Simulations and Imaging Algorithm Development for a Cosmic Ray Muon Tomography System for the Detection of Special Nuclear Material in Transport Containers

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Abstract

The Cosmic Ray Inspection and Passive Tomography (CRIPT) collaboration is developing a cosmic ray muon tomography system to identify Special Nuclear Materials (SNM) in cargo containers. In order to gauge the viability of the technique, and to determine the best detector type, GEANT4 was used to simulate the passage of cosmic ray muons through a cargo container. The scattering density estimation (SDE) algorithm was developed and tested with data from these simulations to determine how well it could reconstruct the interior of a container. The simulation results revealed the ability of cosmic ray muon tomography techniques to image spheres of lead-shielded Special Nuclear Materials (SNM), such as uranium or plutonium, in a cargo container, containing a cargo of granite slabs.

1. Introduction

E.P. George first pioneered the use of cosmic rays to measure the interior of a large structure in 1955, when he used the attenuation of cosmic rays to determine the overburden of the Guthega-Munyang tunnel in New South Wales, Australia [1]. In the late 1960s, Luis Alvarez *et al.* then used measurements of the attenuation of cosmic ray muons to search for chambers within Chephren's pyramid [2]. In response to post 9/11 concerns that terrorists might attempt to smuggle illicit nuclear materials or bombs via ports of entry, groups at Los Alamos National Laboratory and Uppsala University in

Sweden have explored the use of cosmic ray muon tomography to detect Special Nuclear Materials (SNM) hidden within cargo containers [3-[7].

Unlike the X-ray, γ -ray and neutron beams that have been investigated for SNM detection [8], cosmic-ray muons are a naturally occurring, harmless form of penetrating radiation. Muons are produced in the Earth's upper atmosphere when cosmic ray protons collide with air molecules [9]. They are leptons, and have the same properties as electrons, except that they are much more massive (about 207m_e) and have a lifetime of only 2.2 µs. Cosmic ray muons arrive at the surface of the Earth at a rate of about $1/\text{cm}^2/\text{min}$, and have a mean energy of 3-4 GeV. The relativistic speeds of cosmic ray muons thus allow them to live long enough to reach the Earth's surface and to penetrate several metres of rock [10].

When muons travel through a material, such as rock, each one scatters many times from the electrons and protons in the material. The muons that survive passage through the material then emerge with a distribution of cumulative scattering angles that is given by the following equation:

$$\sigma_{\theta} = \frac{13.6 \ MeV}{\beta cp} \sqrt{\frac{H}{X_0}} \left[1 + 0.038 \ln\left(\frac{H}{X_0}\right) \right]$$
(1)

where H is the distance muon traveled through the material, and σ_{θ} is the standard deviation of the cumulative scattering angle distribution. 98% of the cumulative scattering angles of a collection of muons will fall within a Gaussian distribution of standard deviation σ_{θ} . The remaining 2% of the cumulative scattering angles will lie in broad non-Gaussian tails. The quantity X_0 is the radiation length of the material, which decreases with increasing material density and atomic number, Z. Thus, muons traversing a block of low-Z material will tend to have smaller cumulative scattering angles than those that traveled through a lump of high-Z material, like uranium. If one measures the cumulative scattering angles of a collection of muons that passed through a container, one can therefore use their scattering angles to determine whether low, medium or high-Z material is inside the container [10].

By using the Points of Closest Approach Method (PoCA), one can estimate the positions of different materials within a container. The PoCA is the point at which the initial and final trajectory lines are closest to each other. The PoCA is at the midpoint of the shortest line connecting the initial and final trajectory vectors as in Figure 6. This line is perpendicular to the initial and final trajectory lines. Given the initial and final trajectory lines, one can therefore exploit this fact to calculate the PoCA. One can obtain the initial and final trajectory lines of a muon that passed through a container, by placing at least two tracker planes above and at least two tracker planes below the container as Figure 1 shows. The muon hit positions in the trackers, which must be position-sensitive radiation detectors, provide the points that define the two trajectory lines. In reality, a muon scatters many times as it passes through a dense lump of material. However, the points at which the muon scatters most strongly are typically located within the higher-Z, higher

density regions of the object. Thus the PoCA points are good indicators of the locations of high-Z, high density materials, like SNM, if they are surrounded by a lot of much lower density material [11].



Figure 1 The scattering of a muon occurs at many points along its track; however, the greatest amount of scattering occurs near the PoCA, which the black circle on the diagram denotes.

2. The simulation method

GEANT4, which was developed by an international collaboration based at CERN [12], was used to simulate the passage of muons through a cargo container. GEANT4 is a Monte Carlo particle transport simulation code that incorporates three main simulation modules: the Geometry module, the Event Generator module and the Event Recorder module. The Geometry module provides the user with full control over the simulation geometry and the materials used in the creation of that geometry. The Event Generator generates primary particles that are specified by the user, and secondary particles, generated by interactions of the primary particles with matter. The Event Generator provides the user with control not only over the primary particle species, but also over the particles' energies, momentum vectors and positions of origination. Once the Event Generator throws a particle into the simulation, it uses the physics processes associated with that particle to determine its behaviour. Since the user must specify the processes that will apply to each particle species generated, GEANT4 also provides the user with control over the physics that will be simulated. Finally, the Event Recorder module provides output to the screen and/or a file of the data generated by the simulation. As in the cases of the Geometry and Event Generator modules, GEANT4 also gives the user

full control over what information is recorded in an output file or printed to the screen [12].

2.1 Geometry

The simulation geometry, diagrams of which appear in Figure 2 and Figure 3, consisted of two sets of top and bottom tracker modules, a cargo container with 3 mm thick steel walls, and the muon momentum spectrometer. The cargo container held a 26 cm tall stack of granite slabs and two 25 kg and 8 kg Highly Enriched Uranium (HEU) spheres, surrounded by 2.8 cm thick spherical, lead shields. Two 1.7 cm thick, polystyrene slabs of density 1.06 g/cm³ comprised each tracker module. The centres of the tracker modules were 1 m apart, and those closest to the cargo container were 50 cm above and below it. Each tracker module also extended 50 cm beyond the cargo container's front, back and side faces. The momentum spectrometer consisted of four tracker modules and four 10 cm thick iron slabs. Each tracker module/iron slab combination was placed 1 m away from the next.

2.2 The simulated cosmic ray muon distribution

In order to accurately depict the cosmic ray muons that were incident upon the trackers and the cargo container, the polar angles of the initial muon trajectories were drawn randomly from a $\cos^2\theta$ distribution. The initial energy assigned to each muon was then randomly taken from the μ + energy spectrum measured with the Balloon-borne

Experiment with a Superconducting Spectrometer (BESS) at Tsukuba, Japan. A plot of



the BESS spectrum appears in

Figure 5 [14]. As one can see from the plot, most of the muons had energies below 2 GeV. Larger muon energies are more common at larger zenith angles, and the BESS distribution was taken for vertically-oriented muons. Hence, by using the BESS spectrum to represent muons of all zenith angles, the simulations underestimated the number of higher energy muons. Nevertheless, as one can see from Figure 7, since only a small fraction of the muons had zenith angles greater than 45°, the use of the BESS spectrum represented a reasonable approximation [15].



Figure 2 Diagrams of the simulated cargo containers, containing granite slabs and lead-shielded HEU spheres.

2.3 The Muon Processes

The last aspect of the simulations consisted of the processes each muon underwent in its travels through the simulated geometry. The four muon processes that were simulated were GEANT4's G4MuMultipleScattering, G4MuIonisation, G4MuBremsstrahlung and G4MuPairProduction. G4MuMultipleScattering, as the GEANT4 process' name suggests is the process of multiple elastic scattering of muons off of the electrons and nuclei in the object they are traversing. This process is based on the Urban Multiple Scattering Model90 and the Wentzel Model, which contains empirical correction terms to Equation 1. G4MuIonisation is simply the muon energy loss process in which a muon imparts enough energy to an electron to cause it to it leave the atom to which it was attached. This is a form of inelastic muon scattering on atomic electrons. G4MuBremsstrahlung is the process by which a muon radiates energy as it decelerates. The bremsstrahlung energy that a decelerating charged particle emits is in the form of Xrays. Finally, G4MuPairProduction is the process in which a muon creates a positronelectron pair while travelling through an object. All four of these processes have an impact on the energy loss of a muon as it travels through matter; however, the process that has greatest effect on the cumulative scattering angle of the muon is multiple scattering. Since the inelastic scattering of muons on nuclei is important only for muons

above 10 GeV, and most of the simulated muons had energies below that value, this process was also not simulated [16]. Since the muon nuclear spallation reaction cross sections are many orders of magnitude smaller than those for the simulated processes, they were not included in the simulations [16][17].



Figure 3 A wireframe view of the geometry that was used for the detector parameter simulations in GEANT4.



Figure 5 Measured BESS spectra reproduced from M. Motoki. The GEANT4 simulations in this work used the µ+ flux taken at Tsukuba [14].



Figure 7 Cosmic-ray muon energy spectra for different zenith angles [15].

3. The Scattering Density Estimation Algorithm

3.1 What is the scattering density?

The scattering density is a measure of the ability of a material to produce large deflections in a muon's trajectory. A definition of the scattering density, based on Equation 1 is

$$\lambda = \left[\frac{13.6 \ MeV}{(\beta cp)_0}\right]^2 \frac{1}{X_0} \tag{2}$$

 $(\beta cp)_0$ is the average of the product of the muons' speeds and momenta. As one can see, the scattering density, λ , increases with decreasing X₀. Hence, high-Z materials, like uranium, which have shorter radiation lengths, will have larger scattering densities than low to mid-Z materials, like aluminum and iron. Thus, a 3D map of the scattering density inside a container can act as a 3D map of the mass density and atomic number within the container. The logarithmic term in Equation 1 is generally no larger than about 0.17, compared to unity, so one can neglect it. Thus, after neglecting the logarithmic term, if one substitues Equation 1 into Equation 2, one obtains Schultz *et*

al.'s definition for the scattering density in terms of the quantities measured with tracker modules and a momentum spectrometer:

$$\lambda = \frac{\sigma_{\theta}^2}{H} \left[\frac{\beta cp}{(\beta cp)_0} \right]^2$$
(3)

The quantity H, in this case, is an estimate of the length of the muon's path through the material. β cp and is the product of the muon's speed and momentum. The momentum spectrometer provides this information. One can obtain an estimate of H from the muon's scattering angle and the length of the line passing through the PoCA, and connecting its initial and final trajectories. Finally, σ_{θ}^2 is estimated with the muon's cumulative scattering angle, θ^2 [18].

3.2 How the SDE algorithm works

When the scattering density algorithm (SDE) estimates the scattering density map within a cargo container, it first begins by dividing the container into 1000 cm³ cubic volume elements, called voxels. Once the cargo container has been divided into voxels, the SDE Algorithm reconstructs four point tracks for all of the muons that entered the container (see Figure 6). It does this by calculating the cumulative scattering angles, PoCA points and speeds×momenta for each of the muons that entered the container. It then assigns to each voxel an estimated scattering density, based upon the cumulative scattering angles, momenta and distances between the PoCA points of the muons that had PoCAs inside the voxel. This entails taking the averages of these quantities over all of the muons that had PoCAs in a given voxel. The subsequent blurring resulted in errors in the angular resolution which lead to some poorly localized "hot spots" (where the deflection angle and/or POCA distance were too high). The Pitchfork Method reduces the influence of these hotspots by "spreading" them according to the angular resolution of the measuring process. It was found empirically that the "real" points are less affected by errors in localization, and hence they will be less "spread out". Figure 9 illustrates the Pitchfork Method, which involves creating two randomly-oriented, perpendicular vectors of magnitude one, named p1 and p2. It then creates the four outer prongs of the pitchfork by adding and subtracting p1 and p2 from the initial or final trajectory vector, u. The SDE Algorithm then performs its PoCA and scattering angle calculations by using the original u vectors, and also the four outer prong vectors that surround each of them. Based on the initial and final trajectory vectors, and their outer prong or satellite vectors, it thus calculates 25 PoCAs and scattering angles for each muon. When the SDE Algorithm calculates the averages for a given voxel, it gives a smaller weight to PoCAs and scattering angles that were calculated with outer prong (p1 or p2) vectors and larger weights to PoCAs and scattering angles calculated with inner prong (u) vectors. Hence, PoCAs and scattering angles calculated with the two u vectors are given a weight of 1/9. PoCAs and scattering angles calculated with one u and one outer prong vector are given a weight of 1/18 and PoCAs and scattering angles calculated with two outer prong vectors are given a weight of 1/36.

Finally, the SDE Algorithm cuts all of the tracks for which the momentum×scattering angles are greater than those for the other 98%. The algorithm thus removes the non-Gaussian outliers that tend to make discriminating between mid and high-Z materials more difficult.



Figure 6 The reconstructed four-point track of a muon. The hit positions in the top and bottom trackers, the PoCA point, and the shortest line connecting the initial and final trajectories form the reconstructed track.



Figure 9 A diagram of a five-vector "pitchfork". The central vector (black arrow), u, is the measured initial or final trajectory. The four outer prong trajectories (blue arrows) are formed by adding and subtracting a randomly-generated set of two perpendicular vectors to and from u.

4. **Results and conclusions**

The passage of muons through a cargo container, containing a 26 cm tall stack of granite slabs and two HEU spheres, surrounded by lead shielding, was simulated in the Monte Carlo program GEANT4. The GEANT4 simulation printed the muons' hit positions in the eight tracker modules. The hit positions in the first four tracker modules provided the cumulative scattering angles and PoCA points of the muons after they passed through the container. The hit positions in the last five tracker modules, which were parts of the momentum spectrometer, provided the cumulative scattering angles of the muons due to passage through the iron slabs. The scattering angles due to scattering in the iron slabs yielded the individual muons' momenta via Maximum Likelihood fits for the standard deviations of the scattering angles and Equation 1.

The cumulative scattering angles (due to passage through the container), PoCA points and momenta of the muons then went into the SDE Algorithm's calculations of the estimated scattering density map of the cargo container. Plots of the scattering density maps for the simulated cargo container, and for 1 and 3 mm hit position resolutions appear in Figure 10 and Figure 12. As one can see from the figures, application of the SDE Algorithm to the simulated muon scattering data produced 3D images in which the two HEU spheres and their lead shielding are quite visible. This image reconstruction method also worked for simulation data that had hit position resolutions that were similar

to those associated with the real life muon detectors that will ultimately be used to perform measurements. The simulation data and SDE Algorithm results thus support the viability of the use of cosmic ray muon tomography to identify SNM in cargo containers.

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Figure 10 3D colour plots of the estimated scattering density as a function of voxel position for the granite cargo, two HEU+Pb spheres, 3 mm position resolution, and 78% 1/ β cp resolution. The white cubes, which are meant to guide the eyes, denote the positions of the HEU+Pb spheres. While the black pixels indicate the largest scattering densities, the orange pixels denote slightly lower scattering densities.



Figure 12 3D colour plots of the estimated scattering density as a function of voxel position for the granite cargo, two HEU+Pb spheres, 1 mm position resolution, and 63% 1/ β cp resolution.

6. References

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