperature* T by the relation $T_K = \left(\frac{K}{K-\left(\frac{3}{2}\right)}\right) T$. It is to be noted that for $K \to \infty$, kappa speed distribution tends to Maxwellian speed distribution.

(d) Two-temperature pseudo-Maxwellian speed distribution

This is an anisotropic distribution defined in terms of two temperatures $T_\perp$ and $T_{||}$, for which *velocity distribution* is given by the following expression [3]:

$$f(\mathbf{V})=\left(\frac{m}{2\pi T_\perp}\right)\left(\frac{m}{2\pi T_{||}}\right)^{\frac{1}{2}} e^{\frac{-mV_\perp^2}{2T_\perp}} \; e^{\frac{-mV_{||}^2}{2T_{||}}} \tag{10}$$

The corresponding speed distribution is obtained in [4] and is given below:

$$f(V)=\left(\frac{m}{3T}\right)^{3/2}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{(2\beta+1)^{\frac{3}{2}}}{\beta}\right) V^2 \int_0^1 d\mu\, e^{\frac{-mV^2(2\beta+1)(1-\mu^2+\beta\mu^2)}{6\beta T}} \tag{11}$$

where m, T, V are as in Eq. 7, $\beta = \frac{T_\perp}{T_{||}}$, $\frac{3}{2}T = T_\perp + \frac{1}{2}T_{||}$, $\mu = \cos\theta$. This is obtained by writing the *velocity distribution* in Eq.10 in spherical coordinates and performing the integration over the angular variables. It can be easily seen that for β =1, the above speed distribution reduces to the Maxwellian speed distribution.

4. Results and Discussion

4.1 Multi-Group Fusion Reactivities in Relative Velocity Space

In the speed distributions given in Eqs. 7 to 11, if the mass m is replaced by the reduced mass of the two interacting species, we obtain the relative speed distribution, provided both the species follow the same distribution at the same kinetic temperature. Using this property, the group-wise reactivities in the relative velocity space can be obtained using Eq. 6. The calculated results are shown in Tables 1 to 3 for the various distributions given in Section 3.

Table 1 Total and fractional group reactivities for D-T reaction for generalized Maxwellian distribution

Reactivity contribution	T=10 KeV			T= 15 KeV			T= 20 KeV		
	p=1	p=2	p=3	p=1	p=2	p=3	p=1	p=2	p=3
Total reactivity $<\sigma v> *10^{22}$, m^3s^{-1}	**1.483**	**1.159**	**0.9218**	**2.689**	**2.786**	**2.650**	**3.727**	**4.396**	**4.586**
Group-1 contribution	**0.0015%**	**0.040%**	**0.139%**	**0.013%**	**0.169%**	**0.411%**	**0.052%**	**0.429%**	**0.870%**
Group-2 contribution	**1.029%**	**3.430%**	**5.900%**	**3.295%**	**7.407%**	**10.28%**	**7.072%**	**13.04%**	**16.12%**
Group-3 contribution	**98.97%**	**96.53%**	**93.96%**	**96.69%**	**92.42%**	**89.31%**	**92.88%**	**86.53%**	**83.01%**

p =1: enhanced-tail Maxwellian , p=2: Maxwellian , p=3: depleted-tail Maxwellian.

Table 2 Total and fractional group reactivities for D-T reaction for kappa distribution

Reactivity contribution	T=10 KeV			T= 15 KeV			T= 20 KeV		
	K=70	K=100	K=∞	K=70	K=100	K=∞	K=70	K=100	K=∞
Total reactivity $<\sigma v> *10^{22}$, m^3s^{-1}	1.225	1.201	1.178	2.867	2.836	2.805	4.462	4.434	4.407
Group-1 contribution	0.039%	0.039%	0.040%	0.167%	0.167%	0.166%	0.429%	0.427%	0.425%
Group-2 contribution	3.37%	3.38%	3.39%	7.43%	7.40%	7.37%	13.21%	13.12%	13.03%
Group-3 contribution	96.59%	96.58%	96.57%	92.40%	92.43%	92.46%	86.36%	86.46%	86.55%

K= ∞ represents the Maxwellian case.

Table 3 Total and fractional group reactivities for D-T reaction for two-temperature pseudo-Maxwellian distribution

Reactivity contribution	T=10 KeV			T= 15 KeV			T= 20 KeV		
	β=0.1	β=1.0	β=10.0	β=0.1	β=1.0	β=10.0	β=0.1	β=1.0	β=10.0
Total reactivity $<\sigma v> *10^{22}$, m^3s^{-1}	1.540	1.145	1.351	2.741	2.762	2.858	3.721	4.368	4.210
Group-1 contribution	0.0013%	0.04%	0.009%	0.012%	0.166%	0.053%	0.05%	0.42%	0.163%
Group-2 contribution	0.871%	3.398%	2.100%	2.880%	7.318%	5.330%	6.383%	12.88%	10.28%
Group-3 contribution	99.13%	96.56%	97.89%	97.11%	92.52%	94.62%	93.57%	86.70%	89.55%

β =1 represents the Maxwellian case.

From these tables we can clearly understand that the maximum contribution to the total reactivity comes from fastest group of ions, followed by contributions from the intermediate group and the slowest group, respectively. Practically the ions belonging to the slowest group contribute nothing to the total reactivity and the corresponding data are only of academic interest. The intermediate group ions, however, have some contribution to the total reactivity, although the contribution is considerably small compared to the contribution from the fastest group. Another important observation is that the contribution of the fastest group-3, although dominant at all temperatures, reduces appreciably with increasing ion temperature. This gives an idea about how the ion spectrum gets affected with changing ion temperature and about the effect of ion temperature on group-wise fusion reactivity. Moreover, we can also infer that the ion population in the fastest velocity group depletes fast compared to the ions from other velocity groups due to high reaction rate in the fastest group. Therefore, for a sustained fusion system, replenishment of the population of fastest velocity group ions is very much needed.

Table 1 also exhibits the effect of tail-enhancement and tail-depletion on the group-wise contribution. The effect on the contribution from the fastest group of ions is an important aspect to look at. Here, we can clearly see from the table that enhanced-tail distribution (p =1) results in higher percentage of contribution of fusion reactivity from the fastest group compared to the usual Maxwellian case (p = 2); whereas tail-depleted speed distribution (p = 3) results in a lower percentage of reactivity contribution from the fastest group compared to the above mentioned cases. Furthermore, the variation in the total fusion reactivities for the above mentioned types of speed distributions has been shown in the Table 1, which itself is important, because fusion power density is directly proportional to the reactivity and one should always look for higher reactivity, if possible, for a particular ion kinetic temperature. As can be seen in Table 1, reactivity for tail-enhanced Maxwellian distribution is *about 30% more* compared to the Maxwellian distribution at 10 keV. This enhancement of reactivity is somewhat less at 15 keV and disappears at 20 keV.

Kappa distribution takes into account local thermal accelerations that may take place in fusion reactors. Thus it is quite logical to introduce Kappa distribution to model ion velocities in practical fusion reactors. Kappa distribution function resembles a Maxwellian distribution at lower energies but exhibits a power law tail at higher energies.

The total and fractional reactivities for kappa distributions are shown in Table 2. It is evident from the calculated data that group-wise percentage contributions to the total fusion reactivity are pretty much insensitive to the change of power law index parameter K. However, as K decreases, tail part of the distribution in enhanced and as a result the total fusion reactivity increases. For $K \rightarrow \infty$, kappa speed distribution approaches to Maxwellian distribution. Thus, in the context of Kappa distribution, it may be concluded that more is the deviation from Maxwellian distribution, better is the total fusion reactivity.

Finally**,** Table 3 presents the total and fractional reactivities for two-temperature pseudo-Maxwellian distribution. This speed distribution is important because it incorporates anisotropy through introduction of two different temperatures characterizing ion velocities parallel and perpendicular to the applied magnetic field. Here we have studied the effect of the temperature ratio parameter β on the group-wise reactivity contributions as well as on the total fusion reactivity. *It is to be noted that β=1 corresponds to usual Maxwellian distribution*. For other values of β we can observe the effect of anisotropy of speed distribution on the total as well as the fractional reactivities at different ion temperatures. For 10 keV, *extreme anisotropy may enhance fusion reactivity by more than 35%*, compared to the Maxwellian distribution. At 15 keV, the enhancement due to anisotropy is about 3%, whereas at 20 keV isotropic distribution gives better reactivity.

4.2 Multi-Group Fusion Reactivity in Laboratory Frame

Multi-group fusion reactivities in laboratory frame of reference provide more insight about the fusion reactions among different energy groups of interacting ion species. This is of more practical interest, because here multiple groups for each ion species are considered, giving more detailed information about the fractional reactivity contribution due to inter-group interaction between the two species of ions. Fusion reactivities due to inter-group interactions can be evaluated using Eq. 3 or Eq. 4. These results actually help us to identify the ion species and groups to which more energy should be supplied preferably, if one at all has a control over it.

Table 4 Inter-group fusion reactivity contributions for D-T reaction at 10 keV for Maxwellian distribution

[$\langle\sigma v\rangle_{total} = 1.139 * 10^{-22}$ m^3s^{-1}]

Deuteron \ Triton	Group-1	Group-2	Group-3
Group-1	**0.05%**	**0.45%**	**5.35%**
Group-2	**0.728%**	**2.98%**	**14.75%**
Group-3	**12.017%**	**21.494%**	**42.18%**

Table 5 Inter-group fusion reactivity contributions for D-T reaction at 15 keV for Maxwellian distribution

[$\langle\sigma v\rangle_{total} = 2.755 * 10^{-22}$ m^3s^{-1}]

Deuteron \ Triton	Group-1	Group-2	Group-3
Group-1	**0.14%**	**0.98%**	**7.65%**
Group-2	**1.40%**	**5.10%**	**17.10%**
Group-3	**15.0%**	**21.20%**	**31.43%**

Tables 4 and 5 present the results of the calculation for Maxwellian distribution at 10 and 15 keV respectively. In these tables we can clearly see that a major fraction of the reactivity is contributed by the fusion reaction between the fastest deuterons and the fastest tritons. However, the percentage reactivity contribution coming from the interaction of the fastest group of ions decreases with increasing ion kinetic temperature, whereas contributions due to other inter-group reactions may considerably increase.

We can also see that, at both the temperatures reactivity contributions are higher when the participating deuterons are from the fastest group, thus preferably more energy should be supplied to the deuteron population, if possible, which may lead to enhanced total reactivity.

Table 6 Inter-group fusion reactivity contributions for D-T reaction at 15 keV for enhanced-tail Maxwellian distribution (p=1)

[$<\sigma v>_{total} = 2.776 * 10^{-22}$ m^3s^{-1}]

Triton / Deuteron	Group-1	Group-2	Group-3
Group-1	**0.014%**	**0.248%**	**8.913%**
Group-2	**0.469%**	**2.263%**	**15.96%**
Group-3	**16.71%**	**21.90%**	**33.523%**

Table 7 Inter-group fusion reactivity contributions for D-T reaction at 15 keV for depleted-tail Maxwellian distribution (p = 3)

[$<\sigma v>_{total} = 2.706 * 10^{-22}$ m^3s^{-1}]

Triton / Deuteron	Group-1	Group-2	Group-3
Group-1	**0.31%**	**1.55%**	**7.45%**
Group-2	**2.30%**	**6.85%**	**18.40%**
Group-3	**13.90%**	**20.50%**	**28.74%**

Tables 6 and 7 present the results of the calculation for enhanced-tail and depleted-tail Maxwellian distributions respectively at 15 keV. In this case we get essential information about the effect of tail enhancement or tail depletion on the reactivity contribution coming from the fusion reactions between the fastest deuterons and the fastest tritons. From tables 6 and 7, it is evident that the percentage reactivity contribution coming from the fusion reaction between the fastest deuterons and the fastest tritons is higher in case of enhanced-tail speed distribution (p =1) compared to the case of usual Maxwellian speed distribution; whereas, depleted-tail speed distribution results in a lower percentage of reactivity contribution coming from the interaction between fastest population of deuterons and tritons.

Table 8 Inter-group fusion reactivity contributions for D-T reaction at 15 keV for kappa distribution (K = 70)

[$<\sigma v>_{total} = 2.862 * 10^{-22}$ m^3s^{-1}]

Triton / Deuteron	Group-1	Group-2	Group-3
Group-1	**0.143%**	**0.99%**	**7.655%**
Group-2	**1.448%**	**5.128%**	**17.068%**
Group-3	**15.054%**	**21.063%**	**31.451%**

Table 9 Inter-group fusion reactivity contributions for D-T reaction at 15 keV for kappa distribution (K = 100)

[$<\sigma v>_{total} = 2.828 * 10^{-22}$ m^3s^{-1}]

Triton / Deuteron	Group-1	Group-2	Group-3
Group-1	**0.143%**	**0.99%**	**7.648%**
Group-2	**1.45%**	**5.125%**	**17.086%**
Group-3	**15.032%**	**21.087%**	**31.439%**

Tables 8 and 9 present the results of the calculation for kappa distribution for K =70 and K =100, respectively. From these tables it is evident that change in *power law index parameter* K has little or no effect on the fractional reactivity contributions coming from various inter-group fusion reactions. However, lesser K values result in a slightly higher total reactivity.

5. Effect of Unequal Ion Temperatures and Anisotropy on the Total Fusion Reactivity

We have already mentioned in Section 4 that anisotropy can lead to enhanced fusion reactivity. It can also be seen that there is an asymmetry in the inter-group reactivity matrix in the laboratory frame of reference (Tables 4 and 5), indicating that unequal ion temperatures combined with anisotropy, can add to total fusion reactivity enhancement. In this section we have investigated this combined effect using two-temperature pseudo-Maxwellian velocity distribution function (Eq. 10), taking different kinetic temperatures for the interacting species.

Here we have developed a mathematical expression similar to Eq. 3 and Eq. 4. The only significant difference is that this new formula involves 5-dimensional integration. The expression is given below:

$$<\sigma v> = 8\pi \int_0^{\pi/2} d\theta \int_0^{\infty} dV_D^{||} \int_0^{\infty} dV_D^{\perp}\ V_D^{\perp}\ f_D(\boldsymbol{V_D}) \int_0^{\infty} dV_T^{||} \int_0^{\infty} dV_T^{\perp}\ V_T^{\perp}\ f_T(\boldsymbol{V_T})\ .\ \mathrm{S} \tag{12}$$

Where, $\mathrm{S} = \sum_{i,j=1}^{2} v_{ij}\sigma(v_{ij})$, $v_{ij} = \left(v_{||,i}^2 + v_{\perp,j}^2\right)^{\frac{1}{2}}$

Where, $v_{||,1} = \left|V_D^{||} - V_T^{||}\right|$, $v_{||,2} = \left|V_D^{||} + V_T^{||}\right|$, $v_{\perp,1} = \left({V_D^{\perp}}^2 + {V_T^{\perp}}^2 - 2V_D^{\perp}V_T^{\perp}cos\theta\right)^{\frac{1}{2}}$ and $v_{\perp,2} = \left({V_D^{\perp}}^2 + {V_T^{\perp}}^2 + 2V_D^{\perp}V_T^{\perp}cos\theta\right)^{\frac{1}{2}}$

In this case, azimuthal symmetry is there in the ion distributions, thus it is essentially a case of limited anisotropy. Here, with the help of this mathematical expression we have studied effect of unequal ion kinetic temperature on total fusion reactivity for D-T reaction at average ion kinetic temperatures of 10, 15 and 20 keV, respectively. As this model involves 5-dimensional integration, it may be computationally time consuming if one wants to predict reasonably accurate results. Sufficient number of points should be taken for the convergence of the reactivity value. Here, we have presented these results graphically for better understanding.

In the graphs, we have introduced a parameter called x, which denotes *the fractional deviation from the average ion kinetic temperature* (T) of the fusion system. Deuteron and triton kinetic temperatures are expressed as follows:

$$T_D = (1 + x)\ \mathrm{T} \tag{13}$$

$$T_T = (1 - x)\ \mathrm{T} \tag{14}$$

$\beta = \frac{T_{\perp}}{T_{||}}$ is the measure of anisotropy and is kept same for deuterons and tritons.

It can be seen from Figures 1 to 3, that for both isotropic (β =1) and anisotropic (β =0.1 or 10) distributions it is always advantageous to have hotter deuterons than tritons (x > 0) for the same average temperature of the two species. The combined effect of unequal ion temperatures and anisotropy on reactivity enhancement can be as large as 100% as compared to the Maxwellian distribution at 10 keV. This, however, is the case under extreme conditions of anisotropy and fractional deviation from the average kinetic temperature.

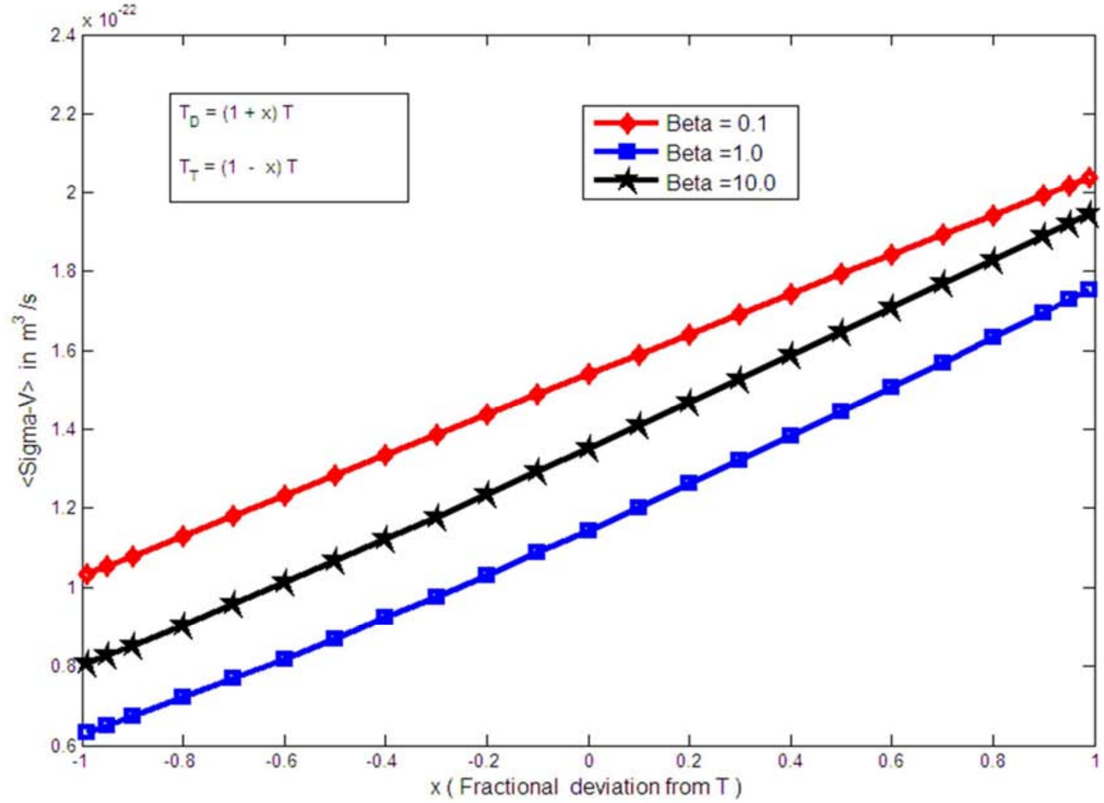

Figure 1 Reactivity versus x (fractional deviation from average kinetic temperature) plot for D-T reaction at T = 10 keV

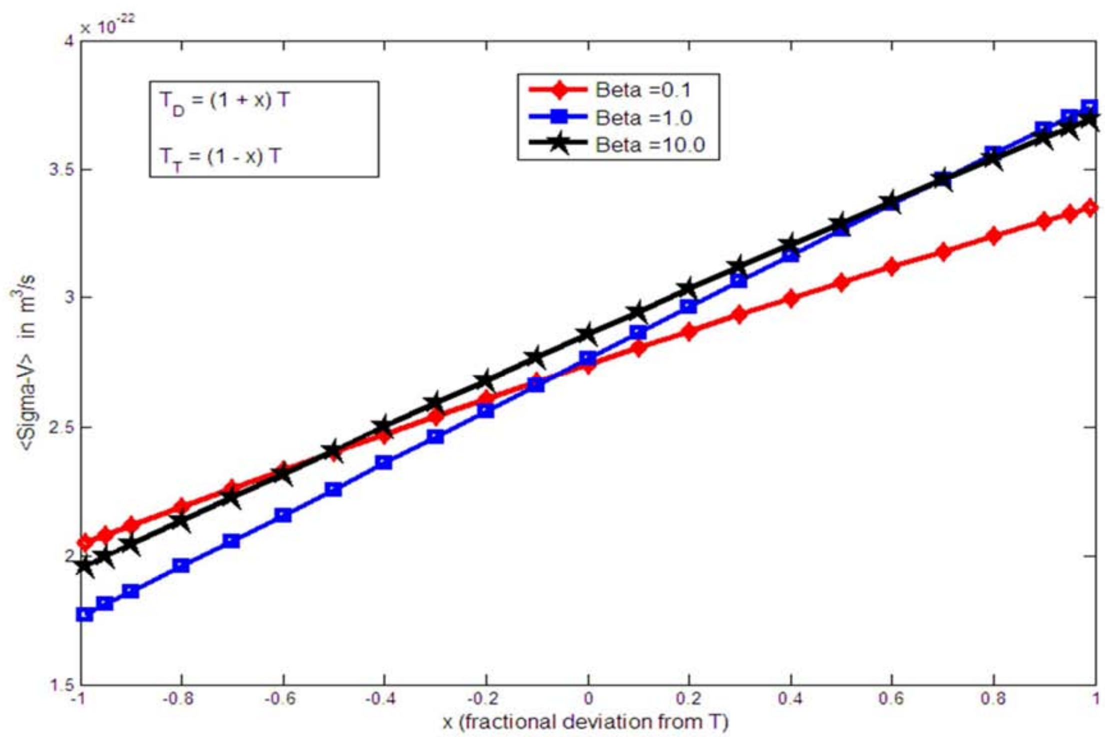

Figure 2 Reactivity versus x (fractional deviation from average kinetic temperature) plot for D-T reaction at T = 15 keV

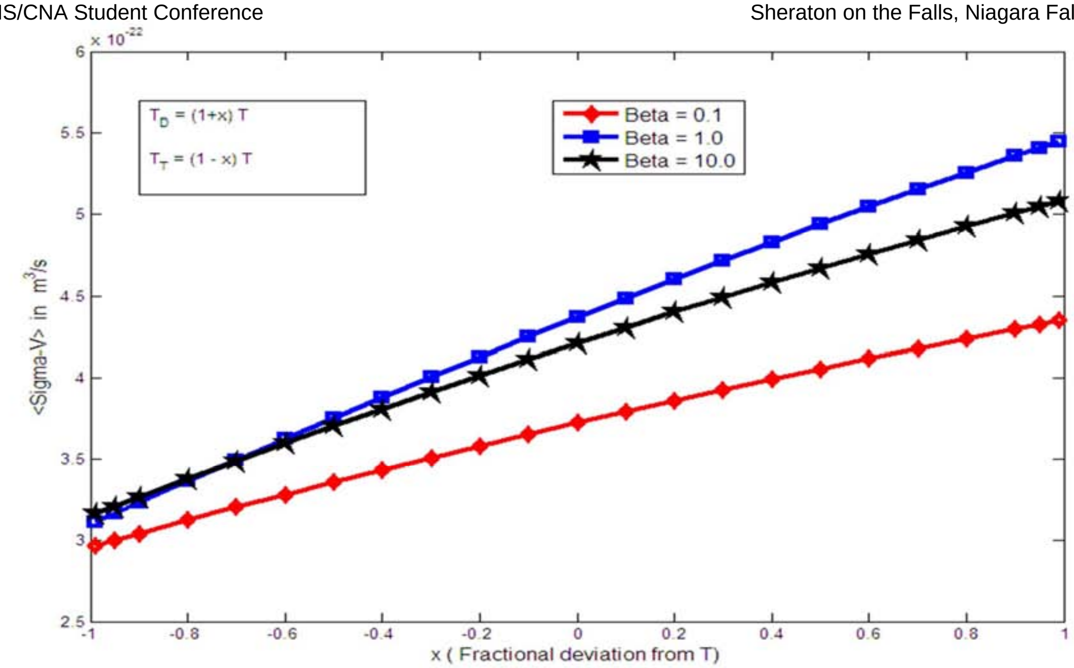

Figure 3 Reactivity versus x (fractional deviation from average kinetic temperature) plot for D-T reaction at T = 20 keV

6. Conclusion

From the computational results already discussed in the present work, we can conclude that anisotropy in ion distributions may result in an enhanced total reactivity for the D-T reaction particularly at 10 keV. The effect of unequal ion temperatures on the reactivity has also been studied and it can be inferred from the results that higher kinetic temperature of deuteron population results in enhancement of total reactivity by a considerable amount. Tail-enhanced Maxwellian distribution can increase or decrease the total reactivity depending on the temperature. Furthermore, from this work, one can get the basic ideas about the speed or energy ranges from which most of the reactivity contributions come as well as about the variations in the group-wise and total reactivities for different ion distributions and temperatures, which actually add to the database required for hot thermonuclear plasma diagnostics and detailed power balances for scientific or engineering break-even studies.

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