# Fuel-Pin Flux Reconstruction for CANDU Applications[1]

**M. Dahmani, W. Shen and B. Phelps**
Atomic Energy of Canada Limited, Mississauga, Ontario, Canada
(dahmanim@aecl.ca)

## Abstract

In this paper, we propose a method for fuel pin/ring flux reconstruction for CANDU applications. This method is based on the modulation method where the core mesh flux is reconstructed by multiplying the form factor and an interpolated mesh flux along with an appropriate normalization. This method was adapted to a finite difference method. The implementation of the method and test results will be presented.

## 1. Introduction

The use of homogenized parameters to predict reactor properties results in an inevitable loss of certain information, which would otherwise be available if the reactor were analysed by methods not involving homogenization. However, a reactor may contain several hundred fuel assemblies (channels), and each assembly/channel may include several hundred of fuel pins. Hence, an explicit representation of heterogeneous assemblies requires tens of thousands different regions. With the actual computing capability, such detailed three-dimensional (3-D) transport calculations are not feasible. However, if we are satisfied knowing only the average values of spatial flux distributions, the 3-D diffusion solution will constitute the final solution. On the other hand, in reactor design analysis we often need direct information about the local pin-flux distribution for the heterogeneous assemblies. For this reason, after having solved the full-reactor-core problem, we have to look into the possibilities of recovering in second step information on local properties of single heterogeneous assemblies (going backward). Such a two-step approach for 3-D reactor calculations has a practical advantage. Assuming sufficient accuracy in solving the global reactor problem one can temporarily forget most of the details and complications of the heterogeneous reactor core which may be re-introduced in the second step where and when necessary. This procedure requires a coupling between the global diffusion solution and the heterogeneous assembly (lattice) calculation.

In CANDU[2] reactor analysis, the highest bundle power obtained is translated into a maximum pin power using a factor representing the ratio of the highest pin power to the average pin power. The ratio is pre-determined (as a function of fuel burnup) by lattice-cell calculation, which does not take into account the effect of any global flux gradient across the cell. Thus the environment effect on the bundle under consideration and its pin power distribution is ignored. A correction or adjustment to allow for possible azimuthal tilt in the pin power distribution would be made if judged required or for conservative reasons. This method is limited since it does not take into account the intranodal (inside the cell) flux at the core calculation level. Hence the method proposed here will improve the current technique for fuel-pin flux/power reconstruction in CANDU core analyses.

In the Light Water Reactor (LWR) industry, most of the modern neutronics codes employ advanced nodal methods. The pin-flux information is obtained through a pin flux “reconstruction” or “dehomogenization” process. The terms reconstruction and dehomogenization arise from the fact that the neutron flux calculation performed in a nodal code is based on homogenized nodal cross sections so that the resulting intranodal flux distribution can not reflect any local heterogeneity within an assembly. A reconstruction process is thus required to incorporate the actual heterogeneous structure of each assembly into the smoothly-varying intranodal flux


[2] CANDU is a registered trade-mark of Atomic Energy of Canada Limited (AECL)

distribution. These reconstruction methods have reached high levels of development in LWR and have become standard analysis tools because they extend the usefulness of computationally efficient nodal methods and eliminate the need to perform full-core fine-mesh calculations. All these methods intuitively assume that the flux shape in an assembly can be approximated by superposing lattice-based "heterogeneous" form functions on a core-based "homogeneous" intranodal shape function. These methods are commonly named Form Function Methods and in some literature these methods are also called Modulation Methods. For consistency and simplicity, in the following we will use the terminology borrowed from the nodal method: the average node flux corresponds to the average mesh flux and the intranodal flux to the spatial mesh flux.

## 2. Theory

The pin-flux reconstruction process involves a fundamental assumption; that is, detailed pin-by-pin distributions within an assembly/bundle can be estimated as the product of a global intranodal distribution and a local heterogeneous form function. The form function accounts for bundle heterogeneities and it is generated for each fuel assembly type by a lattice-physics code at the same time that the homogenized cross sections are generated. The assumption of separability of the global intranodal flux and the local form function is commonly adopted in various pin-flux reconstruction methods that have been extensively researched in the past two decades [1] [2]. The assumption has been shown to be valid such that the reconstruction methods are regularly used in best-estimate reactor safety analysis of non-CANDU power reactors. The flux is then obtained by combining these two functions (fluxes):

$$\Phi(x,y)_{reactor} = \Phi(x,y)_{\text{hom}} \times \Psi(x,y)_{het} \tag{1}$$

where $\Phi(x,y)_{\text{hom}}$ is the "homogeneous" intranodal flux calculated by the nodal diffusion method at core level; and $\Psi(x,y)_{het}$ is the "heterogeneous" form function evaluated at lattice level. $\Psi(x,y)_{het}$ can be obtained by using a single-cell (assembly) model or by a multi-assembly (colorset) model. The form function must account for assembly heterogeneities caused by water holes, burnable absorber pins, enrichment variations, etc. This factorization approximation is justifiable because the neutron free mean path is very small compared to the assembly/bundle dimensions so that the lattice effects and global (whole reactor) effects can be "decoupled".

### 2.1 The Reconstructed Intranodal Flux

Using the surface-average fluxes and currents and corner point fluxes as well as the node-average flux, the intranodal group flux is approximated for every node by assuming that the flux within a node is separable in the *x-y* plane and axial directions $\Phi_g(x,y,z) = \Phi_g(x,y) \times \phi_g(z)$. With this separability assumption, the axial flux distribution can be interpolated by a quadratic polynomial. The expansion coefficients are determined by requiring the node-average flux and the top and bottom surface-average fluxes to be reproduced.

In the *x-y* plane, the flux distribution $\Phi_g(x,y)$ is expanded for each energy group into a linear combination of polynomials of order N:

$$\Phi_g(x,y) = \sum_{n=1}^{N} c_n^g f_n(x,y) \tag{2}$$

The $f_n(x,y)$ functions belong to an N-dimensional polynomial space and represent its basis. The expansion coefficients $c_n^g$ are determined by the node parameters namely, the node average flux, the four surface-average fluxes, and the four corner point fluxes:

$$c_n^g \equiv c_n^g\left(\overline{\phi}^g, \phi_s^g, \phi_c^g\right) \tag{3}$$

For the given node parameters, the problem is reduced to that of finding a unique set of basis functions that will allow the interpolation of the intranodal flux (Eq. 2) by satisfying the constraints imposed on the node parameters. The problem then becomes purely mathematical and the choice of an adequate polynomial space is the key to an accurate solution. In the context of the coarse mesh finite difference (CMFD) method and considering a polynomial space of order 2 (N=2), Eq. (2) can be rewritten as follows:

$$\Phi_g(x,y) = \sum_{i,j=0}^{2} c_{ij}^g x^i y^j \tag{4}$$

where the homogeneous flux is expanded on polynomials in each square node (side length equal to *h*). Knowing the node quantities, the task is to determine the nine coefficients $c_{ij}^g$ that satisfy Eq. (4).

## 2.2 The Form Function

The calculation of the form function $\Psi(x,y)$ is performed at the lattice level. In LWR more detailed and accurate fuel-pin flux distributions can be obtained because the fuel pins are arranged in square assemblies. In contrast, the CANDU fuel pins are arranged in a cylindrical cluster geometry. The fuel-pin flux (or power) distribution is in practice more complicated to apply in the diffusion code with Cartesian geometry. One way to overcome this is to consider the average flux (power) per ring. This suggests the assumption that the fuel pins belonging to the same ring experience similar neutron exposure. In fact, this assumption is reasonable when reflective boundary conditions are used in an infinite lattice calculation. However, when the effect of unlike neighbour lattices is taken into account or when an absorber is present in the vicinity, this assumption is less credible. A practical approach is that the transport flux $\Psi_{nm}^T$ at any discrete position (*n*, *m*) is computed using an infinite lattice calculation with reflective boundary conditions. The $\Psi_{nm}^T$ will depend on the lattice local parameters such as burnup, the presence of devices, and the neighbours (in the case of multicell). The two-group fluxes per ring should be calculated and tabulated at the same time as the generation of the homogenized cross sections. Note that the calculation surface fluxes at the edges might be necessary to obtain an accurate flux distribution. To be consistent with Eq. (1), the form functions should be dimensionless. To do so, a normalization of the form function should be introduced.

Usually the flux-volume-weighting homogenization is performed after the lattice calculation. To be consistent with this, one way to normalize the form function is to use the homogenized average lattice flux in such a way that the form function is obtained from this equation:

$$\Psi_{nm} = \frac{\Psi_{nm}^T}{\overline{\Psi}^T}; \tag{11}$$

where $\overline{\Psi}^T$ is the average lattice transport flux, defined as:

$$\overline{\Psi}^T = \frac{1}{V}\int_V dV \Psi^T(x,y); \tag{12}$$

and *V* is the volume of the lattice.

## 2.3 Reconstruction of the Heterogeneous Node Flux

The heterogeneous flux distribution in each node at point (*n*, *m*) and in each group *g* is obtained by multiplying the form function and the flux expansions for each energy group (Eq. 4):

$$\phi_{nm}^{g} = \Psi_{nm}^{g} \times \Phi_{nm}^{g} = \Psi_{nm}^{g} \times \sum_{i,j=0}^{N} c_{ij}^{g} n^{i} m^{j} \ . \tag{13}$$

There is a shortcoming in using the flux-volume normalization technique for the form function, in that the continuity of the form function (and the reconstructed flux) at the edge of the cell is not assured. To overcome this problem, another way to normalize the form function is to utilize the Selengut factor [3]. The Selengut (i.e assembly discontinuity factor ADF) is defined as:

$$\gamma = \frac{\frac{1}{S}\int_{S} dS \Psi^{T}(x,y)}{\frac{1}{V}\int_{V} dV \Psi^{T}(x,y)} \tag{14}$$

where $S$ is the cell surface. In this case the macroscopic cross sections used in reactor calculations are normalized along with the form function in this way:

$$\phi_{nm}^{g} = \frac{1}{\gamma^{g}} \times \Psi_{nm}^{g} \times \Phi_{nm}^{g} \left(\frac{1}{\gamma^{g}} \Sigma^{g}\right) \tag{15}$$

where $\Sigma^{g}$ symbol represents all the homogenized macroscopic cross sections involved in the reactor core calculations. For all the results shown in this paper only the flux-volume normalization technique was utilized.

## 3. Numerical results

The MINER (Multigroup Iterative Neutronics External Replacement) solver [4] was developed in order to have a flux solver that is more flexible than the flux solver in RFSP [5]. The MINER code includes both a finite-difference [6] solver and a Green's function nodal-expansion [7] solver. Either solver can solve multi-group problems, with and without ADFs. The MINER solvers can be used independently to solve standard benchmark problems, or can be used with RFSP to solve full-core problems based on the RFSP input file. The modulation method presented here has been implemented in the MINER solver with the finite-difference method in 2-D form.

For performing the intranodal flux reconstruction computations, the MINER solver was used in its stand-alone form. The MINER solver currently consists of a collection of codes that work together. The MINER-derived intranodal flux computations in this report proceeded in 3 stages: the parsing of the model input data, the flux solution, and the intranodal flux reconstruction.

### 3.1 The 5x1 Benchmark Problem

The 5x1 benchmark problem is illustrated in Figure 2. The 5x1 geometry included 4 fuel lattice cells and 1 reflector lattice cell. All 4 fuel cells were one uniform fuel type: the CANDU-6 37-element natural-uranium (NU) fuel with a burnup of 1482.75 MWd/Te. A 4x4 mesh splitting in each node was used.

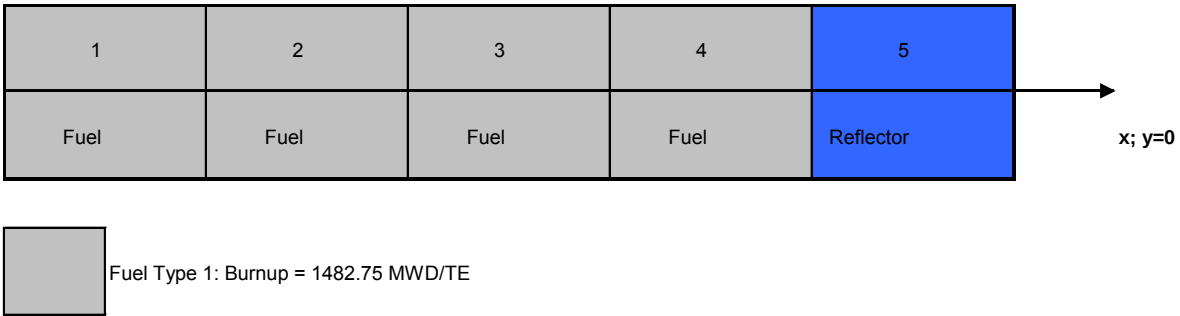

**Figure 2 Problem Geometry for 5x1 Benchmark Problem**

The final fuel-pin reconstructed fast and thermal flux results using the modulation method are plotted in Figure 3 and Figure 4. The homogeneous lattice properties were calculated from the single cell calculation. The WIMS-calculated reference heterogeneous flux solutions are also included in these plots for a comparison. The maximum percent difference for the fast and thermal fluxes between the reconstructed and the reference fluxes is about 5 % for cells 1, 2 and 3. The percent difference is higher at the interface between cell 4 and the reflector: -17% for the fast flux and -10% for the thermal flux.

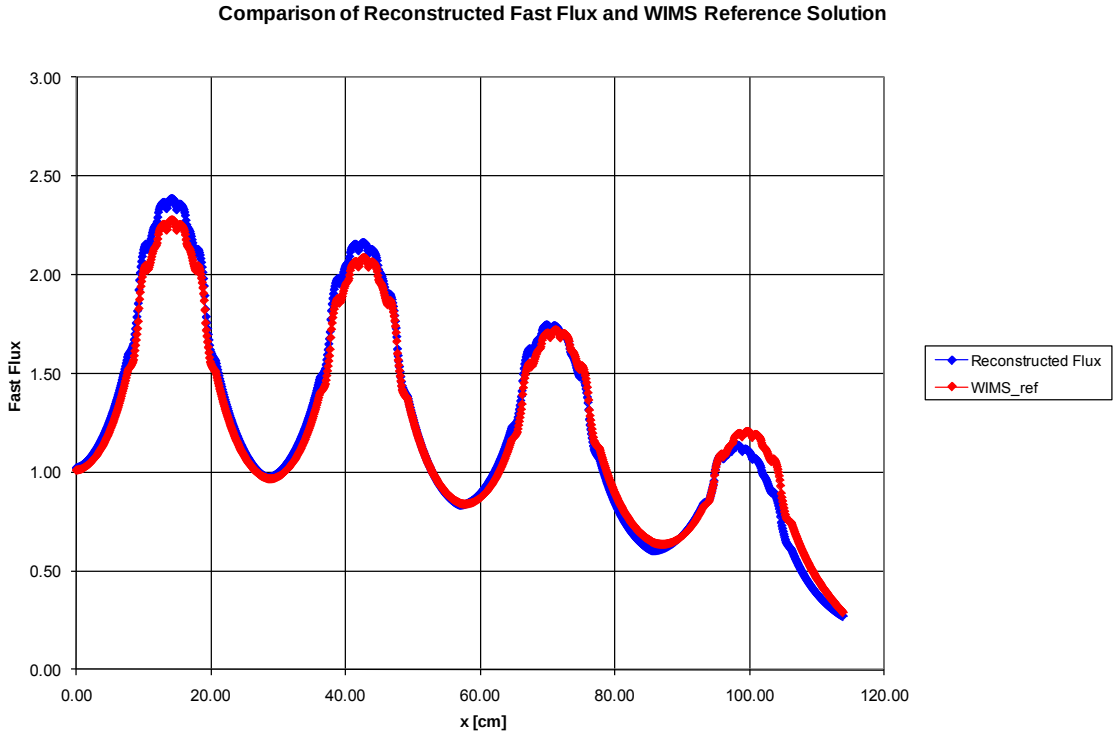


**Figure 3 Reconstructed Fast Flux and WIMS-AECL Reference Solution for the 5x1 Benchmark**

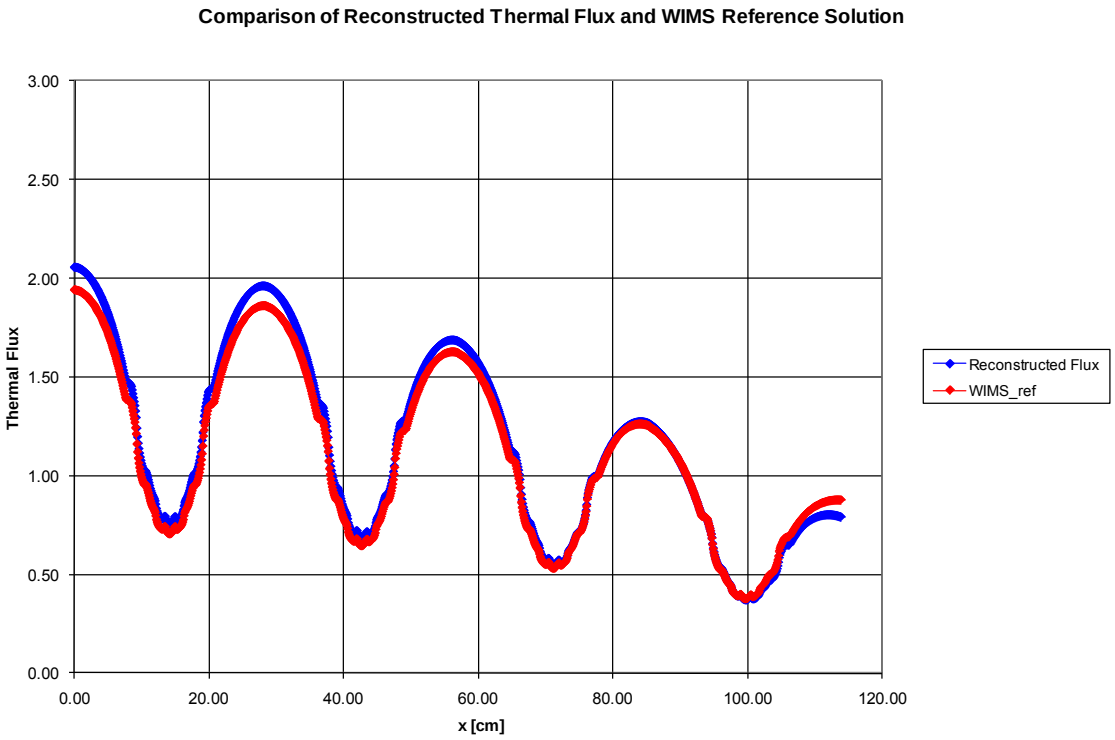


**Figure 4 Reconstructed Thermal Flux and WIMS-AECL Reference Solution for the 5x1 Benchmark**

## 3.2 The 3x3 Benchmark Problem

The 3x3 benchmark problem is illustrated in Figure 5. The 3x3 geometry included 9 lattice cells with three types of fuel: the CANDU-6 37-element NU fuels at 3 different burnup values. These 3 lattice cell types were given the names 'Fuel type 1', 'Fuel type 2', and 'Fuel type 3' with the fuel burnups at 1482.75 MWd/Te, 5860.14 MWd/Te and 8348.67 MWd/Te respectively. The homogeneous lattice properties were calculated from the single cell calculation. For all MINER calculations, 4x4 mesh splitting in each node was used. To compare with the WIMS-AECL reference results, the fluxes were illustrated at 3 different lines through the 2-D geometry. These 3 lines are shown in Figure 12. 'Line 456' and 'line 789' are the two horizontal lines along y=0 and y=28.575 through the centers of the respective cells. 'Line 159' is the diagonal line along y=x through the centers of cells 1, 5, and 9.

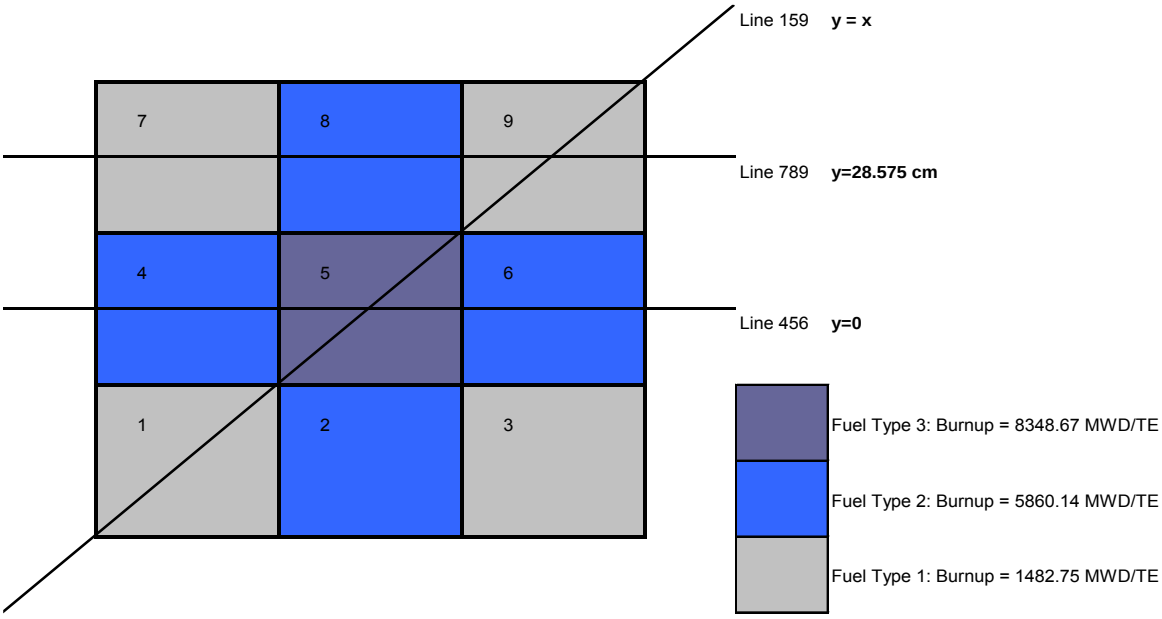


**Figure 5 Problem Geometry for 3x3 Benchmark Problem**

The reconstructed thermal flux, for line 456, using the modulation method is presented in Figure 6. The WIMS-AECL calculated reference heterogeneous flux solution is also included in this plot for comparison purposes. The results for lines 789 and 159 are shown in Figure 7 to Figure 8. For these cases, the maximum percent difference in thermal flux between the reconstructed and the reference flux is in general less than 3%. Some larger differences less than 4% were observed at the boundary.

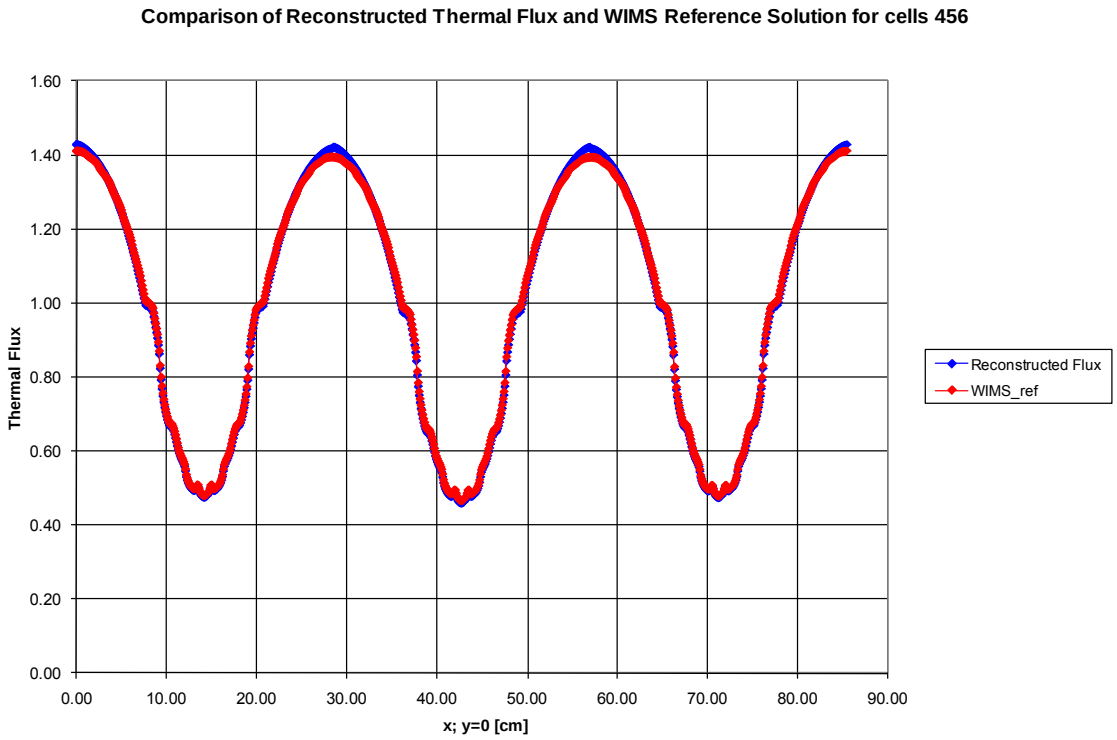


**Figure 6 Reconstructed Thermal Flux and WIMS-AECL Reference Solution for Cells 4,5,6 in the 3x3 Benchmark**

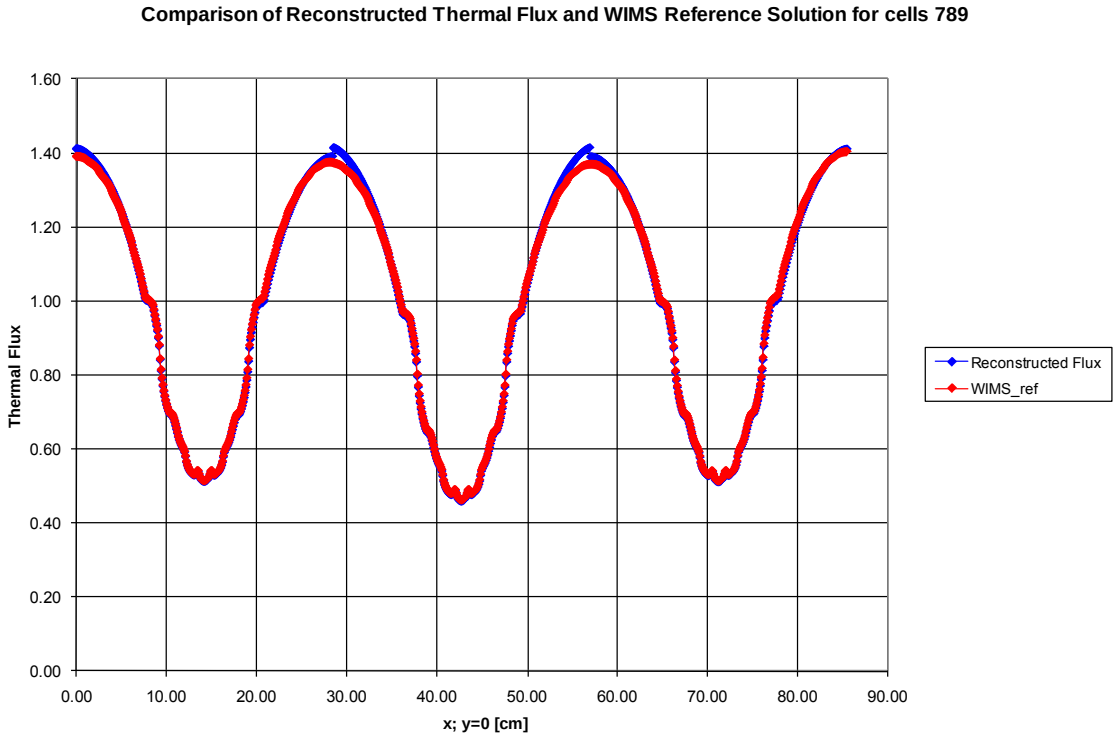


**Figure 7 Reconstructed Thermal Flux and WIMS-AECL Reference Solution for Cells 7,8,9 in the 3x3 Benchmark**

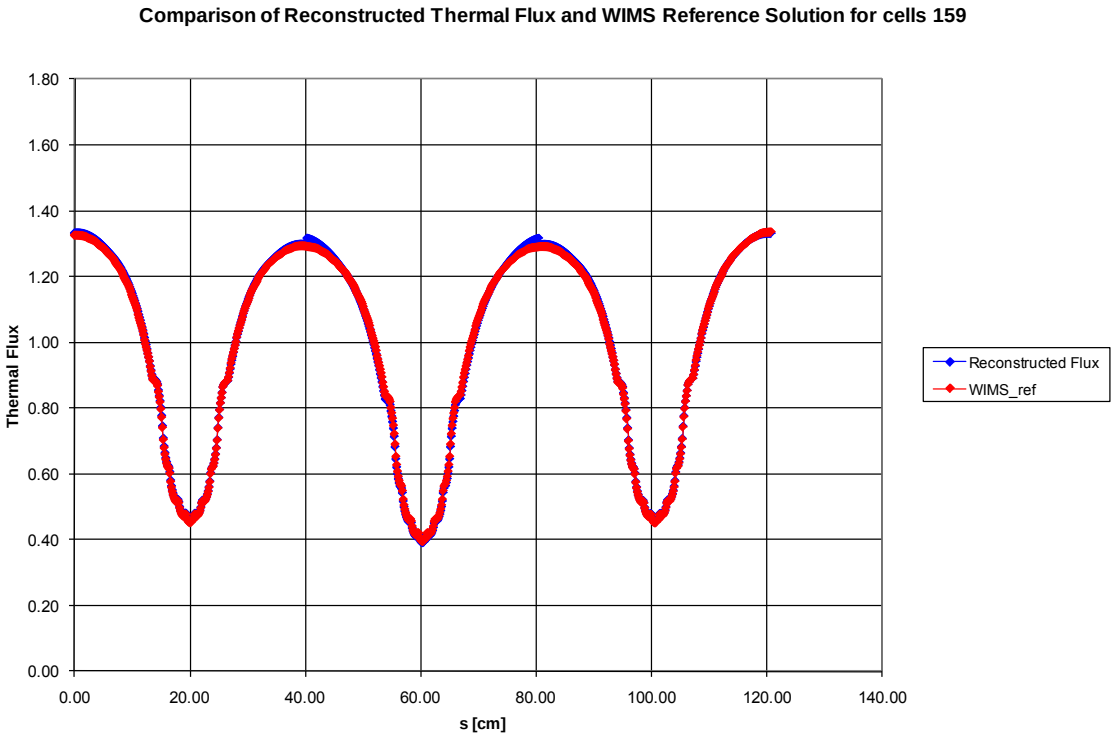


**Figure 8 Reconstructed Thermal Flux and WIMS-AECL Reference Solution for Cells 1,5,9 in the 3x3 Benchmark**

## 4. Conclusions and Future Work

In this paper we have proposed a fuel-pin flux reconstruction method for CANDU-type reactor applications. Preliminary results show a good agreement between the reconstructed flux and the corresponding heterogeneous transport flux for two different 2-D benchmark problems. In the future this method will be extended to 3-D problems where tests including absorber devices will be performed. The Selengut-type normalization will also be implemented.

## 5. References

[1] K.R. Rempe, K. Smith, and A.F. Henry, "SIMULATE-3 Pin Power Reconstruction: Methodology and Benchmarking", Proc. Int. Reactor Physics Conference, Jackson Hole, Wyoming, September, 1988.

[2] R. Boer and H. Finnemann, "Fast Analytical Flux Reconstruction Method for Nodal Space-Time Nuclear Reactor Analysis", Annals of Nuclear Energy, Vol. 19, pp. 617-628, 1992.

[3] E. Girardi et al., "Pin-by-Pin Power Reconstruction in the Future EDF Calculation Scheme", Proceedings of ICAPP '08. Anaheim, CA USA, June 8-12, 2008.

[4] B. Phelps, W. Shen, and E. Varin, "MINER: Three-Dimensional Multi-Group Finite-Difference and Nodal Method for CANDU Applications", *Proceedings of the 30th Annual Conference of Canadian Nuclear Society*, Calgary, May 31 - June 3, 2009.

[5] B. Rouben, "RFSP-IST, The Industry Standard Tool Computer Program for CANDU Reactor Core Design and Analysis", Proceedings of the 13th Pacific Basin Nuclear Conference, Shenzhen, China, Oct. 21-25, 2002.

[6] W.M. Stacey, *Nuclear Reactor Physics*, John Wiley & Sons, Inc., pp. 81-87, 2001.

[7] Wei Shen, Zhongsheng Xie, and Banghua Yin, "The Green's Function Nodal Expansion Method for Light Water Reactor Diffusion Calculations", *Nuclear Science and Engineering* 121, 130-135, 1995.