An analytical approximation for the prediction of transients with temperature feedback

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Abstract

A new approximation solution of the point kinetics equation system for a group of precursors with temperature feedback is presented. The formulation proposed consists of the expansion of neutron density in power of the generation time for prompt neutrons and presents explicit dependence in time. The results obtained displayed small deviations in relation to the numerical reference solution and systematically better than other analytical methods found in the literature, despite the simplicity of its functional form and ease of implementation.

1. Introduction

The analytical solution of point kinetics equations with a group of delayed neutrons is useful in predicting the variation of neutron density during the operation of a nuclear reactor. Apart from neutron density, other variable quantities in time such as the reactor operating temperature and the concentration of delayed neutron precursors are essential for the control and design of reactor core [1].

In order to predict how these quantities will vary in time some methods, numerical [2,3] and analytical [5,6,7] have been under development to solve the point kinetics equations also considering the effect of the temperature feedback to the adiabatic and Newtonian models.

The point kinetics equations with a group of precursors considering that the temperature feedback is adiabatic are written as follows:

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{l} n(t) + \lambda C(t) \tag{1}$$

$$\frac{dC(t)}{dt} = \frac{\beta}{l} n(t) - \lambda C(t) \tag{2}$$

$$\rho(t) = \rho_0 - \alpha\left[T(t) - T_0\right] \tag{3}$$

$$\frac{dT(t)}{dt} = K_c n(t), \tag{4}$$

where $n(t)$ is the neutron density, $\rho(t)$ is the reactivity, β is the total fraction of the delayed neutrons, l is the generation time for prompt neutrons, λ is the decay constant for

delayed neutron precursors, $C(t)$ is the mean density for delayed neutron precursors, α $(\alpha > 0)$ is the reactivity coefficient and K_c is the thermal capacity of the reactor.

In the present paper a new approximated solution for the point kinetics equation system with temperature feedback is presented considering the adiabatic model as in Nahla's paper [7]. Those solutions are valid for PWR and are based on the expansion of the neutrons density in terms of the generation time of prompt neutrons and presents the advantage of being explicit in time and having a simple functional form in comparison with other existing formulations.

2. A New Solution of the Point Kinetics Equations with Temperature Feedback

From equations (1) and (2) it is possible to eliminate the explicit dependency of the concentration of delayed neutrons and write the following equation for the neutron density:

$$l\frac{d^2n(t)}{dt^2}+\left[\beta+\lambda l-\rho(t)\right]\frac{dn(t)}{dt}-\left[\frac{d\rho(t)}{dt}+\lambda\rho(t)\right]n(t)=0\,, \tag{5}$$

subjected to the initial conditions,

$$\begin{cases} n(0)=n_0 \\ \dfrac{dn(0)}{dt}=0 \end{cases}. \tag{6}$$

In expanding the concentration of neutrons in terms of powers of the generation time for prompt neutrons and considering that $l >> l^2$ and $l >> \alpha l$ one can write, until the first order:

$$n(t)=n_1(t)+ln_2(t)\,, \tag{7}$$

and

$$\frac{d\rho(t)}{dt}=-\alpha K_c n_1(t). \tag{8}$$

By replacing equations (7) and (8) in equation (5) one obtains that functions $n_1(t)$ and $n_2(t)$ are expressed, respectively, as solution for the differential equations below:

$$\left[\beta-\rho(t)\right]\frac{dn_1(t)}{dt}=\left[\frac{d\rho(t)}{dt}+\lambda\rho(t)\right]n_1(t) \tag{9}$$

$$\left[\rho(t)-\beta\right]\frac{dn_2(t)}{dt}+\left[\frac{d\rho(t)}{dt}+\lambda\rho(t)\right]n_2(t)=\frac{d^2n_1(t)}{dt^2}+\lambda\frac{dn_1(t)}{dt}\,. \tag{10}$$

The expression for can be obtained from equations (8) and (9) [7], allowing one to write the following differential equation:

$$\frac{dn_1}{d\rho} = \frac{\alpha K_c n_1 - \lambda\rho}{\alpha K_c(\beta - \rho)}, \tag{11}$$

which, when integrated with the initial conditions $\rho(0) = \rho_0$ and $n(0) = n_0$ provides the following expression for function $n_1(t)$:

$$n_1 = \frac{\lambda}{2\alpha K_c}\left(\frac{\sigma^2 - \rho^2}{\beta - \rho}\right), \tag{12}$$

where $\rho_0 < \beta$ and $\sigma = \sqrt{\rho_0^2 + \frac{2}{\lambda}\alpha K_c n_0(\beta - \rho_0)}$ is a constant that can be approximated using Taylor series, by:

$$\sigma = \rho_0\sqrt{1 + \frac{2\alpha K_c n_0}{\lambda\rho_0^2}(\beta - \rho_0)} \approx \rho_0 + \frac{\alpha K_c n_0}{\lambda}\left(\frac{\beta}{\rho_0} - 1\right). \tag{13}$$

The approximation expressed by equation (13) is valid for parameters that are typical for PWR reactors. By replacing equation (12) in equation (8) one obtains the following differential equation:

$$\frac{d\rho(t)}{dt} = -\frac{\lambda}{2}\left[\frac{(\sigma - \rho)(\sigma + \rho)}{\beta - \rho}\right]. \tag{14}$$

Let us consider the scenario where the core of the nuclear reactor operates near the condition in that $\rho_0 \approx \beta$. This condition implies, from equation (13), that $\sigma \approx \beta$. Thus, it is possible to simplify equation (14), rewriting it as follows:

$$\frac{d\rho(t)}{dt} = -\frac{\lambda}{2}(\sigma + \rho). \tag{15}$$

In integrating equation (15) and imposing the initial condition $\rho(0) = \beta$ one obtains an expression for system reactivity with an explicit time dependency:

$$\rho(t) = (\sigma + \beta)e^{-\frac{\lambda t}{2}} - \sigma. \tag{16}$$

When the core of the nuclear reactor operates in a condition in that $\rho_0 \approx \beta$ it is possible to obtain a simple and explicit solution in the time. From equation (16) it is possible to estimate the asymptotic behaviour of the reactivity from the expression:

$$\rho_{ass}(t) = \lim_{t\to\infty} \rho(t) = -\rho_0 + \frac{\alpha K_c n_0}{\lambda}\left(1 - \frac{\beta}{\rho_0}\right). \tag{17}$$

By replacing equation (16) in equation (3) one can write an expression for the variation of the temperature of reactor core:

$$T(t) = T_0 + \left(\frac{\sigma + \rho_0}{\alpha}\right)\left(1 - e^{-\frac{\lambda t}{2}}\right). \tag{18}$$

From equation (18) it also possible to estimate the asymptotic behaviour of the temperature in the reactor core from the expression:

$$T_{ass}(t) = \lim_{t\to\infty} T(t) = T_0 + \left(\frac{\sigma + \rho_0}{\alpha}\right). \tag{19}$$

To calculate the concentration of neutrons according to the approximation proposed one should first calculate function $n_1(t)$ from equation (12):

$$n_1(t) = \frac{\lambda(\sigma + \rho_0)}{2\alpha K_c} e^{-\frac{\lambda t}{2}}. \tag{20}$$

Replacing the equations (16) and (20) and their derivates in equation (10) it is possible to write the following differential equation for the $n_2(t)$ function:

$$\frac{dn_2(t)}{dt} + \frac{\lambda}{2} n_2(t) = \frac{\Delta_1}{\Delta_2 - e^{\frac{\lambda t}{2}}}, \tag{21}$$

where $\Delta_1 = -\frac{\lambda}{8\alpha K_c}\Delta_2$ and $\Delta_2 = \left(1 + \frac{\rho_0}{\sigma}\right)$ are constant.

Equation (21) is of the ordinary kind and not homogeneous and can be written thus:

$$\frac{dn(t)}{dt} + f(t)n(t) = g(t), \tag{22}$$

being solved using the integrating factor method [8] and providing solutions written by:

$$n(t) = e^{-\int f(t)dt}\left[\int e^{\int f(t)dt} g(t)\,dt + \Omega\right], \tag{23}$$

where $f(t) = \frac{\lambda}{2}$, $g(t) = \frac{\Delta_1}{\Delta_2 - e^{\frac{\lambda t}{2}}}$ and Ω is an integration constant.

Therefore, solving equation (21) imposing the initial condition $n_2(0)=0$ one can write the following expression for the $n_2(t)$ function:

$$n_2(t)=\frac{(\sigma+\rho_0)}{4\alpha K_c\sigma}e^{-\frac{\lambda t}{2}}\ln\left[\frac{\sigma}{\rho_0}\left(1-e^{-\frac{\lambda t}{2}}\right)+1\right]. \tag{24}$$

Finally, by replacing equations (20) and (24) in equation (7) one obtains the expression for neutron density $n(t)$:

$$n(t)=A_1e^{-\frac{\lambda t}{2}}\left\{A_2+\ln\left[\frac{\sigma}{\rho_0}\left(1-e^{-\frac{\lambda t}{2}}\right)+1\right]\right\}, \tag{25}$$

where $A_1=l\frac{(\sigma+\rho_0)}{4\alpha K_c\sigma}$ and $A_2=\frac{2\lambda\sigma}{l}$.

The concentration of neutron precursors $C(t)$ can be calculated from equation (1), to produce:

$$C(t)=\frac{1}{\lambda}\left\{\frac{dn(t)}{dt}-\left[\frac{\rho(t)-\beta}{l}\right]n(t)\right\}, \tag{26}$$

where reactivity $\rho(t)$ is calculated from equation (16) and neutron density $n(t)$ and its derivate are obtained from equation (25).

3. Results

The results obtained in this paper represent a solution of the point kinetics equations with a group of precursors considering the temperature feedback in a transient situation that nears the condition $\rho_0\approx\beta$.

In all of the simulations the supercritical process in a PWR reactor will be considered, using as fuel material ^{235}U and assuming that $\beta=0.0065$, $l=0.0001s$, $\lambda=0.07741s^{-1}$, $K_c=0.05K/MW.s$, $T_0=300K$ and $\alpha=5\times10^{-5}{}^0C^{-1}$. The numerical method of finite differences will be employed as the reference.

Graphs 1 until 4 show, respectively, the behaviour of the variation of the reactivity, of the temperature, of neutron density and of neutron precursors, as obtained from the formulations proposed in this paper, equations (16), (18), (25) and (26), and the method proposed by Nahla (2009) for $\rho_0=0.95\beta$.

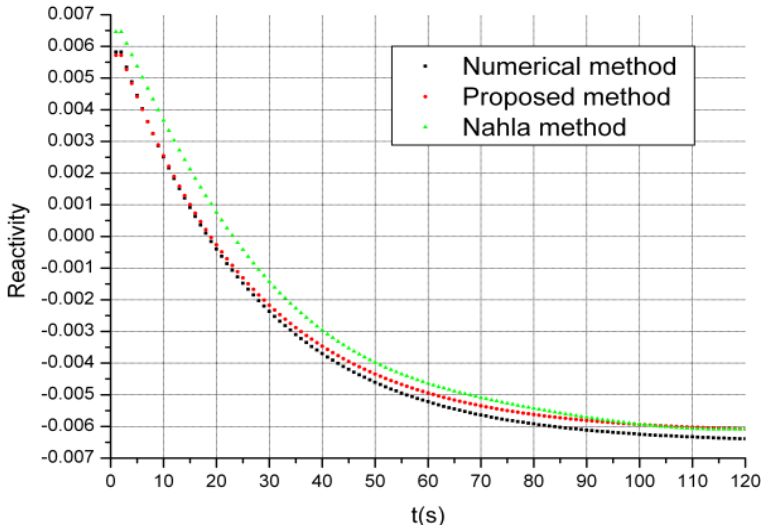

Figure 1 The reactivity calculated by equation (16).

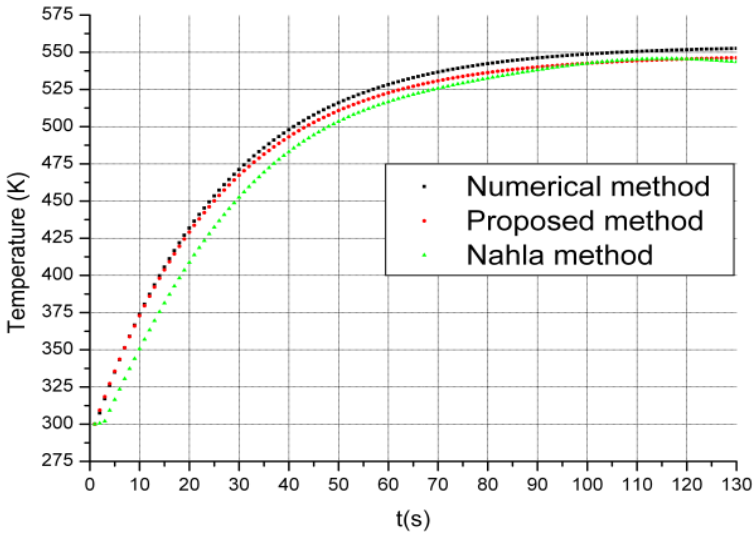

Figure 2 The temperature calculated by equation (18).

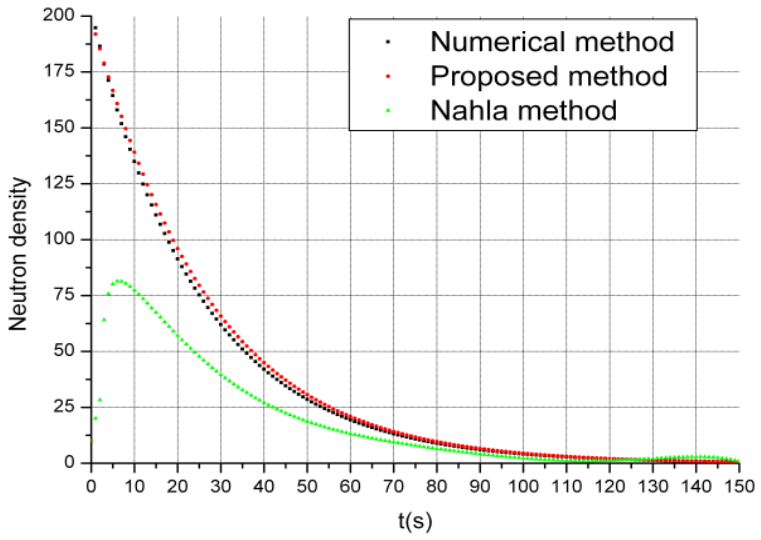

Figure 3 The neutron density calculated by equation (25).

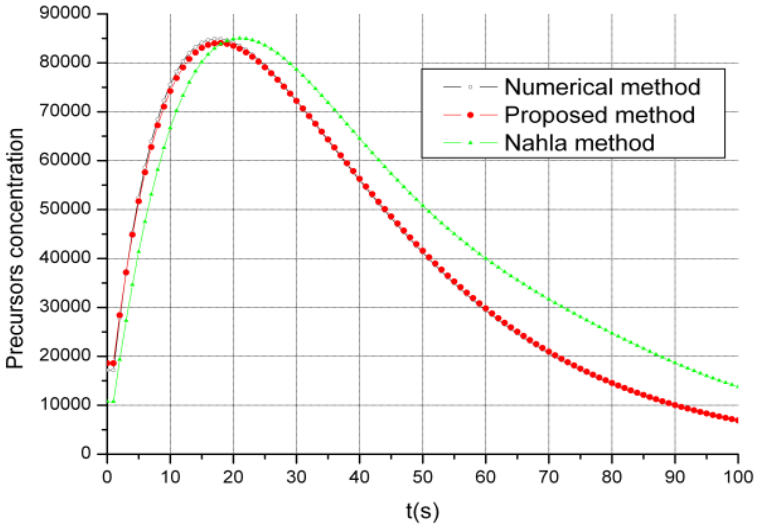

Figure 4 The precursors neutron density calculated by equation (26).

In all of the cases the method proposed showed to be accurate with the advantage of being simpler in its implementation due to the explicit dependency of the time in its functional form. It is possible to see that the method proposed overlaps the reference numerical method in the calculation of function $C(t)$.

4. Conclusions

An analytical approximation was developed in this paper in the attempt to predict the neutron density $n(t)$ during a condition in that $\rho_0 \approx \beta$, considering a group of delayed neutron precursors and temperature feedback. The formulation proposed consists of the solution of the point kinetics equations for a group of precursors from the expansion of neutron density in power of parameter l. Although there are different solutions for point kinetics equations with temperature feedback, one of the advantages of the method proposed is to provide analytical approximations with an explicit time dependency for important quantities as regards the design and operation of the nuclear reactor such as neutron density, concentration of delayed neutron precursors and temperature of the reactor core. One of the applications of the expressions presented consists of the prediction of transients near to $\rho_0 \approx \beta$ considering temperature feedback and the adiabatic model. The results obtained present small deviations in relation to the numerical reference solution.

5. References

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