NEUTRON DIFFUSION WAVES IN CANDU® REACTORS

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Abstract

Neutron noise, i.e. the difference between the time dependent neutron flux and its average, is used for diagnostic and monitoring purposes in nuclear reactors. Neutron diffusion waves, which can be used for noise analysis, describe the propagation of neutron flux disturbances in a nuclear reactor.

Neutron diffusion waves allow, in principle, the determination of sources of neutron flux variations in a nuclear reactor, so they can be used as an additional diagnostic tool. A model was developed to explore the properties of such waves.

Preliminary results on neutron diffusion waves in CANDU®[1] reactors are presented. The results include information on dispersion relations for the neutron waves and interference effects involved in neutron wave propagation.

1. Introduction

Neutron noise is defined as the difference between the time-dependent neutron flux and its time-averaged value. The neutron noise is already used to investigate internal processes in nuclear reactor cores. Noise is often Fourier analyzed in the frequency domain. The traditional methods of noise analysis make it hard to detect and analyze the origins of the neutron noise.

A theory that connects the origin of the noise to the noise itself has twofold importance: before an experiment, it can give an estimation of how a perturbation induces a measurable response and the characteristics of such a response. This paper explores such a theory, that is, the properties of a space dependent reactor response function similar to the one introduced in References [1] and [2]. Behaviour of the response function is investigated at frequencies ranging from 50 Hz to 500 Hz, comparable to the sampling frequency of existing noise analysis systems. The reactor lattice cell inhomogeneity has to be taken into account for estimating the effects of neutron wave interference [4].

A frequency-dependent response function is defined as the term $R(\omega,\vec{r},\vec{r}\,')$ in the long-time response at position $\mathbf{r}$, $\Phi(\omega,\mathbf{r}) = R(\omega,\mathbf{r},\mathbf{r}')\exp(i\omega t)$ of a system to harmonic perturbation, $s = \exp(i\omega t)$ at position $\mathbf{r}'$. Such a function can be considered as the response of the neutron flux at a given position in the reactor to a harmonic perturbation at another position in the reactor. This response function is the Fourier transform of the time-dependent Green's function, that is, the function describing the propagation of a neutron wave from a point source at $\mathbf{r}'$ to another location at point $\mathbf{r}$.

Frequency analysis of the neutron noise is common, and the results are often present using the Fourier transforms of the noise at a given point in the reactor. The Fourier transform neutron flux

[1] CANDU is a registered trademark of AECL.

response at several points and the space-dependent reactor response function can be used to estimate the perturbations in the reactor causing the neutron noise, which is a means to analyse noise.

The response function approach is a particular case of the to the neutron transport/diffusion perturbation method, that is, the source of noise is assumed to be small. The method described here applies well in the cases when the response to the perturbation is hard to factor in space-dependent and time dependent factors, such as the cases when the reactor dynamics is hard to reduce to a point reactor model or a flux harmonics model. The source of the perturbation is small, but the perturbation to the diffusion/ transport operator may be significant.

This paper addresses the results obtained for the reactor response function using a simplified model of the CANDU® reactor core. The objective of the reactor response function analysis is to explore the options available to investigate the sources of noise using flux measurements.

2. On the Calculation of the Response Function

The neutron transport equation and the delayed fractions equations:

$$\begin{aligned}&\frac{1}{\mathrm{v}(E)}\frac{\partial\Psi(\mathbf{r},\mathbf{\Omega},E,t)}{\partial t}+\mathbf{\Omega}\nabla\Psi(\mathbf{r},\mathbf{\Omega},E.t)+\Sigma_t(\mathbf{r},E)\Psi(\mathbf{r},\mathbf{\Omega},E,t)=\\&\int_{\mathbf{\Omega}',E'}\Psi(\mathbf{r},\mathbf{\Omega}',E',t)\Sigma_s(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega},E'\to E)d\mathbf{\Omega}'dE'\\&+\frac{\nu(E)}{k_{eff}}\left[(1-\beta)\int_{\mathbf{\Omega}',E'}\Psi(\mathbf{r},\mathbf{\Omega}',E',t)\Sigma_f(\mathbf{r},E')d\mathbf{\Omega}'dE'+\sum_{k=1}^{g}\lambda_kC_k(\mathbf{r},t)\right]+S(\mathbf{r},\mathbf{\Omega},E.t)\end{aligned}\tag{1}$$

$$\frac{\partial C_k}{\partial t}=-\lambda_kC_k++\frac{\beta_k}{k_{eff}}\int_{\mathbf{\Omega}',E'}\Psi(\mathbf{r},\mathbf{\Omega}',E')\Sigma_f(\mathbf{r},E')d\mathbf{\Omega}'dE'\tag{2}$$

are Fourier transformed, resulting in the following equation:

$$\begin{aligned}&\mathbf{\Omega}\nabla\Psi(\omega,\mathbf{r},\mathbf{\Omega},E)+\underline{\left(\frac{i\omega}{\mathrm{v}}+\Sigma(\mathbf{r},E)\right)}\Psi(\omega,\mathbf{r},\mathbf{\Omega},E)=\\&\int_{\mathbf{\Omega}',E'}\Psi(\omega,\mathbf{r},\mathbf{\Omega}',E')\Sigma(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega},E'\to E)d\mathbf{\Omega}'dE'\\&+\frac{\nu(E)}{k_{eff}}\left(1-i\omega\sum_{k=1}^{g}\frac{\beta_k}{i\omega+\lambda_k}\right)\int_{\mathbf{\Omega}',E'}\Psi(\omega,\mathbf{r},\mathbf{\Omega}',E')\Sigma_f(\mathbf{r},E')d\mathbf{\Omega}'dE'+S(\omega,\mathbf{r},\mathbf{\Omega},E)\end{aligned}\tag{3}$$

which is formally equivalent to the steady-state transport equation if the total cross-section $\Sigma(\mathbf{r},E)$ is replaced by the underlined quantity, $\left(\frac{i\omega}{\mathrm{v}}+\Sigma_t(\mathbf{r},E)\right)$, in Equation (3), and the fission cross-section $\Sigma_f(\mathbf{r},E')$ is replaced by $\Sigma_f(\mathbf{r},E)\left(1-i\omega\sum_{k=1}^{g}\frac{\beta_k}{i\omega+\lambda_k}\right)$.

The quantities used in the above equations are defined as follows:

$\Psi(\mathbf{r},\mathbf{\Omega},E,t)$	Angular neutron flux	$\Sigma_t(\mathbf{r},E)$	Total macroscopic cross section
$\mathrm{v}(E)$	Neutron velocity	$\Sigma_s(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega},E'\to E)$	Macroscopic scattering differential cross-section
$\mathbf{r}$	Position vector	k_{eff}	Effective neutron multiplication factor
$\mathbf{\Omega}$	Neutron path direction	$\nu(E)$	Fission neutron spectrum
E	Neutron energy	$C_k(\mathbf{r},t)$	Density of the *k*-th delayed fraction
t	Time	$S(\mathbf{r},\mathbf{\Omega},E.t)$	External perturbation (source)

The multi-group neutron diffusion equation is obtained from the neutron transport equation by averaging the appropriate functions of the above neutron cross sections and neutron fluxes. Applying formally to equation (3) the standard procedure of obtaining the steady-state diffusion equation from the steady-state transport equation (see references [3]) yields results identical to those described in the work of C. Demazière ([1], [2]).

If a higher accuracy of the reactor response function is desired, the full treatment of the problem should involve the same sequence of calculations as it is currently performed when going to detailed transport models to less detailed multi-group diffusion models. This may include, as appropriate, averaging over the energy spectrum to obtain group cross-sections and group diffusion coefficients, homogenisation over reactor lattice cell(s), discontinuity factors and so on. Everywhere in the sequence, the total cross-section $\Sigma_t(\mathbf{r},E)$ is replaced by $\left(\frac{i\omega}{\mathrm{v}}+\Sigma_t(\mathbf{r},E)\right)$ and the fission cross-section $\Sigma_f(\mathbf{r},E)$ is replaced by $\Sigma_f(\mathbf{r},E)\left(1-i\omega\sum_{k=1}^{g}\frac{\beta_k}{i\omega+\lambda_k}\right)$.

3. A Simplified CANDU® Model

To explore the applicability of the space dependent response function, a simplified model of a CANDU® core was used. The model has two important characteristics:

3.1 Homogenized Regions of a Reactor Lattice Cell

A four zone-zone homogenization of the reactor lattice cell geometry (shown in Figure 1) is used instead of the commonly used homogenization of the lattice cell, to allow exploration of the lattice cell inhomogeneity on the propagation of neutron waves. For a reasonably accurate representation, a lattice cell is divided into four homogenized regions, numbered from 1 to 4. Region 1 contains all of the fuel, pressure tube and coolant and some small amount of moderator. Regions 2 and 3 contain moderator between Region 1 and the lattice cell boundary, and have to have identical characteristics. Regions 2 and 3 are computed separately as an additional check for the accuracy of the homogenization procedure. Meanwhile, Region 4 contains the moderator at the corners of the lattice cell. The lattice cell itself is represented by the blue square.

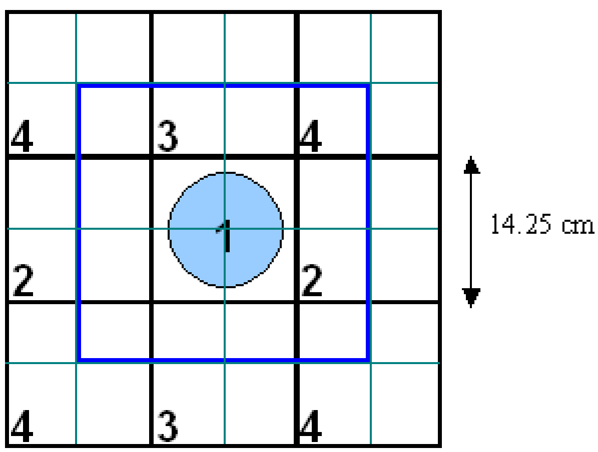

Figure 1 Mesh Structure of One Lattice Cell. The thin green lines were added to the figure to indicate the mesh used in the calculation. Both black and green lines indicate computational meshes for the diffusion calculation.

3.2 Two Fuel Types Core Model

A two fuel types reactor model (shown in Figure 2) is used because this is the simplest model that yields a computed neutron flux distribution similar to the one observed in the CANDU® reactor core. The dark red and the yellow meshes contain the two types of fuel, while the light blue and orange meshes denote moderator regions. As the properties of region 4 in Figure 1 were found to be close to the properties of Region 2, a single type of homogenized moderator is assumed for each of the corresponding fuel type

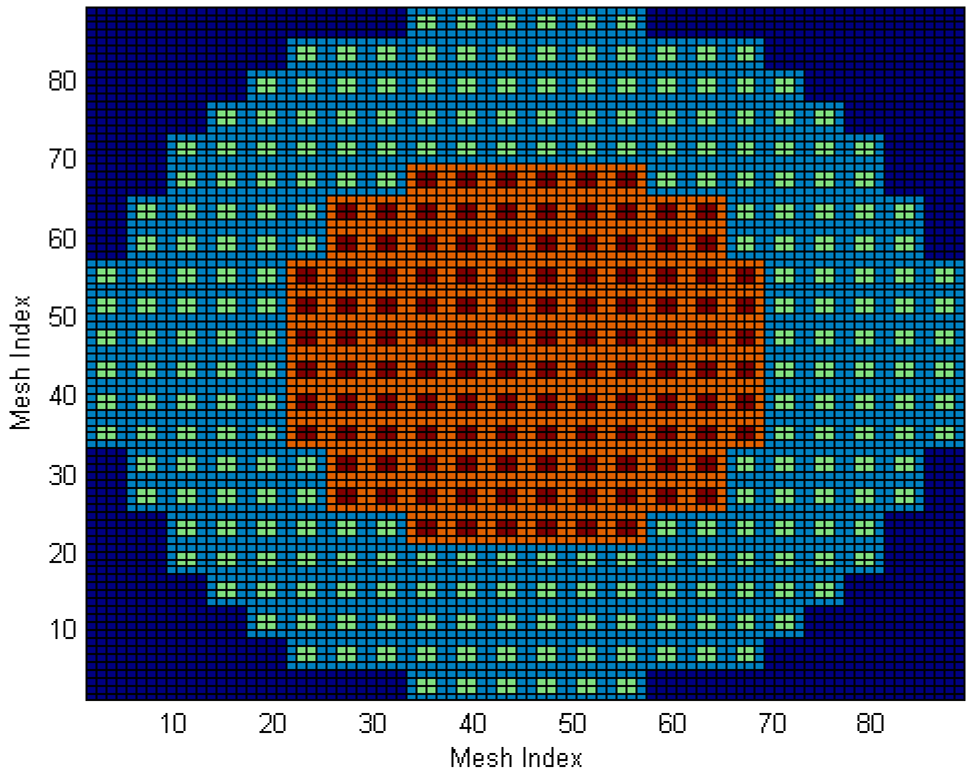

Figure 2 Material Map for the Whole Reactor Model.

4. Results for a Simplified CANDU® Model

To illustrate the behaviour of the response function over the whole frequency range, results for the following frequencies ν=50 Hz, ν=250 Hz, ν =500 Hz and ν=600 Hz are presented. The first frequency (ν=50 Hz) is chosen to illustrate the low-frequency behavior of the reactor response function, while the second (ν=250 Hz) is chosen to illustrate the intermediate frequency. The behaviour of the response function at a high frequency (ν =500 Hz) illustrates the situation before the lattice inhomogeneity is being felt. The last frequency (ν=600 Hz) reactor response function illustrates the situation when the frequency is in the range where lattice inhomogeneity dominates the response.

4.1 Response Function at ν=50 Hz

In this case, the response function decays fast with distance, about a decade per reactor lattice cell pitch, as seen in Figure 3. As the Fourier analysis of reactor noise signals works well up to 10^{-3} to10^{-4} relative intensity of a spectral component, the influence of a perturbation source can be felt three to four lattice cells away.

As shown in Figure 3, the phase varies slowly about 4π from one reactor boundary to the other. The decimal logarithm of the absolute value of the response function and the response function phase vary smoothly with position. The details of the lattice do not visibly influence the propagation of neutron waves, except for some positional modulation due to the reactor lattice cell inhomogeneity, as see in the left image in Figure 3. At the frequency of ν = 50 Hz, and up to about 500 Hz, the homogenization of the entire lattice cell model (with the lattice cell completely homogenized) can be used to describe the propagation of neutron waves.

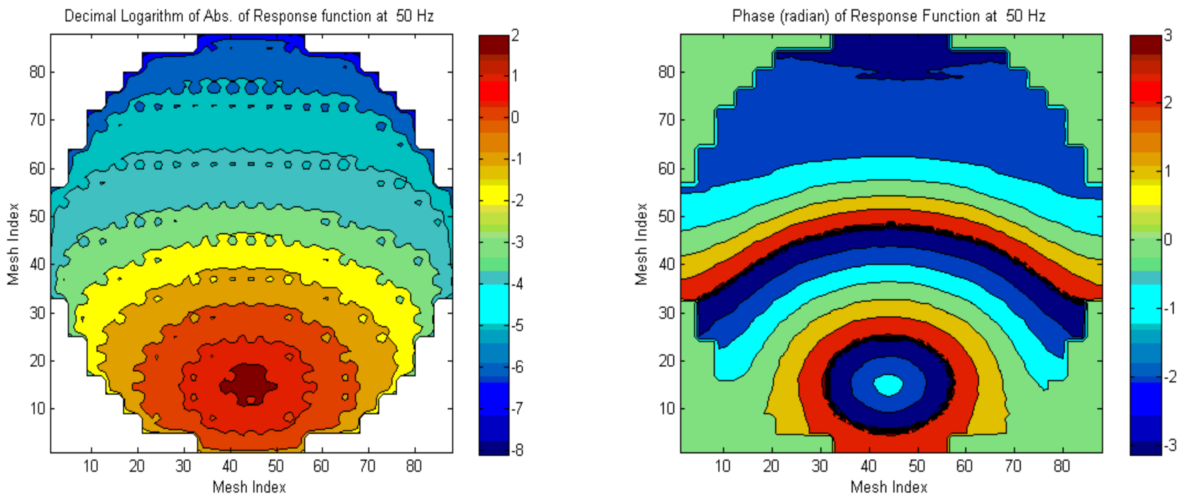

Figure 3 Decimal logarithm and phase of the response function at ν=50 Hz.

4.2 Second Case: ν=250 Hz and ν=500 Hz

The 250 Hz frequency is an intermediate and shows that the phase variation and the exponent of the decay with distance have comparable values, that is, the neutron waves satisfy a dispersion relation close to that for pure diffusion waves. The decay of the response function with distance is fast, about one decade per lattice pitch.

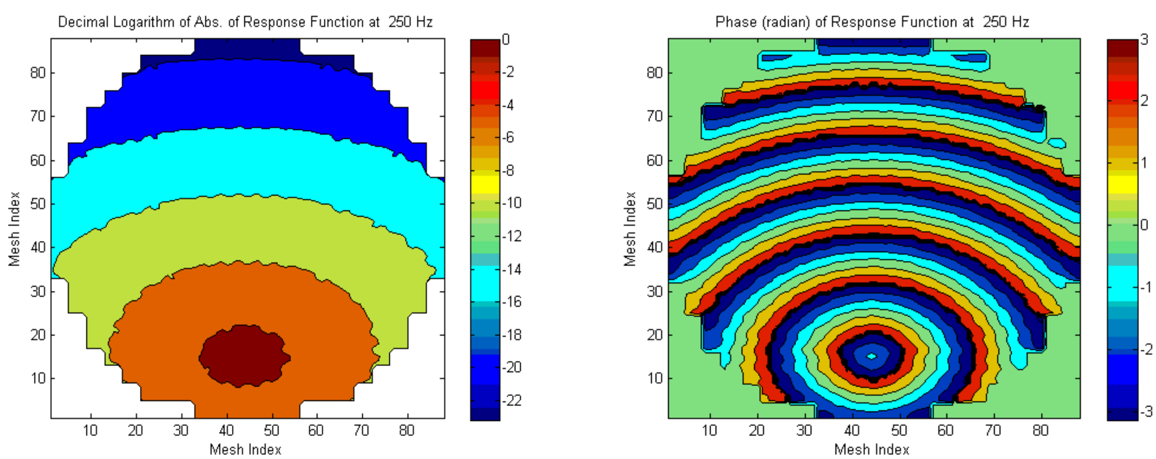

Figure 4 Decimal logarithm and phase of the response function at ν=250 Hz.

The above pattern survives up to ν=500 Hz. At this frequency, effects of the lattice inhomogeneity start to influence the result close to the source of perturbation

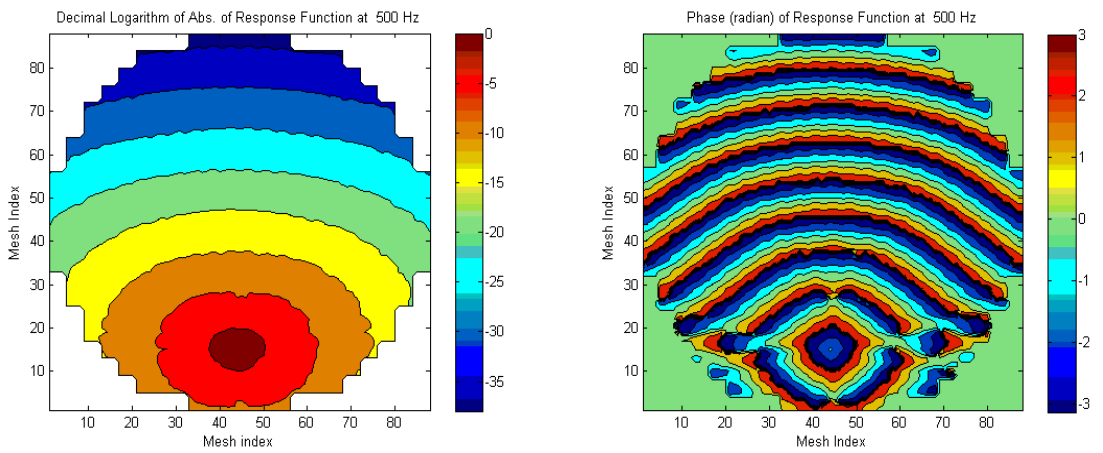

Figure 5 Decimal logarithm and phase of the response function at ν=500 Hz.

4.3 Third Case: ν=600 Hz

Figure 6 shows the plots of the decimal logarithm of the response function and of the phase of the response function for a 600 Hz perturbation. This frequency is important because it is above the first frequency at which the effects of the non-homogeneity in the reactor core lattice [4] starts to influence the propagation of the diffusion waves.

The phase has a complicated variation around the source and then a slow variation in the rest of the reactor core. The decay of the response function is fast, almost two decades per lattice pitch.

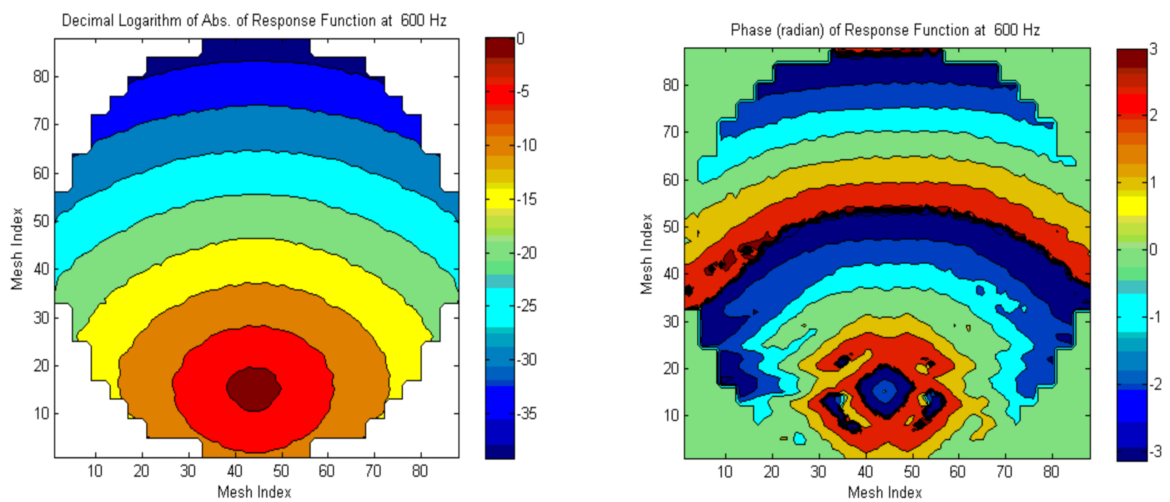

Figure 6 Phase of the response function at ω=765 π Hz (thermal part)

5. Discussion and Conclusions

The analysis of frequency-dependent response functions and neutron diffusion waves concludes with the following:

a) A simplified neutronic model for the CANDU® reactor core can account for lattice cell inhomogeneity. The model can be used towards exploratory studies on the neutron wave effects in the reactor core.

b) For frequencies above 50 Hz, the effects of a perturbation can be felt, at most, 5 to 6 reactor lattice cell pitches away; therefore, a regional model such as the multi-cell model can work and be accurate.

c) The decay of the response function with distance from the source increases with frequency. The inverse of the decay length is roughly proportional to the square root of the perturbation frequency. For frequencies in the range of hundreds of Hertz, regional models for the reactor core are reasonably accurate.

d) The propagation of neutron waves in a CANDU® reactor core are sensitive to the cell lattice inhomogeneity, resulting in wave interference effects for frequencies larger than 500 Hz.

e) Above 500 Hz, it is hard to distinguish reactor-relevant features in the response function, as the interference pattern becomes more complex. Investigations at higher frequencies may be less effective in identifying phenomena inside the core. At a frequency of 500 Hz, the influence of a perturbation may be felt about 2 cell pitches away.

f) Identification of any of the sources of noise in the reactor core (such as vibrations of fuel bundles pressure tubes, control rods, boiling of coolant etc.) may serve to guide actions for improving the maintenance of reactors, reactor safety and reactor operations.

6. Further Work

The method described here is being generalized to the more realistic three-dimensional case and to lower frequencies:

a) In the three-dimensional case, models of the current CANDU core detectors will be used, that is, relatively fast ones (platinum clad or Inconel) that are 3 lattice pitches long. In two dimensions, this case cannot be modelled properly. The current noise measurements use a maximum sampling of 500 Hz i.e. a maximum useable frequency of about 200 Hz, hence some of the wave effects may be detectable.

b) The algorithms used in the program will be optimized so that very low frequency noise such as 1 Hz can be analyzable. The Fourier domain and the time domain behave in opposite manners: in the Fourier domain, high frequency phenomena are easier to analyze, while in the time domain low frequency (slow) phenomena are easier to analyze.

7. References

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[4] H. Takeda, K, Yoshino, “Diffusion band gaps of thermal neutron diffusion in two-dimensional structures with periodic moderators”, Journal of Physics D, vol. 36, pp. 1890-1895, 2003.