Inverse kinetics for subcritical systems with variating external sources

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Abstract

With the rise of a new generation of ADS (Accelerator-Driven System) nuclear reactors it is important to have a rapid and accurate prediction of the variation in reactivity during a possible variation in the intensity of external sources. This paper presents a formulation for the calculation of reactivity in subcritical systems using the inverse method as related only to nuclear power derivatives. One of the applications of the proposed method is the possibility of developing reactimeters that allow the continuous monitoring of subcritical systems.

1. Introduction

The analytical solution of point kinetics equations with a group of delayed neutrons is useful in predicting neutron density variation during the start-up of existing nuclear reactors. With rise of a new generation of ADS (Accelerator-Driven System) nuclear reactors [1], the rapid and accurate prediction of the variation in reactivity during a possible variation in the intensity of external sources becomes necessary [2]. Results found in the literature show that, the closest a multiplicative system is to criticality, the less dependent of the source it becomes, and that, although they describe in a reasonable manner the temporal behaviour of a subcritical system, conventional point kinetics equations [3,4,5] do not have good accuracy in such analysis.

This paper presents a formulation of inverse kinetics for subcritical systems based on nuclear power derivatives. This formulation is based on the set of point kinetics equations proposed by Gandini & Salvatores [6] specifically to describe subcritical systems.

2. Point Kinetics for Subcritical Systems

Considering the time-dependent neutron transport equation,

$$V^{-1}\frac{\partial}{\partial t}\left|\Psi\right\rangle = -A\left|\Psi\right\rangle + \left(1-\beta\right)\frac{\chi_p}{4\pi}F\left|\Psi\right\rangle + \sum_{i=1}^{I}\frac{\chi_{d,i}}{4\pi}\lambda_i\left|c_i\right\rangle + \left|q_{ext}(t)\right\rangle \ , \tag{1}$$

and the following function importance associated to this equation

$$A_o^{\dagger}\left|n_o^{+}\right\rangle + F_o\frac{F_o^{\dagger}}{4\pi}\left|n_o^{+}\right\rangle + \frac{\gamma}{W_o}\left|\Sigma_f^{(o)}\right\rangle = \left|0\right\rangle . \tag{2}$$

This function importance is the same proposed by Gandini and Salvatores [6], though with a slightly different notation and taking the angular dependence into account.

Weighing equation (1) with vector $\langle n_o^+|$, one has that

$$\frac{\partial}{\partial t}\left\langle n_o^+\left|V^{-1}\right|\Psi\right\rangle = \left\langle n_o^+\right|-A\left|\Psi\right\rangle + \left\langle n_o^+\right|q_{ext}(t)\left.\right\rangle + \left\langle n_o^+\left|(1-\beta)\frac{\chi_p}{4\pi}F\right|\Psi\right\rangle + \sum_{i=1}^{I}\lambda_i\left\langle n_o^+\left|\frac{\chi_{d,i}}{4\pi}\right|c_i\right\rangle . \tag{3}$$

Supposing that the system is disturbed thus:

$$-A \to -A_o + \delta A \tag{4}$$

$$F \to F_o + \delta F \tag{5}$$

$$q_{ext}(t) \to q_{ext}^{(o)} + \delta q_{ext}(t) \;, \tag{6}$$

where $-A_0$, F_0 and $q_{ext}^{(0)}$ correspond to the undisturbed stationary states. By replacing these disturbances in equation (3) and adding and subtracting the term

$$\sum_{i=1}^{I}\left\langle n_o^+\left|\frac{\chi_{d,i}}{4\pi}\beta_i\left(F_o+\delta F\right)\right|\Psi\right\rangle , \tag{7}$$

in the right side of the resulting equation, one has that:

$$\begin{aligned}\frac{\partial}{\partial t}\left\langle n_o^+\left|V^{-1}\right|\Psi\right\rangle &= \left\langle n_o^+\left|\delta A\right|\Psi\right\rangle + \left\langle n_o^+\left|\left[(1-\beta)\frac{\chi_p}{4\pi}+\sum_{i=1}^{I}\frac{\chi_{d,i}}{4\pi}\beta_i\right]\delta F\right|\Psi\right\rangle + \left\langle n_o^+\left|\delta q_{ext}(t)\right.\right\rangle + \sum_{i=1}^{I}\lambda_i\left\langle n_o^+\left|\frac{\chi_{d,i}}{4\pi}\right|c_i\right\rangle - \\ &\sum_{i=1}^{I}\left\langle n_o^+\left|\frac{\chi_{d,i}}{4\pi}\beta_i F\right|\Psi\right\rangle + 1 + \left\langle n_o^+\left|-A_o\right|\Psi\right\rangle + \left\langle n_o^+\left|F_o\left[(1-\beta)\frac{\chi_p}{4\pi}+\sum_{i=1}^{I}\frac{\chi_{d,i}}{4\pi}\beta_i\right]\right|\Psi\right\rangle .\end{aligned} \tag{8}$$

where, in the calculations made to obtain equation (8) the relation of reciprocity of source $\left\langle n_o^+\left|q_{ext}^{(o)}\right.\right\rangle = 1$ was used. Weighing equation (2) with vector $\langle\Psi|$ and taking into account the properties of the linear operators, one has that:

$$\left\langle n_o^+\left|-A_o\right|\Psi\right\rangle + \left\langle n_o^+\left|F_o\left[(1-\beta)\frac{\chi_p}{4\pi}+\sum_{i=1}^{I}\frac{\chi_{d,i}}{4\pi}\beta_i\right]\right|\Psi\right\rangle = -\frac{\gamma}{W_o}\left\langle\Sigma_f^{(o)}\left|\Psi\right.\right\rangle . \tag{9}$$

Replacing the right side of equation (9) in equation (8), one finds

$$\frac{\partial}{\partial t}\left\langle n_o^+ \left| V^{-1} \right| \Psi \right\rangle = \left\langle n_o^+ \left| \delta A + \frac{\chi}{4\pi}\delta F \right| \Psi \right\rangle - \sum_{i=1}^{I}\left\langle n_o^+ \left| \frac{\chi_{d,i}}{4\pi}\beta_i F \right| \Psi \right\rangle + \sum_{i=1}^{I}\lambda_i \left\langle n_o^+ \left| \frac{\chi_{d,i}}{4\pi} \right| c_i \right\rangle + 1$$
$$- \frac{\gamma}{W_o}\left\langle \Sigma_f^{(o)} \middle| \Psi \right\rangle + \left\langle n_o^+ \middle| \delta q_{ext}(t) \right\rangle, \tag{10}$$

where

$$F_o^\dagger = \chi = (1-\beta)\chi_p + \sum_{i=1}^{I}\chi_{d,i}\beta_i \ . \tag{11}$$

Considering the following factorisation,

$$\Psi \approx P(t)\cdot\phi_o \tag{12}$$

and replacing equation (12) in equation (10) one obtains that:

$$\left\langle n_o^+ \left| V^{-1} \right| \phi_o \right\rangle \frac{dP}{dt} = \left\langle n_o^+ \left| \delta A + \frac{\chi}{4\pi}\delta F \right| \phi_o \right\rangle P(t) - \sum_{i=1}^{I}\left\langle n_o^+ \left| \frac{\chi_{d,i}}{4\pi}\beta_i F \right| \phi_o \right\rangle P(t) + \sum_{i=1}^{I}\lambda_i \left\langle n_o^+ \left| \frac{\chi_{d,i}}{4\pi} \right| c_i \right\rangle$$
$$+ \left[1 - P(t)\right] + \left\langle n_o^+ \middle| \delta q_{ext}(t) \right\rangle, \tag{13}$$

where consideration was given to identity

$$\frac{\gamma}{W_o}\left\langle \Sigma_f^{(o)} \middle| \phi_o \right\rangle \equiv 1 \ . \tag{14}$$

Dividing all the members of equation (13) by a normalization factor $I = \left\langle n_o^+ \left| \left(\frac{\chi}{4\pi}\right) F \right| \phi_o \right\rangle$, one has

$$\frac{\left\langle n_o^+ \left| V^{-1} \right| \phi_o \right\rangle}{I}\frac{dP}{dt} = \frac{\left\langle n_o^+ \left| \delta A + \frac{\chi}{4\pi}\delta F \right| \phi_o \right\rangle}{I} P(t) - \sum_{i=1}^{I}\frac{\left\langle n_o^+ \left| \frac{\chi_{d,i}}{4\pi}\beta_i F \right| \phi_o \right\rangle}{I} P(t) + \sum_{i=1}^{I}\lambda_i \frac{\left\langle n_o^+ \left| \frac{\chi_{d,i}}{4\pi} \right| c_i \right\rangle}{I}$$
$$+ \frac{1}{I}\left[1 - P(t)\right] + \frac{\left\langle n_o^+ \middle| \delta q_{ext}(t) \right\rangle}{I}. \tag{15}$$

Applying the following definitions of the integral parameters

$$l_{eff} = \frac{\left\langle n_o^+ \left| V^{-1} \right| \phi_o \right\rangle}{I} \ , \tag{16}$$

$$\rho(t) = \frac{\left\langle n_o^+ \middle| \delta A + (\chi/4\pi)\,\delta F \middle| \phi_o \right\rangle}{I} \ , \tag{17}$$

$$\beta_{eff} = \sum_{i=1}^{I} \frac{\left\langle n_o^+ \middle| \left(\chi_{d,i}/4\pi\right) \beta_i F \middle| \phi_o \right\rangle}{I} \ , \tag{18}$$

$$\xi_i = \frac{\left\langle n_o^+ \middle| \left(\chi_{d,i}/4\pi\right) \middle| c_i \right\rangle}{I} \ , \tag{19}$$

$$\frac{1}{I} = \zeta \ , \tag{20}$$

$$q_{ext}(t) = \frac{\left\langle n_o^+ \middle| \delta q_{ext}(t) \right\rangle}{I} \ , \tag{21}$$

equation (15) can be written thus:

$$l_{eff}\,\frac{dP}{dt} = \left(\rho(t) - \beta_{eff}\right) P(t) + \sum_{i=1}^{I} \lambda_i \xi_i + \zeta\left[1 - P(t)\right] + q_{ext}(t). \tag{22}$$

The delayed neutron precursor equation is:

$$\frac{\partial}{\partial t}\left| c_i \right\rangle = \beta_i F \left| \Psi \right\rangle - \lambda_i \left| c_i \right\rangle \ . \tag{23}$$

Multiplying equation (23) by $\left\langle n_o^+ \middle| \left(\chi_{d,i}/4\pi\right)\right.$, applying the factorisation represented by equation (12) and dividing by normalization factor I , one has that:

$$\frac{d}{dt}\frac{\left\langle n_o^+ \middle| \frac{\chi_{d,i}}{4\pi} \middle| c_i \right\rangle}{I} = \frac{\left\langle n_o^+ \middle| \frac{\chi_{d,i}}{4\pi}\,\beta_i F \middle| \phi_o \right\rangle}{I} P(t) - \lambda_i \frac{\left\langle n_o^+ \middle| \frac{\chi_{d,i}}{4\pi} \middle| c_i \right\rangle}{I}, \tag{24}$$

where, by the definitions of the integral parameters, one has the following equation:

$$\frac{d}{dt}\xi_i = \beta_{i,eff}\,P(t) - \lambda_i \xi_i \ . \tag{25}$$

The system of point kinetics equations formed by equations (22) and (25) is similar to the system of equations defined by Gandini and Salvatores [6], being different however in the integral parameter α that multiplies the second term of the right side of equation (22) that, although it appears in the system of reference equations [6], is not found in the system of equations (22) and (25) presented here.

3. Inverse Kinetics for Subcritical Systems

Equation (25) can be formally integrated in time, subjected to the condition that the ith generalized concentration of precursors $\xi_i(t)$ is null in an instant infinitely before instant t:

$$\xi_i(t) = \beta_{i,eff} \int_{-\infty}^{t} P(t') e^{-\lambda_i (t-t')} dt' . \tag{26}$$

Supposing that $P_0(t<0) = P_0$ one can separate equation (26) into two parts:

$$\xi_i(t) = \beta_{i,eff} \left[\int_{-\infty}^{0} P_0 e^{-\lambda_i (t-t')} + \int_0^t P(t') e^{-\lambda_i (t-t')} dt' \right], \tag{27}$$

or still:

$$\xi_i(t) = \frac{\beta_i P_0}{\lambda_i} e^{-\lambda_i t} + \beta_i \int_0^t P(t') e^{-\lambda_i (t-t')} dt' . \tag{28}$$

Replacing equation (28) in equation (22) and resolving for reactivity $\rho(t)$, one has the following expression:

$$\rho(t) = \beta_{eff} + l_{eff} \frac{d}{dt}\left[\ln P(t)\right] + \varsigma \left(1 - \frac{1}{P(t)}\right) - \frac{q_{ext}(t)}{P(t)} - \frac{1}{P(t)} \sum_{i=1}^{I} \lambda_i \beta_{i,eff} H(t). \tag{29}$$

where H(t) is the history of power as expressed by:

$$H(t) = \frac{P_0}{\lambda_i} e^{-\lambda_i t} + \int_0^t e^{-\lambda_i (t-t')} P(t') dt' . \tag{30}$$

Thus, the expression for reactivity based on the nuclear power can be written thus:

$$\rho(t) = \beta_{eff} + l_{eff} \frac{d}{dt}\left[\ln P(t)\right] + \varsigma \left(1 - \frac{1}{P(t)}\right) - \frac{q_{ext}(t)}{P(t)} - \frac{1}{P(t)} \sum_{i=1}^{I} \lambda_i \beta_{i,eff} \left[\frac{P_0}{\lambda_i} e^{-\lambda_i t} + \int_0^t e^{-\lambda_i (t-t')} P(t') dt' \right]. \tag{31}$$

Equation (31) represents the reactivity obtained by the inverse method for subcritical systems considering the set of point kinetics equations proposed by Gandini and Salvatores. The expression for the proposed reactivity, i.e. equation (31), is exact, and considers external sources that vary in time and will be used as a reference in the validation of the new formulation that will be presented in the next section.

4. Formulation for Determining Reactivity in Subcritical Systems

In a recent paper, Diaz et. al. [7] proposed a new formulation to deal with power history in critical systems using conventional point kinetics equations. This formulation can be extended to the case of the inverse kinetics of subcritical systems.

The integral found in equation (31) can be re-written integrating it in parts n times:

$$\int_0^t e^{-\lambda_i\left(t-t'\right)} P\left(t'\right) dt' = -\sum_{n=0}^{k}\left(-\frac{1}{\lambda_i}\right)^{n+1} P^{(n)}\left(t\right) + \sum_{n=0}^{k}\left(-\frac{1}{\lambda_i}\right)^{n+1} P^{(n)}\left(0\right) e^{-\lambda_i t} + \left(-\frac{1}{\lambda_i}\right)^{k+1} \int_0^t e^{-\lambda_i\left(t-t'\right)} P^{(k+1)}\left(t'\right) dt', \quad (32)$$

where $P^{(n)}(t)$ represents the n-order derived from nuclear power and $P^{(0)}(t) = P(t)$.

Equation (32) can be written in a more convenient manner, assuming that the nuclear power meets the following conditions:

$$P^{(2n-1)}(t) = r^{n-1} P^{(1)}(t), n \in N \quad (33)$$

$$P^{(2n)}(t) = r^{n} P^{(0)}(t), n \in N, \quad (34)$$

where the variable r is written by:

$$r \equiv \frac{P^{(2)}(t)}{P^{(0)}(t)}. \quad (35)$$

Independently from the index k being even or odd, Diaz et. al. demonstrated that equation (32) can be written by the following expression:

$$\int_0^t e^{-\lambda_i\left(t-t'\right)} P\left(t'\right) dt' = -\left[\frac{\lambda_i P(0) - P^{(1)}(0)}{\lambda_i^2 P(0) - P^{(2)}(0)}\right] P(0) e^{-\lambda_i t} + \left[\frac{\lambda_i P(t) - P^{(1)}(t)}{\lambda_i^2 P(t) - P^{(2)}(t)}\right] P(t), \quad (36)$$

where $\langle P_0 \rangle = P(0) = P_0$ is the initial normalized power of the operation period considered in the reactor, the following expression for reactivity is obtained:

$$\rho(t) = \beta_{eff} + l_{eff}\frac{d}{dt}\left[\ln P(t)\right] + \varsigma\left(1 - \frac{1}{P(t)}\right) - \frac{q_{ext}(t)}{P(t)}$$
$$-\frac{1}{P(t)}\sum_{i=1}^{I}\lambda_i \beta_{i,eff}\left\{\frac{P_0}{\lambda_i} e^{-\lambda_i t} - \left[\frac{\lambda_i P(0) - P^{(1)}(0)}{\lambda_i^2 P(0) - P^{(2)}(0)}\right] P(0) e^{-\lambda_i t} + \left[\frac{\lambda_i P(t) - P^{(1)}(t)}{\lambda_i^2 P(t) - P^{(2)}(t)}\right] P(t)\right\}. \quad (37)$$

Equation (37) is a simple and easy-to-implement expression for the calculation of reactivity in subcritical systems.

5. Applications

This section studies reactivity behaviour using equation (37) for different variations of the nuclear power.

5.1 Constant power

Let us consider that in an arbitrary period of the commercial operation the nuclear power is kept constant. It can be expected that during this operating period the external neutron source is also constant and equal to q_0. In order to study the variation of reactivity associated to this regime one makes $q(t) = q_0$ and $P(t) = P_0$ in equation (37), obtaining the following expression for reactivity associated to the system:

$$\rho(t) = \varsigma\left(1 - \frac{1}{P_0}\right) - \frac{q_0}{P_0}, \tag{38}$$

where ζ is defined in equation (20). From equation (38) one sees that system reactivity is strongly dependent on the source as well as on the level of power required.

5.2 Linear variation of the nuclear power

During the start-up process of a nuclear reactor one can represent a linear power rise thus:

$$P(t) = P_0 + \omega t, \tag{39}$$

where P_0 is the initial reactor power and ω is the rise rate for the nuclear power. Let us consider an external source that varies linearly, written by:

$$q_{ext}(t) = q_0 + \varepsilon t, \tag{40}$$

where q_0 is the initial intensity of the source of neutrons inserted in the system and ε is the rate of linear insertion for external neutrons.

The main motivation of this source model is the possible practical application of the obtaining of a smooth and linear power rise during reactor start-up that does not harm the structural composition of its components in the case of ADS reactors [1].

Where the normalized nuclear power is written by equation (39), one has that $P^{(1)}(t) = \omega$ and $P^{(2)}(t) = 0$. Replacing these results in equation (37) one has, considering a group of generalized neutron precursors, the following expression for reactivity:

$$\rho(t)=\beta_{eff}+\frac{l_{eff}\,\omega}{P_0+\omega t}+\varsigma\left(1-\frac{1}{P_0+\omega t}\right)-\frac{\beta_{eff}\left[\left(\lambda P_0-\omega\right)+\omega e^{-\lambda_i t}+\lambda\omega t\right]}{\lambda\left(P_0+\omega t\right)}-\frac{q_0+\varepsilon t}{P_0+\omega t}. \tag{41}$$

For a slow power rise the reactivity tends to remain on a constant level given by:

$$\rho^{ass.}(t)=\lim_{t\to\infty}\rho(t)=\varsigma-\frac{\varepsilon}{\eta\omega} \tag{42}$$

The asymptotic behavior $\rho^{ass.}(t)$ for reactivity as expressed by equation (42) depends on the level of sub-criticality of the system and on the velocity of insertion for the neutrons coming from the external source. For a system that nears criticality and a constant source ($\varsigma\approx 0$ and $\varepsilon=0$) the result obtained from equation (42) reproduces the expected result $\rho^{ass.}(t)=0$. Results obtained from equation (41) will be presented in the results section.

5.3 Exponential variation of the nuclear power

In this case we consider a steep nuclear power rise considering an external neutron source that varies linearly in time as the one described by equation (40). A simple model for a situation of this kind is that of the exponential variation of the nuclear power, represented by the following expression:

$$P(t)=P_0e^{\omega t}. \tag{43}$$

In this case $P^{(1)}(t)=P_0\omega e^{\omega t}$ and $P^{(2)}(t)=P_0\omega^2e^{\omega t}$. By replacing the derivatives in equation (43) in equation (37) one has the following expression for reactivity, considering a group of generalized neutron precursors:

$$\rho(t)=\beta_{eff}+l_{eff}\,\omega+\varsigma\left(1-\frac{e^{-\omega t}}{P_0}\right)-\frac{\left(q_0+\varepsilon t\right)e^{-\omega t}}{P_0}-\frac{\lambda\beta_{eff}}{\lambda+\omega}\left[\omega e^{-(\lambda+\omega)t}+1\right]. \tag{44}$$

The asymptotic behavior $\rho^{ass.}(t)$ for the reactivity, as expressed by equation (44) is provided by the following expression:

$$\rho^{ass.}(t)=\lim_{t\to\infty}\rho(t)=\beta_{eff}+l_{eff}\,\omega+\varsigma-\frac{\lambda\beta_{eff}}{\lambda+\omega}. \tag{45}$$

From equation (45) once concludes that the external source loses importance with the passing of time and ceases to drive the system, as expected. The results obtained from equation (45) are shown in the next section.

6. Numerical Method

As a reference in the validation of the expression for the reactivity obtained in this paper, equation (37), equation (31) will be numerically calculated, with its discretisation being written thus:

$$\rho_g^j = \beta_{eff} + \frac{l_{eff}}{P^j}\frac{P^j - P^{j-1}}{\delta t} + \varsigma\left(1 - \frac{1}{P^j}\right) - \frac{q_s^j}{\eta P^j} - \frac{P_0}{P^j}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t_j} - \frac{1}{P^j}\sum_{i=1}^{6}\lambda_i\beta_i\tilde{H}_{i,j}, \tag{46}$$

Where $\delta t \equiv t_j - t_{j-1}$, $P^j \equiv P(t_j)$, $q_s^j \equiv q_{ext}(t_j)$, $\rho_g^j \equiv \rho(t_j)$ and t_j is the time passed in the jth iteration and

$$\tilde{H}_{i,j} = H(t) \equiv \int_0^{t_j} e^{-\lambda_i(t_j - t')} P(t')\,dt'. \tag{47}$$

For the numerical implementation of function $\tilde{H}_{i,j}$ it is necessary that a recursive relation is obtained from equation (47). For that, let us consider an instant $t_j \equiv t_{j-1} + \delta t$ such that:

$$\tilde{H}_{i,j+1} = \int_0^{t_j+\delta t} e^{-\lambda_i(t_j+\delta t - t')} P(t')\,dt' = e^{-\delta t}\left[\int_0^{t_j} e^{-\lambda_i(t_j - t')} P(t')\,dt' + \int_{t_j}^{t_j+\delta t} e^{-\lambda_i(t_j - t')} P(t')\,dt'\right]. \tag{48}$$

Recognizing in equation (48) the very function $\tilde{H}_{i,j}$ it is possible to establish the following recurrence relation for the pseudo history of power:

$$\tilde{H}_{i,j+1} = e^{-\delta t}\left[\tilde{H}_j + R(t_j + \delta t)\right], \tag{49}$$

where function $R(t_j + \delta t)$, written thus:

$$R(t_j + \delta t) = \int_{t_j}^{t_j+\delta t} e^{-\lambda_i(t_j - t')} P(t')\,d\,'. \tag{50}$$

Equation (50) can be numerically integrated at every instant in time using Simpson 3/8 method [8], which consists of approximating integrals, defined from the following expression:

$$\int_a^b F(x)\,dt \approx \frac{(b-a)}{8}\left[F(a) + 3F\left(\frac{2a+b}{3}\right) + 3F\left(\frac{a+2b}{3}\right) + F(b)\right]. \tag{51}$$

Thus, the reactivity can be calculated from equation (46) where function $\tilde{H}_{i,j}$ is updated according to the following recurrence relation:

$$\tilde{H}_{i,j+1} = e^{-\delta t}\left\{\tilde{H}_j + \frac{\delta t}{8}\left[P(t_j)e^{\lambda_i t_j} + 3P\left(t_j + \frac{\delta t}{3}\right)e^{\lambda_i\left(t_j + \frac{\delta t}{3}\right)} + 3P\left(t_j + \frac{2\delta t}{3}\right)e^{\lambda_i\left(t_j + \frac{2\delta t}{3}\right)} + P(t_j + \delta t)e^{\lambda_i(t_j + \delta t)}\right]\right\}. \tag{52}$$

All numerical simulations used mesh point $\delta t = 10^{-6} s$.

7. Results

Figure 1 shows reactivity variation as obtained from a linear variation in nuclear power, represented by P(t)=1+0.02t, using equation (41) and the detailed numerical method of reference represented by equation (46). The external source considered is represented by Q(t)=1+0.002t.
The following nuclear parameters were used, valid for all numerical simulations in this paper: λ=0.00127, ζ=0.0493, β_{eff}=0.00009 and $l_{eff.}$=6.25x10^{-5}.

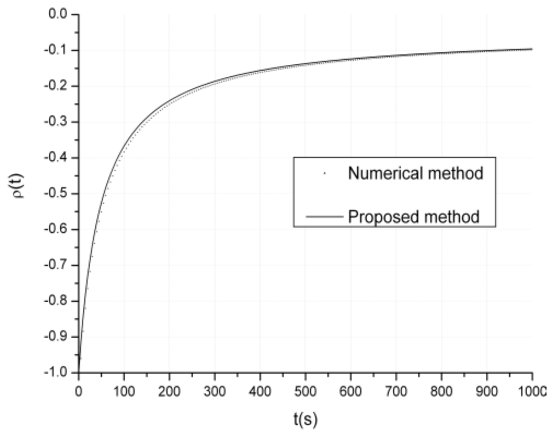

Figure 1 P(t)=1+0.002t calculated from reactivity $\rho(t)$ written by equation (41) and Q(t)=1+0.002t.

Figure 2 shows the variation in reactivity as obtained from an exponential variation of the nuclear power written by P(t)=exp(0.12353t) using equation (44) also taking the numerical method as reference, equation (46). The external source used is represented by Q(t)=1+0.002t.

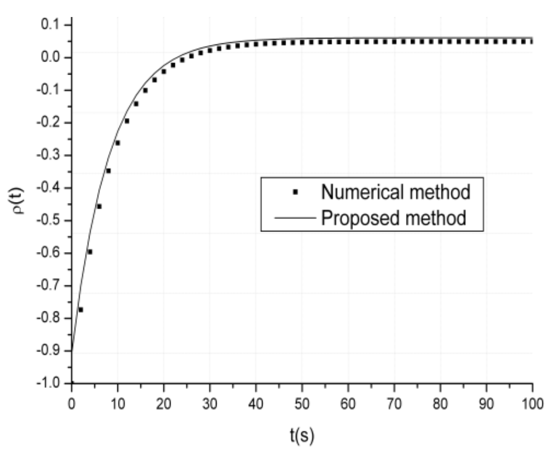

Figure 2 P(t)=exp(0.12353t) calculated from reactivity $\rho(t)$ written by equation (44) and Q(t)=1+0.002t.

Through the comparison of the results shown in Figure 2 it is possible to see the concordance between the analytical method to calculate the reactivity for subcritical systems with the inverse method, equation (37), and the numerical reference method. Thus, a study can be made on the influence of the sources of neutrons on the reactivity in subcritical systems via equation (37).
The variation of reactivity obtained from equation (41) for different variation of the source of neutrons is found in Figure 3 for a nuclear power represented by P(t)=1+0.2t.

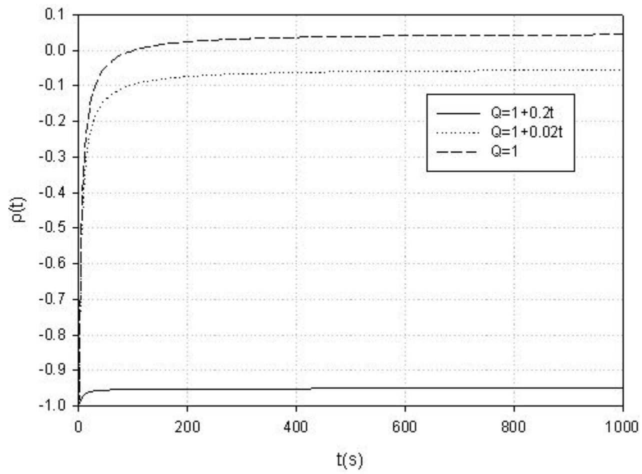

Figure 3 Calculation of reactivity $\rho(t)$ from equation (41) for different external sources and P(t)=1+0.2t.

Through the results shown in Figure 3 one can see the influence of the external neutron source in the variation of the reactivity when it rises linearly according to equation (39).

The variation of reactivity obtained from equation (44) for an exponential power increase for different profiles of external neutron sources can be seen in Fig. 4.

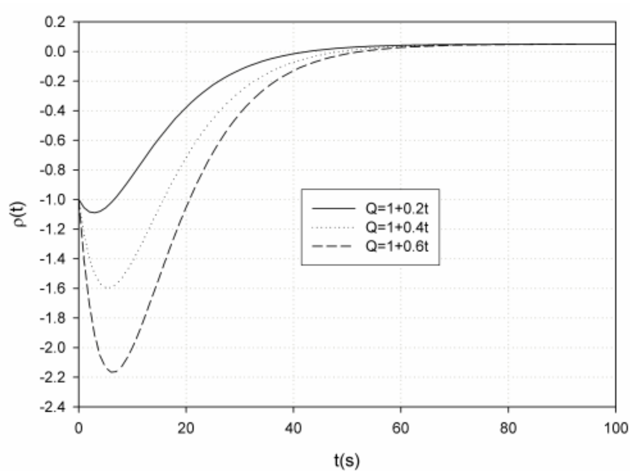

Figure 4 Calculation of reactivity $\rho(t)$ from equation (44) for different external sources and P(t)=exp(0.12353t).

8. Conclusions

The inverse point kinetic equation for subcritical systems presented in this paper, equation (37), shows the possibility of considering arbitrary sources, inclusive those variable in time, to determine reactivity in subcritical systems. The formulation presented is simple and allows one to obtain reactivity behaviour data as resulting from variations in time of the nuclear power and of the external neutron source. The analytical results obtained have shown to be precise through comparison with results found in the literature and numerical reference data.

One of applications of the proposed method is the possibility of developing reactimeters that allow the continuous monitoring of subcritical systems, independently from the nuclear power history.

9. References

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