Consideration of Inspection Uncertainties in the Probabilistic Assessment of Steam Generator Tubing

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Abstract

Steam generators (SG) are periodically inspected to maintain high safety and integrity of the heat transport system in the nuclear plant. The integrity of SG tubing is affected by various degradation mechanisms, such as wear and stress corrosion cracking (SCC). SG tubing integrity assessment is periodically performed to ensure the tubing degradation does not exceed the structural limit in the upcoming operation cycles. This paper presents a probabilistic approach for integrity assessment of SG tubing with fretting wear degradation and highlights the importance of correct modeling inspection uncertainties, such as flaw sizing error, in the prediction. A case study is presented using actual data from a nuclear station, which illustrates the effectiveness of the proposed method.

1. Introduction

Steam generator (SG) tubing is an integral part of the reactor coolant pressure boundary, which maintains the pressure of the primary system and isolates the radioactive fission products in the primary coolant from the secondary system. The SG tubing suffers from various degradation mechanisms, such as corrosion, fretting wear and SCC. The SG tubing degradation can have an adverse impact on safety and reliability of the nuclear plant. For example, the SG tube break can cause a containment bypass accident. The tubing integrity assessment is thus required for the assurance of high safety and performance margins in short term [1], and optimizing the life cycle management of SG fleet in the long term.

The operational assessment of SG tubing is intended to demonstrate that the tube integrity performance criteria will be met throughout the next inspection interval. The operational assessment considers the projected future condition of the tubes based on the latest inspection results and predicted future growth rate. If the future state of any existing flaw projected to the end of evaluation period does not meet the specified acceptance criteria, the tube needs to be removed from the service by plugging.

The nuclear industry in Canada and the U.S. has undertaken extensive research to develop fitness for service criteria and guidelines for inspection and plugging of tubes in steam generators. The operating experience suggests that flaw size and flaw growth rate in steam generators tend to be highly variable or uncertain, which confounds the prediction of future growth of flaws. Another element that affects the accuracy of growth prediction is the flaw sizing error associated with the inspection method. The sizing error is a results of (1) the error associated with the probe (e.g., hardware limitations), and (2) subjectivity associated with the data interpretation (e.g., calibration curves).

To account for uncertainties associated with degradation and flaw inspection process, the nuclear industry has gradually moved towards adopting the probabilistic methods for the assessment of SG tubing. The flaw growth rate is an important element of the assessment. The reason is that conservative predictions will result in frequent inspection/maintenance outages resulting in loss of efficiency, while non-conservative prediction may lead to potential safety problems. The basis for the growth rate estimation is a set of flaw sizes measurements obtained from periodic inspection of SG tubing. Since measurements are contaminated by random sizing error, a precise estimation of the flaw growth rate becomes a challenging task.

A full Monte Carlo simulation based reliability analysis of steam generators is not practical and the input required for simulations model, such as the growth, still needs to be estimated from the contaminated data set. The industry has developed simple approximate methods for considering the impact of sizing error in predicting the flaw growth rate, flaw repair limit, and the time for next inspection. However, the degree of conservative associated with these methods in comparison to more precise solutions has not been evaluated. Although simple approximate methods may be adequate for short term operational assessment, their use in long term life cycle management can lead to erroneous results.

The objectives of this paper are to (1) Develop an accurate probabilistic method for the evaluation of flaw growth rate, and (2) Evaluate the accuracy of currently used method of the flaw growth rate estimation.

2. Industry Practice

2.1 Background

There are three basic elements of the SG tubing integrity assessment [2, 3]: (1) degradation assessment, which documents status of the plant specific degradation mechanisms, such as SCC, pitting, and/or wear, in support of planning future inspection outages, (2) condition assessment, which is a backward assessment to confirm that adequate tube integrity was maintained during the previous inspection interval. It is an evaluation of the as-found condition of tubing relative to integrity acceptance standards, and (3) operational assessment, which is a forward looking prediction of the tubing condition to ensure that acceptance standard will be met during the next inspection interval. In contrast, life cycle management (LCM) is a long term assessment that includes the prediction of the condition of tubing over next several outages, and optimization of future inspection, maintenance and replacement activities.

The structural integrity performance criteria are typically based on the concept of maximum tolerable flaw size (MTFS) or the structural limit (SL). The MTFS (a_{MTFS}) is the maximum size of a part through-wall flaw that satisfies the acceptance criterion including the required safety factors on the load. The MTFS for various types of flaws are derived from mechanistic models of tube burst and leakage [3]. For example, MTFS for fretting/wear flaw is 71% tw (through wall) [3]. A similar concept of structural limit is used to define an acceptable flaw size based on a tube burst model [4].

In the operational assessment, acceptability of an existing flaw is judged based on the repair limit. The repair limit (a_{REP}) is the NDE measured flaw size at or beyond which the tube must be repaired or removed from service by plugging so that the performance criteria will be met at the end of next inspection interval [2]. The repair limit for a specified inspection interval of t EFPY is calculated as [3]

$$a_{REP} = a_{MTFS} - a_{ERR} - a_G(t) \tag{1}$$

where a_{ERR} is the sizing error associated with the non-destructive evaluation (NDE) tool and $a_G(t)$ is the flaw growth during the next inspection interval, t.

The operational assessment (OA) can be approached in two ways. If the time of next inspection, t_{NP}, is fixed, then all existing flaws with size $a \geq a_{REP}$ must be removed from tube plugging [3]. On the other hand, t_{NP} can be back calculated such that the worst flaw (a_{WRS}) in the population does not grow beyond the MTFS [2].

2.2 Assessment approaches

Operating experience and historical inspection data suggest that the degradation growth rate is not a constant in a population of tubes within a steam generator or across a fleet of SG. Similarly, sizing error associated with an NDE probe varies randomly from flaw to flaw, and its actual magnitude is not known. To account for these uncertainties, a_{ERR} and $a_G(t)$ are modeled as random variables with appropriate probability distribution. It means that the flaw repair limit, a_{REP}, is also a random variable with a distribution that depends on those in the right hand side of Eq.(1). Thus, a probabilistic criterion is needed to define acceptance standard. One such criterion is that the OA is equivalent to demonstrating that the probability of meeting the performance criteria is at least 95% at 50% confidence [2].

2.2.1 Approach 1

Since full uncertainty analysis using detailed Monte Carlo simulations including different random variables is not always practical, the industry has developed simplified approaches based on bounds (or percentiles) in place of full distribution of random variables. For example, the use of 95% percentile of the growth rate distribution as an upper bound rate, r_{UP}, and 95% percentile of sizing error distribution as the bounding error, a^U_{ERR}, is recommended in [2]. Using these bounds, the time to next inspection, defined as the time during which the worst case flaw will not grow to exceed the MTFS, can be estimated as [4]

$$t_{NP} = \frac{a_{MTFS} - a^U_{ERR} - a_{WS}}{r_{UP}} \tag{2}$$

The projected worst case degraded tube is defined as the degraded tube with 5% percentile of the flaw size distribution at 50% confidence [2].

In a technical sense, this approach is analogous to random variable (RV) growth rate model of degradation [5], which is frequently used in engineering due to its simplicity. The basic idea of RV model is that since different tubes in the population experience varying rates of degradation (R), it can be treated as a random variable with an appropriate probability distribution, such as lognormal distribution recommended in [2]. The flaw growth with time is typically modeled as a linear function of time, i.e.,

$$X(t) = Rt \tag{3}$$

The random rate (RV) model implies that although the tube specific growth rates in a group of j tubes, $r_1, r_2, \cdots, r_j$, come from a distribution, the rate r_i for a specific i^{th} tube is a fixed number. A specific growth rate r_i is obtained by dividing the difference of wall thickness measurement between two inspections by the time interval. In other words, each tube locks into a fixed rate with which the degradation progresses through out the operating cycle of the tube.

2.2.2 Approach 2

There is an alternate formulation for predicting the flaw growth distribution, $f_X(x;\ t)$, at the end of a time interval (t) as a sum of the two random variables, namely, current flaw size (X) distribution, $f_{X0}(x)$, and that of the flaw growth in this time interval, $f_G(y;\ t)$. The sum of two random variables is written as the following convolution [3, 6]:

$$f_X(x;t) = \int_0^x f_{X0}(x-y)\, f_G(y;t)\,dy \tag{4}$$

This approach implies a cumulative stochastic (or random) process model of the flaw growth. If suppose the interest is in predicting the flaw growth, $X(k)$, after k EFPY (Effective Full Power Year), then it is written as a sum of k variables:

$$X(k) = X_1 + X_2 + ... + X_k \tag{5}$$

where X_i can be modelled as independent and identically distributed random variables. The distribution of the total growth will be a convolution) of k distributions. It was suggested to use the exponential distribution for modeling flaw growth in a unit time interval, such that the growth in k intervals follows a gamma distribution [6]. The time dependent changes in the flaw growth can be easily included in this model.

When the flaw growth per interval is modeled as a gamma distributed with a constant scale parameter and time dependent shape parameter, this model (Eq. 5) is analogous to stochastic gamma process (GP) [5].

2.3 Estimation of the flaw growth rate

Suppose a steam generator is inspected at inspection at times t_1 and t_2 and the measured flaw sizes are denoted by random variables, Y_1 and Y_2, respectively. Suppose the corresponding random sizing errors are denoted as E_1 and E_2, and actual flaw sizes as X_1 and X_2, then the following relations hold:

$$Y_1 = X_1 + E_1 \quad \text{at time } t_1$$

$$Y_2 = X_2 + E_2 \quad \text{at time } t_2 \tag{6}$$

Because of random sizing error, the actual flaw size (x) corresponding to a measured value of y is not known, and it remains a random variable in a technical sense. Here, the measured (R_M) and actual growth rates (R) are defined as

$$R_M = \frac{(Y_2 - Y_1)}{t} \quad \text{and} \quad R = \frac{(X_2 - X_1)}{t} \tag{7}$$

The measured rate can have negative values because of random nature of the sizing error, whereas the actual rate cannot be negative. Since the distribution of measured size is a convolution of the distributions of actual flaw size and the NDE error, a precise statistical estimation of actual growth rate distribution from the measured data is not a straightforward task. The nuclear industry has therefore developed simplified approaches for this purpose, and they are discussed below.

2.3.1 Approximate Method 1

The first approach is simply to ignore the NDE sizing error and estimate the rates from the measured values. In this case, negative rate values are deleted from the sample and then a suitable probability distribution is fitted to the data, such as the exponential or gamma distribution [3, 6].

2.3.2 Approximate Method 2

A more refined simulation-based approach was proposed in [2], assuming that actual rate and the sizing error follow lognormal and normal distributions, respectively. The sizing error has zero mean (i.e., unbiased measurements), $\mu_E = 0$, and a standard deviation of σ_E. From Eq.(7), the measured and actual rates are related as

$$R_M = R + \frac{(E_2 - E_1)}{t_2 - t_1} \tag{8}$$

Firstly, the measured rates are calculated using a sample of periodic inspection data. Assuming that that negative measured rates correspond to very small flaw growth, R is ignored such that the lower tail of the measured rate can be approximated by the normal distribution with a standard deviation of $\sigma_E \sqrt{2}/(t_2 - t_1)$. The following formula is recommended to calculate the standard deviation of the sizing error

$$\sigma_E \approx \frac{(r_{M50} - r_{M05})}{2.33(t_2 - t_1)} \tag{9}$$

Because the sizing error has zero mean, the mean of actual rate is the same as that of the measured rate ($\mu_R = \mu_{RM}$).Assuming a value of the standard deviation of the actual rate (σ_R), samples of R and E are simulated from lognormal and normal distributions, respectively, and added appropriately (Eq.8) to come up with a simulated sample of measured rate. The cumulative distribution function of the simulated sample is graphically compared with that of the actual measured data, and σ_R is varied till the two sets match fairly closely. The details of this method are given in [2].

3. Proposed method

Current methods for the estimation of growth rate are based on heuristic approaches, which may provide acceptable approximations in the near term. However, their accuracy for long term life-cycle management planning is not known. This is a main motivation for the current study.

This paper presents a cumulative degradation model, Eq.(5), to model flaw growth in the SG tubing, which is similar in the spirit to that proposed by in [3, 6]. The assumption of cumulative damage model is that the flaw growth in different time intervals is random and independent of other intervals. Furthermore, a sound method based on maximum likelihood method is presented for incorporating the NDE sizing error in the estimation of the growth rate distribution.

3.1 Gamma process (GP) model

A special case of cumulative damage model, the stationary gamma process (GP), is proposed in which the flaw growth in a unit interval is modelled as a gamma distributed random variable. The total flaw size at the end of a t year interval is a gamma distributed with probability density function (PDF) [5]

$$f_{X(t)}(x) = \frac{(x/\beta)^{\alpha t-1}}{\beta\Gamma(\alpha t)} \exp(-x/\beta) = \mathrm{ga}(x;\alpha t,\beta) \tag{10}$$

where α and β are shape and scale parameters, respectively. Also, $\mathrm{ga}(x;a,b)$ is a concise notation for the gamma PDF with α and β as shape and scale parameters, respectively. The mean and the coefficient of variation (COV) of the flaw size $X(t)$ is given as

$$\mathrm{E}[X(t)] = \alpha\beta t = \mu t, \quad \mathrm{COV}[X(t)] = \frac{1}{\sqrt{\alpha t}} = \frac{\nu}{\sqrt{t}} \tag{11}$$

The distribution of flaw growth rate (for t = 1) is also gamma distributed with mean, $\mu = \alpha\beta$, and COV $\nu = 1/\sqrt{\alpha}$.

3.2 Statistical parameter estimation

This section presents the maximum likelihood approach to estimate the parameters of the GP model (α and β) given periodically obtained sample of flaw size (Y). The likelihood function is formulated using the property of independent gamma distributed increments [7].

The actual inspection data usually consist of a series of successive measurements of a flaw sizes in the SG tube population, denoted as $\mathbf{y} = \{y_1, y_2, \cdots, y_n\}$. Here, $y_i = x_i + e_i$ is an i^{th} measurement of the flaw size is contaminated by the sizing error e_i, which is usually assumed to be normal distributed with zero mean and a given standard deviation σ_E. The actual flaw in the i^{th} inspection is denoted as x_i. Increments are denoted as: $\Delta y_i = y_{i+1} - y_i$, $\Delta x_i = x_{i+1} - x_i$, $\Delta e_i = e_{i+1} - e_i$, and their samples are denoted as $\mathbf{\Delta y} = \{\Delta y_1, \Delta y_2, \cdots, \Delta y_n\}$, $\mathbf{\Delta x} = \{\Delta x_1, \Delta x_2, \cdots, \Delta x_n\}$ and $\Delta\mathbf{e} = \{\Delta e_1, \cdots, \Delta e_n\}$. Obviously, $\mathbf{\Delta y} = \mathbf{\Delta x} + \mathbf{\Delta e}$.

The likelihood function of parameters α and β in terms of true flaw growth increments, $\mathbf{\Delta x}$, can be written as [7]

$$L_X(\alpha, \beta \mid \mathbf{\Delta x}) = \prod_{i=1}^{n} \mathrm{ga}(\Delta x_i; \alpha\Delta t_i, \beta) = \prod_{i=1}^{n} \frac{(\Delta x_i / \beta)^{\alpha\Delta t_i - 1}}{\beta\Gamma(\alpha\Delta t_i)} \exp(-\Delta x_i / \beta)$$

In order to incorporate the effect of sizing error, the likelihood function including the measured data is written in a conditional form, and then using the total probability theorem, it can be written as

$$L_Y(\alpha, \beta \mid \mathbf{\Delta y}) = \int_{\mathbf{\Delta X}} L_X(\alpha, \beta \mid \mathbf{\Delta x}) f(\mathbf{\Delta y} \mid \mathbf{\Delta x}) d\mathbf{\Delta x}$$

Note that $f(\mathbf{\Delta y} \mid \mathbf{\Delta x})$ is the conditional probability density function of $\mathbf{\Delta y}$ given $\mathbf{\Delta x}$. Using the relation $\mathbf{\Delta y} = \mathbf{\Delta x} + \mathbf{\Delta e}$ and the independence of $\mathbf{\Delta x}$ and $\mathbf{\Delta e}$, it can be rewritten in a form that is amenable to the solution by Monte Carlo integration method (See Appendix 7.1):

$$L_Y(\alpha, \beta \mid \mathbf{\Delta y}) = \int_{\mathbf{\Delta E}} L_X(\alpha, \beta \mid \mathbf{\Delta y} - \mathbf{\Delta e}) f(\mathbf{\Delta e}) d\mathbf{\Delta e} \qquad (12)$$

The maximum likelihood method is considered as the most versatile and rational approach for parameter estimation, and its application has become easier with the availability of advanced scientific computation packages.

4. Results

4.1 Simulation-based analysis

The impact of sizing error is illustrated using inspection data simulated from a gamma process model of flaw growth. Consider an example in which a SG has 10 tubes with one fretting wear flaw in each tube. The flaw growth follows a gamma process with mean growth rate of $\mu = 2$% tw/EFPY and the COV of $\nu = 0.4$ (Gamma process parameters are $\alpha = 6.25$ and $\beta = 0.32$). It is assumed that each tube is inspected 4 times at a 2 EFPY interval. In the simulation, 10 growth curves are randomly generated and then 4 points, each at 2 EFPY interval, are selected from each curve as true flaw sizes. Corresponding to each flaw size, the NDE sizing error is simulated from a normal distribution with zero mean and a specified value of σ_E, and it is added to true flaw size to come up with a measured flaw size. This way, one sample of 40 flaw measurements is simulated in one cycle of the Monte Carlo analysis.

The maximum likelihood method is applied to estimate the gamma process parameters, first considering the sizing error and then ignoring it from the likelihood function. From one estimated set of parameters, the 95th percentile of flaw growth in a 2 EFPY future interval is computed. Suppose it is denoted as z_{EST}. Since the parameters of the gamma process generating the data in simulations are known, the exact 95th percentile of flaw growth in the same period can be exactly computed, which is z_{EX} =6 %tw. Thus, he prediction error in one simulated sample is defined as $e = (z_{EST} - z_{EX})$. In summary, an i^{th} simulated sample provides an estimate of the prediction error, e_i. Simulations were repeated 200 times, and root mean-square error (RMSE) of the prediction was evaluated.

A plot of RMSE of predicted flaw growth as a function of the standard deviation of sizing error is shown in Figure 1. It is clear that ignoring sizing error will increase the statistical error associated with future flaw growth prediction. For example, when sizing error σ_E = 5 %tw, the RMSE associated with predicted flaw growth in 2 EFPY can be as high as 8 %tw. This prediction error will lead to further errors in estimating future inspection schedule and the number of tube plugging.

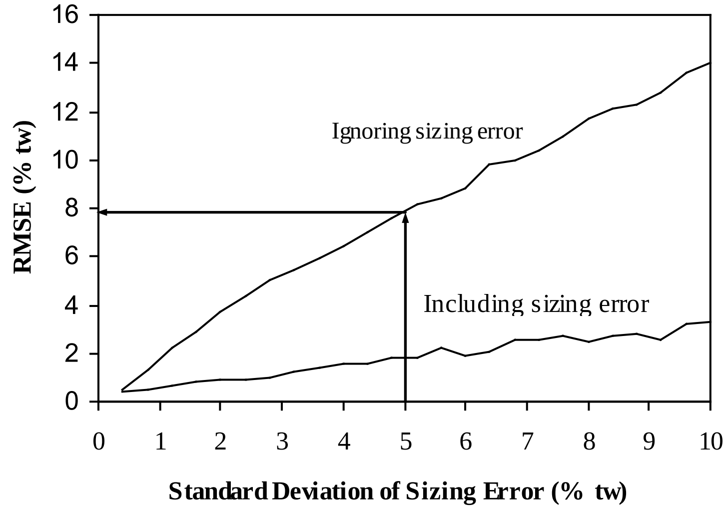

Figure 1: Root mean-square error (RMSE) associated with flaw growth prediction

4.2 Practical case study: Wear of SG tubing

Fretting and wear degradation is caused by mechanical interactions between neighbouring tubes, tubes and anti vibration bars and supports. Flow induced vibrations, which can be highly variable at a local scale, contributes to fretting and wear of SG tubing.

This section presents a case study involving the assessment of fretting wear of SG tubes in a nuclear station. The periodic inspections of 4 SGs were carried out over a 15 year period, which revealed 120 tubes with fretting wear. Several tubes with wear were inspected 2 to 4 times, which provides good data for growth rata analysis. Details of Inspection data are shown in Figure 2 and Table 1. in case of wear, the maximum tolerable flaw size is taken as 71% tw [3].

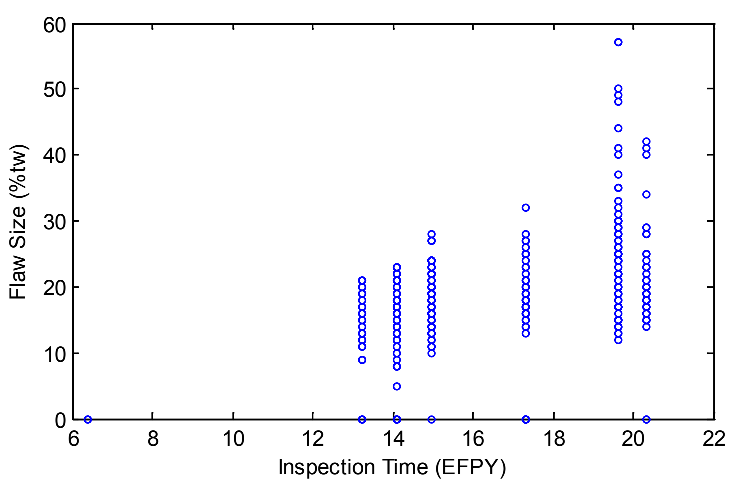

Figure 2: SG tubing wear data from a nuclear reactor

Table 1: Detected worst flaws in four steam generators

SG No.	SG1	SG2	SG3	SG4
Number of tubes with fretting wear	17	29	32	42
worst-case flaw depth (%tw)	42%	31%	29%	57%

4.2.1 Prediction of flaw growth

Table 2: Features of the methods for analyzing NDE sizing error

Method	Model	Data Required	Method to account for the sizing error
Approximate method 1	Growth rate - gamma distribution	Only the first and the last measurements are used	all negative rates are set to zero
Approximate method 2	Growth rate - lognormal distribution	Only the first and the last measurements are used	Monte Carlo simulations and visual comparison
Proposed method	Gamma process model	All periodic measurements are used	Maximum likelihood method

Table 3: Summary of annual flaw growth distributions obtained from the three methods

Method	Statistics of annual flaw growth (%tw/EFPY)	95th percentile (upper bound) of annual flaw growth (%tw/EFPY)
Approximate method 1 (Gamma distribution)	mean: 2.32 cov: 0.82	6.05
Approximate method 2 (Lognormal distribution)	mean: 1.75 cov: 1.45	5.72
Proposed method (Gamma process)	mean: 1.43 cov: 2.10	7.08

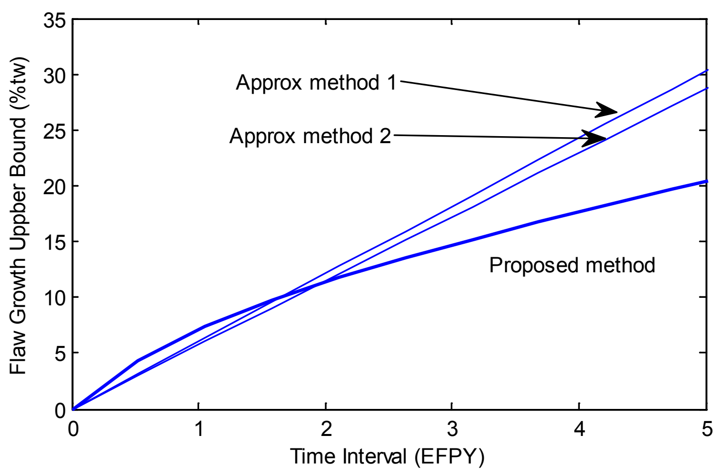

Figure 3: Comparison of 95th upper bound of predicted flaw growth over time

Three methods were used to analyse the data obtained from a nuclear station. Features of the three methods are compared in Table 2. The statistics of annual flaw growth obtained from the three methods as summarized in Table 3, and the upper bound of predicted flaw growth (95% percentile) over a time interval (0-*t*] is plotted in Figure 3. In the maximum likelihood analysis, the standard deviation of sizing error was assumed as σ_E=5 %tw. It is the same value as that estimated by the lognormal model in Approximate Method-2 [2].

It is interesting that in short term ($t \leq 2$ EFPY), predictions of all the three methods are reasonably close. However, in medium term, $t \geq 4$ EFPY, predictions of the two approximate methods can be as high as 30 %tw in comparison to that predicted by the proposed model. The reason is that in long term, the standard deviation of the flaw growth in gamma process model is a function of $\sqrt{t}$, whereas the other two methods use the RV model in which the standard deviation is a linear function of time *t*.

It is interesting that the results of the first approximate method are quite close to that of the second, more refined approximation. It is in spite of the fact that the first method ignores the sizing error and negative values of the measured rate from the data analysis.

4.2.2 Prediction of the time for next inspection

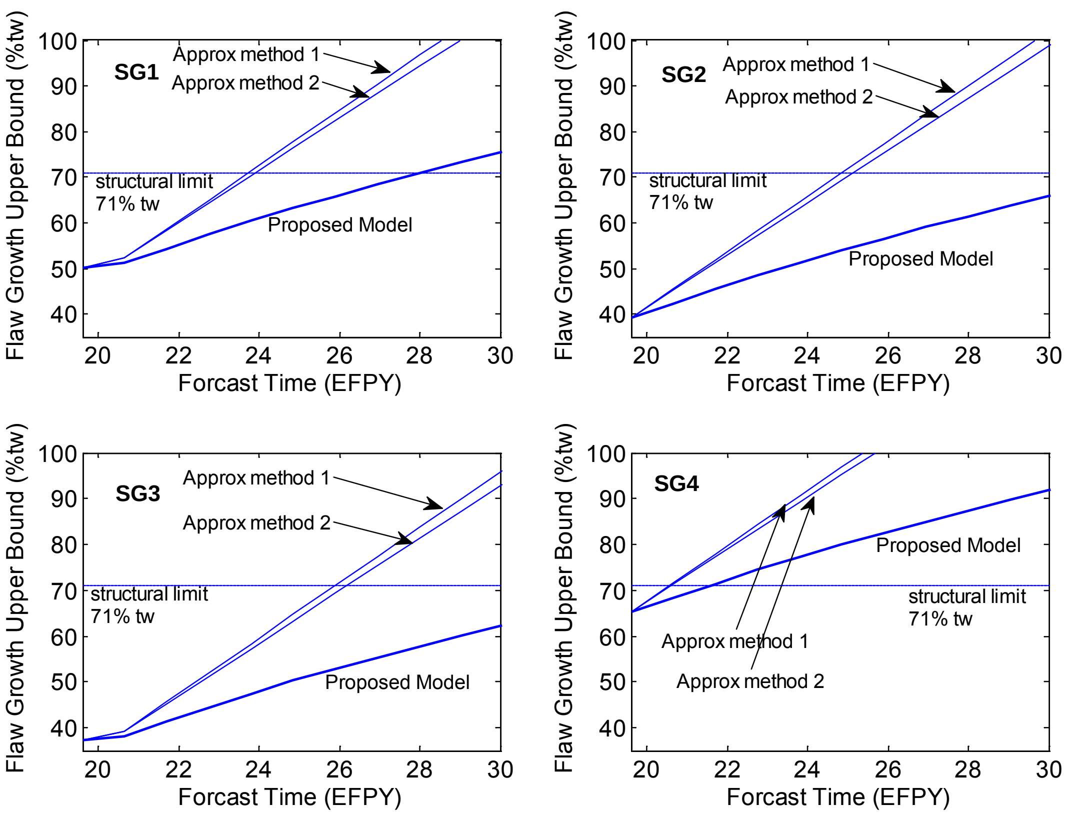

Figure 4: Upper bound growth of the worst-case flaw in four steam generators

The time for next inspection is calculated as the time during which an existing worst flaw in a SG will grow to reach the specified MTFS of 71 %tw. The approximate methods use the following approach to predict the upper bound growth of the worst flaw:

$$a_{WS}^{U}(t) = a_{WS} + r_{UP}t + a_{ERR}^{U} \tag{13}$$

It is should be noted here that the sum of the two 95th percentiles (r and a_{ERR}) is not necessarily equal to the true 95th percentile of the sum of the variables (i.e., a_{WS}). In this sense, Eq.(13) is another approximation associated with the methods currently used by the industry. In the proposed gamma process model, the 95th percentile (upper bound) of the worst flaw growth is obtained in a correct way by evaluating a convolution equation, as shown in Appendix 7.2.

Using the data for the measured size of worst flaws (denoted as a_{WS}) given in Table 1, the growth curves obtained from the three methods are plotted in Figure 4, and predictions for the time for next inspection are compared in Figure 5.

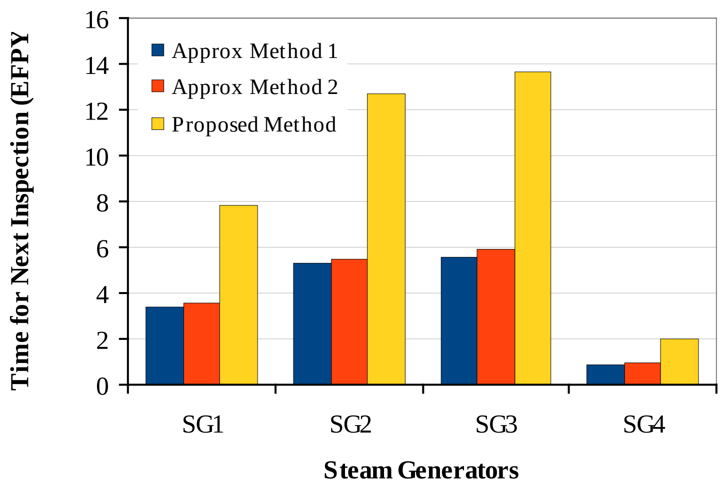

Figure 5: Predicted time for next inspection

The inspection interval predicted by the proposed model is almost twice than that predicted by the approximate methods. Figure 5 shows that approximate methods are quite conservative in nature. The two approximate methods lead to are fairly close predictions, which indicates that the extra analysis involved with the second approximate method [2] has a limited payoff in terms of improving the accuracy of analysis.

5. Summary and conclusions

In this paper, a probabilistic approach is presented for the prediction of future flaw growth in support of operational assessment and life cycle management of steam generators.

The paper first discusses two existing approximate methods used by the nuclear industry for predicting the flaw growth and the time for next inspection. The approximate methods account for the NDE sizing error in a heuristic manner. The first method ignores the negative rates and evaluates the growth rate based on measured data. The second method approximately estimates the standard deviation of the sizing error (σ_E) based on heuristic assumptions.

The paper presents a gamma process model for the flaw growth in SG tubing, which is similar in spirit of the approach suggested in the fitness for service guidelines [3] and it generalizes the formulation of Carmacho and Pagan [6]. The proposed method accounts for the NDE sizing error in a probabilistically consistent manner in the maximum likelihood formulation.

A simulation-based study shows that a relatively small sizing error can lead to large error in the flaw growth prediction. For example, when sizing error σ_E = 5 %tw, the RMSE associated with predicted flaw growth in 2 EFPY can be as high as 8 %tw. Such an error can increase the cost of tube plugging or shorten the inspection interval.

A detailed case study is presented based on the periodic inspection data for fretting wear in SG tubing collected from a Canadian nuclear station. The case study shows that in short term ($t \leq 2$ EFPY), flaw growth predictions by all the three methods are reasonably close (Figure 2). In medium term however, $t \geq 4$ EFPY, the predictions of the two approximate methods can be as high as 30 %tw in comparison to that predicted by the proposed method. A comparison of the predictions of the time for next inspection further highlights the impact of NDE sizing error. The inspection interval predicted by the proposed model is almost twice than that predicted by the approximate methods (Figure 5).

It is interesting that the results of the first approximate method [6] are quite close to that of the second, more refined approximation [2] in this case study. It is in spite of the fact that the first method ignores the sizing error and negative rate data from the analysis. Within the context of the data used in the case study, the results indicate that the extra analysis involved with the second approximate method [6] has a limited payoff in terms of improving the accuracy of the analysis.

It is concluded that approximate methods used by the industry to model inspection uncertainties are conservative, which can impact adversely on the effectiveness of life cycle management plans. The prediction accuracy can be significantly improved by the proposed gamma process model including the NDE sizing error.

6. References

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7. Appendices

7.1 Evaluation of the likelihood equation

Likelihood function (12) is a high dimensional integration because Δe_i are dependent. In fact, the correlation coefficient between Δe_i and Δe_{i-1} is

$$\rho_i = \frac{\operatorname{cov}(\Delta e_i, \Delta e_{i-1})}{\sigma_{\Delta e_i}\sigma_{\Delta e_{i-1}}} = \frac{\operatorname{cov}(e_{i+1}-e_i, e_i-e_{i-1})}{2\sigma_e^2} = -\frac{1}{2}.$$

Eq (12) can then be approximated by the following summation:

$$\begin{aligned} L_Y(\alpha,\beta\,|\,\mathbf{\Delta y}) &\approx \frac{1}{N}\sum_{j=1}^{N}\left[L_X(\alpha,\beta\,|\,\mathbf{\Delta y}-\mathbf{\Delta e}^{(j)})\right] \\ &= \frac{1}{N}\sum_{j=1}^{N}\left[\prod_{i=1}^{n}\frac{(\Delta y_i-\Delta e_i^{(j)}/\beta)^{\alpha\Delta t_i-1}}{\beta\Gamma(\alpha\Delta t_i)}\exp-(\Delta y_i-\Delta e_i^{(j)})/\beta\right], \end{aligned} \tag{13}$$

where $\mathbf{\Delta e}^{(j)} = \{\Delta e_1^{(j)}, \Delta e_2^{(j)}, \cdots, \Delta e_n^{(j)}\}$, $j = 1, 2, \cdots, N$, are N set of samples of measurement error increments.

Using the relation between Δe_i and e_i, equation (13) can be calculated as follows:

1. Generate N sets of inspection error samples $\{e_1^{(j)}, e_2^{(j)}, \cdots, e_{n+1}^{(j)}\}$, $j = 1, 2, \cdots, N$, from normal distribution with zero mean and standard deviation σ_e,

2. Samples of measurement error increments are calculated using $\Delta e_i^{(j)} = e_{i+1}^{(j)} - e_i^{(j)}$;

3. Substitute $\Delta e_i^{(j)}$ in equation (13) to obtain an approximate value $L_Y(\alpha,\beta\,|\,\mathbf{\Delta y})$.

The above steps are repeated for a number of times to ensure that the number of samples *N* is sufficiently large to achieve the required accuracy for the likelihood function.

7.2 Prediction of future flaw growth in proposed model

Assume that the size (or depth) of the worst flaw existing in a steam generator is measured as y_w in the latest inspection at time *t*. In order to calculate the time for next inspection, the future growth of this flaw needs to be predicted. The true size of this measured flaw at current time time *t* is $X(t) = y_w - E$, where *E* is the normal distributed sizing error. The true flaw size, *X*(*t*), is now a random quantity, because the actual sizing error added to this measurement is not known. The flaw growth after a time interval, Δt, is given by:

$$X_w(t+\Delta t) = y_w - E + X(\Delta t)$$

where $X(\Delta t)$ is the flaw growth during time interval Δt.

Define a new ransom variable, $P = y_w - E$, where *P* is normal distributed with mean y_w and standard deviation σ_E. Since *P* and $X(\Delta t)$ are independent, the future flaw growth is a sum of two random variables, i.e., $X_w(t+\Delta t) = P + X(\Delta t)$. In case of gamma process model, the cumulative distribution of $X_w(t+\Delta t)$ is given as

$$\begin{aligned} F_{X_w(t+\Delta t)}(x) &= \iint\limits_{P+X(\Delta t)\le x} f_{X(\Delta t)}(u)\, f_P(v)\,\mathrm{d}u\mathrm{d}v \\ &= \iint\limits_{u+v\le x} ga(u;\alpha\Delta t,\beta)\phi(v;y_w,\sigma_e)\,\mathrm{d}u\mathrm{d}v \end{aligned}.$$

Here $\phi(x;\mu,\sigma)$ is PDF of normal distribution with mean ' and standard deviation σ. The PDF of the growth of the worst flaw $X_w(t+\Delta t)$ is given as

$$f_{X_w(t+\Delta t)}(x) = F'_{X_w(t+\Delta t)}(x) = \int_{-\infty}^{\infty} \mathrm{g}\,a(u;\alpha\Delta t,\beta)\phi(x-u;y_w,\sigma_e)\,\mathrm{d}u\,. \tag{14}$$

The 95th percentile of this distribution is taken as the predicted flaw growth including the sizing error. Note that this value will be different from the algebraic sum of 95th upper bounds of the sizing error and growth distribution, as adopted by approximate methods used by the industry.