Theoretical and Experimental Analysis of Ultrasonic Cross Correlation Flow Measurement Technology

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Abstract

Advanced Measurement and Analysis Group Inc. (AMAG) is a developer and manufacturer of the non-intrusive ultrasonic cross correlation flow meter CROSSFLOW, which is installed in nuclear reactors in Canada and around the world. To meet growing demand for better accuracy, AMAG and De Montfort University started a joint project to develop a mathematical model of the meter, based on turbulence dynamics. In the first year, a simplified model was developed. Predictions made by this model were confirmed by conducted experiments and computational flow simulations. The model can be used to predict the effect of flow disturbances on meter readings.

1. Introduction

Measuring turbulent pipe flow is of great importance in the nuclear power industry. The ultrasonic cross correlation flow meter can be used in environments of high radiation, temperature, and temperature variation, and it is non-intrusive. The technology was originally developed in the UK for multi-phase flows [1]. Canadian General Electric developed the first cross correlation meter for a single-phase flow [2]. Advanced Measurement and Analysis Group Inc. (AMAG) is a developer and manufacturer of the non-intrusive ultrasonic cross correlation flow meter CROSSFLOW, which is used in Canadian reactors and all over the world for feed-water and for reactor coolant flow measurements, and other applications [3,4]. The ultrasonic cross correlation flow meter measures the velocity of turbulent eddies in the flow, and not the average flow velocity. There are methods for determining the average flow velocity from information provided by the ultrasonic cross correlation flow meter, though these methods are mostly empirical [3,4,5]. To expand the capability of cross correlation flow measurement technology, it is necessary to derive a theoretical model that is based on turbulence in a pipe. To meet growing demand for higher accuracy and reliability of flow measurements in nuclear power plants, in 2009 AMAG and De Montfort University (DMU) started a joint project to develop a mathematical model of the cross correlation flow meter, based on accurate description of the dynamics of turbulent eddies, and on the effect of the eddies on the ultrasonic wave generated by the meter.

This paper explains how cross correlation flow measurement technology works, and then gives a brief description of research conducted in the first year of the project, to explore the possibility of predicting the cross-section average pipe flow velocity based on measured velocity of turbulent eddies. A simplified theoretical analysis of cross correlation flow measurement and related fluid dynamics phenomena has been conducted. Predictions of the theoretical analysis were compared with results of tests conducted in the AMAG laboratory, and with conducted computational simulations of the AMAG flow loop. The results of the flow tests and computational simulations confirmed theoretical predictions. Results of the simplified theoretical model can be used to predict the effect of flow disturbances, such as upstream elbows, on meter readings.

The results of the study presented in the paper could not be obtained without support from many people from AMAG and DMU. Especially, authors express their gratitude to Samir Fahs, Yuri Gurevich, Varun Kanda, Brendan Sharp, and Armando Lopez from AMAG, and Professor Goman from DMU.

2. Underlying Principles of Ultrasonic Cross Correlation Flow Meter

The simplest design of an Ultrasonic Cross Correlation Flow Meter consists of the following components:

- Four ultrasonic probes (two transmitters and two receivers)
- A Signal Conditioning Unit (SCU)
- Flow measurement computer, called Signal Processing Unit (SPU)
- Cables connecting SCU to probes and SPU

Setup of the system is shown in Figure 1.

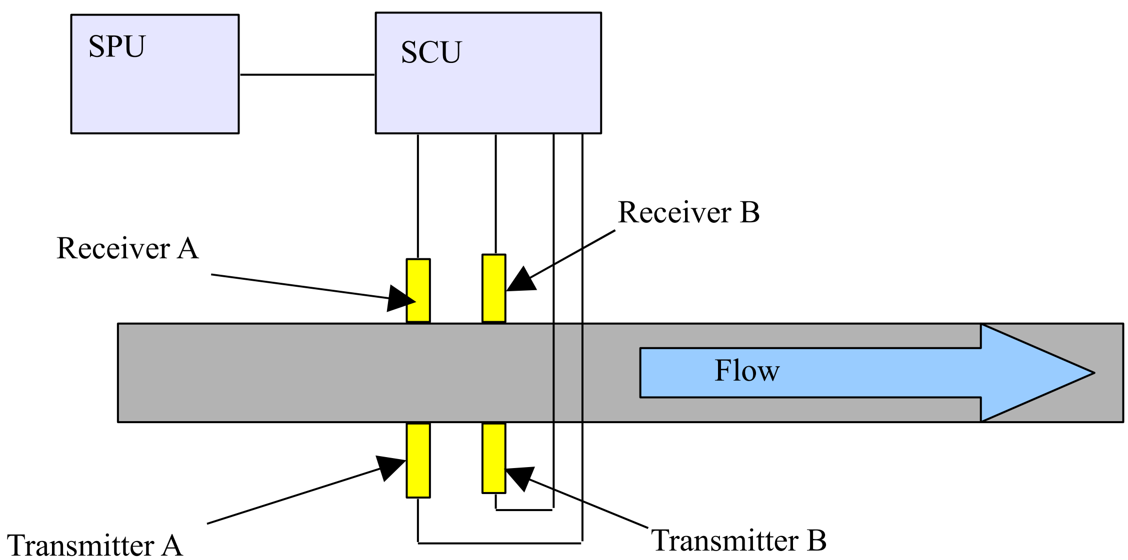

Figure 1 The ultrasonic cross correlation flow meter.

The Transmitter and Receiver upstream of the flow are called Transmitter A and Receiver A respectively, while the Transmitter and Receiver downstream of the flow are called Transmitter B and Receiver B, as is shown in Figure 1. Transmitter A and Receiver A are set up opposite of each other at the same pipe cross-section. The distance between them is hence the outer diameter of the pipe. Transmitter B and Receiver B are set up in the same manner at a downstream cross-section. The two cross-sections are called Cross-Section A and Cross-Section B respectively. (See Figure 2)

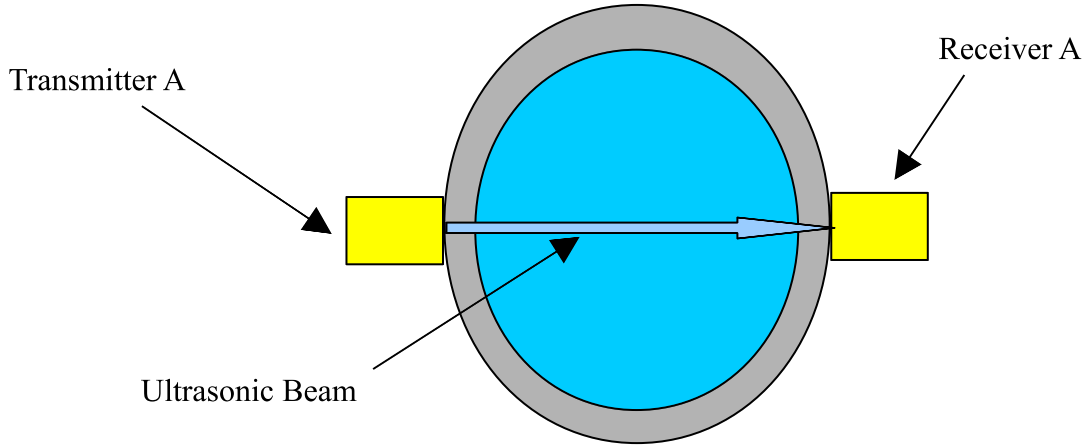

Figure 2 Pipe cross-section with attached transmitter and receiver.

The four probes along with the mechanism that holds them on the pipe is called the transducer, the distance between Cross-Section A and Cross-Section B is called the transducer spacing. The cross-section of the pipe located at the midpoint between Cross-Section A and Cross-Section B is referred to as the cross-section at which the meter is installed.

The flow measurement process works as follows (Note: The description is simplified.):

1 – Flow parameters, such as pipe diameter, fluid density, temperature, etc are entered into the SPU.

2 – Suitable ultrasonic frequencies for the particular pipe, called Carrier Frequencies, are determined such that the desirable amplitude of ultrasonic signals is obtained by the receivers. The process of determining the carrier frequencies is not described in this report.

3 – Transmitters A and B continually send two different ultrasonic signals with Carrier Frequencies.

As turbulent eddies pass through the ultrasonic beams, the beam's frequency slightly alters due to a Doppler shift caused by the eddies. The signal received by a receiver is hence different than that being sent by a transmitter. (See Figure 3)

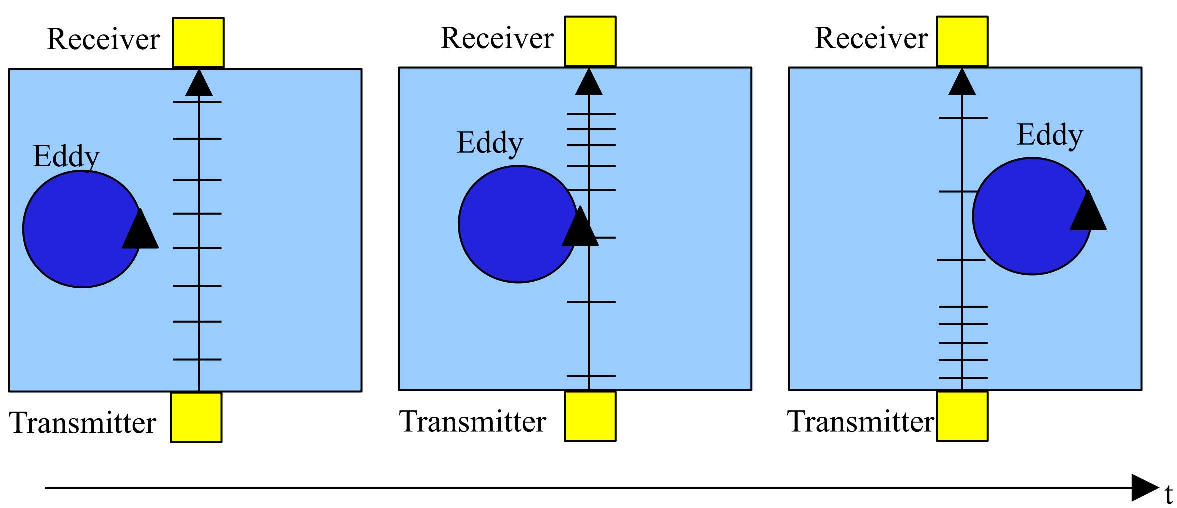

Figure 3 Frequency shift of an ultrasonic beam as a rotating eddy passes.

When eddies passing though the pipe pass through both ultrasonic beams, both beams will be altered in frequency nearly identically. The reason the two beams will not be altered identically is that eddies can deform while between the two beams [6].

2.1 Cross Correlation Function

Let the signal received by Receiver A be called x'(t), and the signal received by Receiver B be called y'(t). These received signals are sent to the SCU and demodulated to remove carrier frequencies, producing demodulated signals x(t) and y(t) respectively. The two demodulated signals will be nearly identical, except for a shift in time. Specifically, y(t) will be delayed in time by the amount of time it took the frequency altering eddies to move from Cross-Section A to Cross-Section B. This amount of time is called the time delay, and is usually represented by τ.

Time delay τ is calculated as the position of the maximum of the cross correlation function $R_{xy}(x)$, defined by Equation (1).

$$R_{xy} = \frac{1}{T}\int_0^T x(t)y(t+\xi)dt \tag{1}$$

R_{xy} is effectively a measure of the similarity between x(t) and y(t) when y(t) is shifted in time by a value of ξ. Consider the square of the difference between x(t) and $y(t+\xi)$. The average of the square of the difference between x(t) and $y(t+\xi)$ over a specified time interval T will be a measure of the similarity between these two signals.

$$\frac{1}{T}\int_0^T (x(t) - y(t+\xi))^2 dt = \frac{1}{T}\int_0^T (x^2 + y^2)dt - 2\frac{1}{T}\int_0^T x(t)y(t+\xi)dt \tag{2}$$

The first term on the right side of (2) is positive, and virtually independent of ξ when ξ is much less than T, as it usually is. The minimum of the integral on the left side then corresponds to the maximum of the last term on the right side, which corresponds to the maximum of the cross correlation function defined by (1).

The location of the global maximum of R_{xy} is the time is takes turbulent eddies to pass the distance of the transducer spacing at the location of the pipe where the transducer is set up. As mentioned above, it is called the time delay. From the time delay, a mass flow rate is calculated by the formula

$$Q = \frac{d}{\tau}\rho\pi r^2 \tag{3}$$

Where d is the transducer spacing, τ is the time delay, ρ is the fluid density, and r is the pipe inner radius.

With the current level of cross correlation flow measurement technology, it is possible, under proper conditions, to measure flow with an accuracy of 0.5%. It is unclear though, when such proper conditions exist.

3. Theoretical Analysis

The demodulated signal derived from a received signal can be represented as

$$\phi(x,t) = \frac{f}{c^2}\int_{-R}^{R} v(x,y,t)\,dy \tag{4}$$

where f is the carrier frequency, c is the speed of sound in the fluid, R is the pipe radius, and v is the y-component of turbulent velocity fluctuations [3]. Here, the x-direction is defined as the direction of the flow, and the y-direction is the direction of the ultrasonic beam. Integration is performed along the path of the beam from the transmitter to the receiver. The result of integration is a function of position x and time t.

3.1 Curl Transport

Since the cross correlation flow meter measures the transportation of turbulent structures, its measurements are more related to the curl field than the velocity field. A Mathematical connection between curl transport and fluid transport will now be demonstrated using the Navier-Stokes Equations.

The Navier-Stokes equations for the transport of velocity of a fluid are:

$$\frac{d\vec{u}}{dt} + (\vec{u}\cdot\nabla)\vec{u} = \frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} \tag{5}$$

[7]. By taking the curl of every term in the Navier-Stokes Equations, one obtains the equations for the transport of the curl of a fluid:

$$\frac{d\vec{\omega}}{dt} + (\vec{u}\cdot\nabla)\vec{\omega} - (\vec{\omega}\cdot\nabla)\vec{u} = \nu\nabla^2\vec{\omega}\ , \quad \vec{\omega} = \nabla\times\vec{u} \tag{6}$$

The above equation, (6), may be written in index summation notation as

$$\frac{d\omega_j}{dt} + u_i\frac{d}{dx_i}\omega_j - \omega_i\frac{d}{dx_i}u_j = \nu\frac{d\omega_j}{dx_i dx_i} \tag{7}$$

A two dimensional approximation will now be considered, where

$$\vec{u}(x,y,t) = (\, u(x,y,t)\, ,\, v(x,y,t)\, ,\, 0\,)\ , \quad \vec{\omega} = \nabla\times\vec{u} = (\, 0\, ,\, 0\, ,\, \frac{dv}{dx} - \frac{du}{dy}\,) \tag{8}$$

Since the curl has only a z-component, we will define

$$\vec{\omega} = (\, 0\, ,\, 0\, ,\, \omega\,) \tag{9}$$

With the approximations made above, and setting the viscosity term equal to zero because it is very small, the curl of the Navier-Stokes equations becomes

$$\frac{d\omega}{dt} + u\frac{d\omega}{dx} + v\frac{d\omega}{dy} = 0 \tag{10}$$

3.2 Deriving A Velocity Field From A Known Curl Field

The next stage in analysis is deriving the velocity field from a known curl field. When particular criteria are met, it is possible to determine a velocity field from a given curl field by the following formula [8].

$$\vec{u} = \int\int\int \frac{\vec{\omega} \times (x-\xi, y-\eta, z-\zeta)}{((x-\xi)^2+(y-\eta)^2+(z-\zeta)^2)^{3/2}} d\xi d\eta d\zeta \tag{11}$$

The formula (11) gives the velocity as a function of coordinates x, y, and z. ω is a function of coordinates ξ, η, and ς. Integration is performed over these coordinates, summing the influence of the curl field at every location.

A two dimensional approximation would set

$$\vec{\omega} = (0,\ 0,\ \omega_\zeta) \tag{12}$$

By substituting (12) into (11), considering only the y-component of the velocity since that is the component the demodulated signal is dependent on, and integrating over ς, one obtains the formula

$$v = 2\int\int \frac{\omega_\zeta (x-\xi)}{(x-\xi)^2+(y-\eta)^2} d\xi d\eta \tag{13}$$

for determining the y-component of a velocity field, v, corresponding to the curl field ω_ς.

Since integration is performed over the entire domain, a curl field must be assigned over the entire domain, that would simulate the conditions of pipe flow. Since the integrated term is inverse proportional to the square of the distance between the location of the contributing curl and the location at which one would like to know the velocity, it may be the case that at sufficient enough a distance, the value of curl will be insignificant, and one may assign a curl value of zero. It can be demonstrated that a curl field simulating conditions of pipe flow can be assigned such that such a sufficient distance exists, and the contribution of curl from non-turbulent aspects of the velocity field are canceled out during integration. The details of how such a curl field can be assigned is not covered in this report.

Below the integral in (4) is given.

$$\int_{-R}^{R} v(x,y,t)dy \tag{14}$$

By defining ω_ς in a way that meets the criteria described in the previous paragraph, the v in (14) is equal to the v in (4). By then substituting (13) into (14) and integrating over y, one obtains the formula

$$\int_{-R}^{R} v(x,y)dy = 2\int\int \omega_\zeta(\tan^{-1}(\frac{R-\eta}{x-\xi})+\tan^{-1}(\frac{R+\eta}{x-\xi}))\,d\xi\,d\eta \tag{15}$$

The factor that ω_ς is multiplied by in (15) is simply the angle α in Figure 10, with a positive sign for ξ less than x, and a negative sign for ξ greater than x, where x is the x-coordinate of the location of the beam, and ξ is the x-coordinate of the location of the contributing curl source (See figure 10).

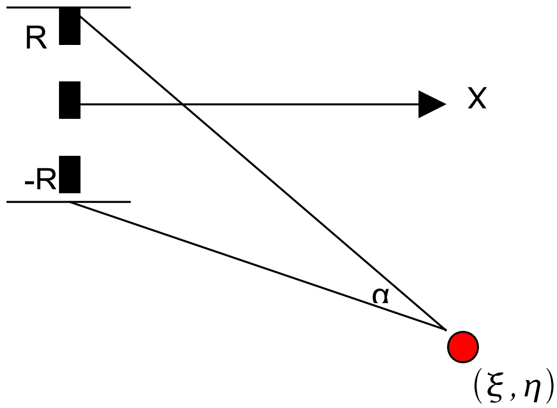

Figure 10 Angle formed by connecting curl source with ends of ultrasonic beam.

In polar coordinates, the formula for α is

$$\alpha(r,\theta,x) = \tan^{-1}(\frac{R-r\sin(\theta)}{x-r\cos(\theta)}) + \tan^{-1}(\frac{R+r\sin(\theta)}{x-r\cos(\theta)}) \tag{16}$$

For the polar coordinate equivalent to Figure 10, see figure 11.

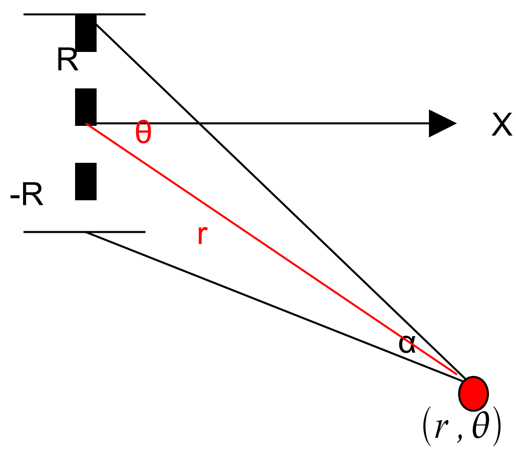

Figure 11 Angle α in polar coordinates.

For any given r value, the absolute value of α is largest for values of θ that are integer multiples of π. For such values of θ, it can be shown with limit analysis that the absolute value of α tends to zero as 2R/r. For other values of θ, α tends to zero even faster.

To demonstrate that the integration on the left side of (15) has a solution, we will consider the integral

$$\int_0^{\infty} \frac{\omega_\zeta}{r} dr \tag{17}$$

It is obvious that (15) has a solution if and only if (17) has a solution. If $\omega_ς$ is a constant, (15) is undefined. But, the nature of $\omega_ς$ is that it is not a constant. Recall that $\omega_ς$ is assigned such that non-turbulent aspects of the velocity field are canceled out during integration. The integral of $\omega_ς$ will only be effected by random turbulent perturbations. Hence, integrating $\omega_ς$ from 0 to a value *a* should give a value of the same order of magnitude as the value obtained by integrating $\omega_ς$ from 0 to a value *b*, for all *a* and *b*. Hence, the term under the integral in (15) tends to zero an order of magnitude faster than 1/r, and hence, (15), and (17), are defined. By substituting (15) into (4), the formula for the demodulated signal, derived from fluid dynamics theory, is

$$\phi(x,t) = \frac{2f}{c^2}\int_0^{\infty}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \omega_\zeta(r,\theta,t)\alpha(r,\theta,x)\,d\theta\, dr \tag{18}$$

3.4 Demodulated Signal Transport

By changing the names of some variables, the formula for the demodulated signal may be rewritten as

$$\phi(x_{0,}t)\frac{2f}{c^2}\int\int \alpha(x_0,x,y)\omega(x,y,t)\,dx\,dy \tag{19}$$

where integration is performed over the entire domain. This form of the demodulated signal will be used for the remainder of this report.

By multiplying (10) by α, 2f, and 1/c^2, integrating over x and y, and performing Taylor series approximations, one obtains the formula

$$\frac{d\phi(x_0,t)}{dt} + U\frac{d\phi(x_0,t)}{dx_0} + g(x_0,t) = 0 \tag{20}$$

where U is the cross-section average flow velocity, and g is a function related to curl transport and depends on both axial and radial components of the velocity. Conducting farther Taylor series approximations, one may obtain the formula

$$\phi(b,t) = \phi\left(a,\, t-\frac{(b-a)}{U}\right) - \frac{(b-a)}{U}g(x_0,t) \tag{21}$$

where a and b are the locations of cross-sections A and B respectfully.

If g would equal zero, the demodulated signal would move along a pipe with velocity U without changing its state, and the velocity measured by the cross correlation flow meter would always be equal to U. The right most term in (21) is responsible for the deviation of the measured velocity from U. This term is proportional to the transducer spacing (b-a), and is larger for larger values of g. From these theoretical results, three predictions may be made:

1 – Larger values of g coincide with larger deviations of measured velocity from U.

2 – Larger values of g coincide with larger dependence of measured velocity on transducer spacing.

3 – Since U is effectively a flat approximation of the flow velocity profile, where the axial component of the velocity is predominant, g should be smaller in situations where the flow velocity profile is flatter, and hence, deviation of the measured velocity from U should grow with non-flatness of the flow velocity profile.

4. Experimental Results

Tests were conducted on the AMAG flow loop, measuring flow at 8 locations along a straight pipe run downstream of a 90-degree elbow, with three different transducer spacings at each location . The 8 locations along the pipe were spread between 6 and 50 pipe diameters from the upstream elbow, as shown in figure 12.

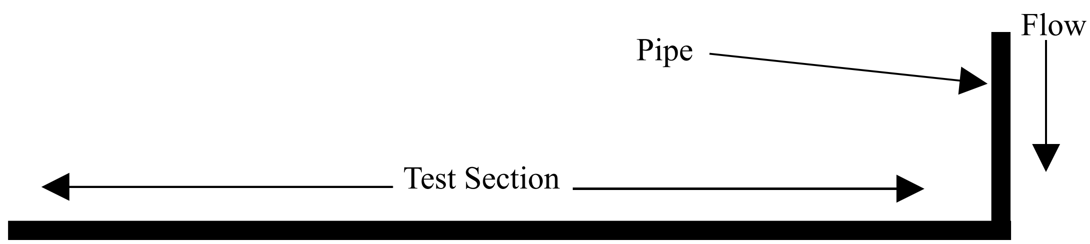

Figure 12 Piping configuration for test.

The actual cross-section average flow velocity for these tests was calculated from previously developed methods [3]. For each location along the pipe, three values for measured velocity were obtained, one for each transducer spacing. The average of these three measured velocity values was taken and will hence fourth be referred to as *average measured velocity*. The three measured velocity values were also plotted against transducer spacing, and a linear regression was made. The magnitude of the slope of the linear regression, hence fourth referred to as *slope*, is an indicator of dependence of measured velocity on transducer spacing.

The graph in figure 13 plots the normalized difference between average measured velocity and U, against the slope, for each of the 8 locations along the pipe. Each point on the plot is generated by data from one of the locations along the pipe where measurements were conducted. The normalized difference between the average measured velocity and U is hence fourth referred to as *deviation from U*. If the first two predictions are true, experimental results should show that deviation from U increases as slope increases. This plot clearly illustrates that this is in fact the case.

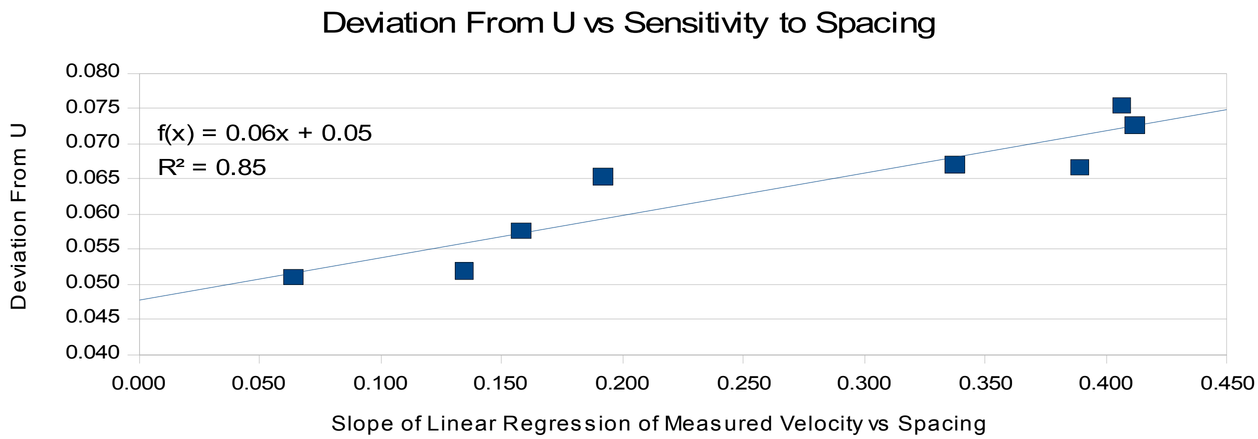

Figure 13 Relation between deviation of average measured velocity from U, and dependence of measured velocity on transducer spacing.

5. Computational Fluid Dynamics

Computational simulations of the AMAG flow loop were conducted. The flow velocity profile was calculated at 6 locations along the section of the pipe that flow measurements were performed at during experimental tests. For each of these six locations, a measure of non-flatness of the flow velocity profile was determined as follows: The velocity values close to the pipe wall where the velocity goes to zero were discarded. The mean value of the remaining velocity values was calculated, the deviation of remaining velocity values from the mean was determined, and the root mean square of these deviations was calculated. This final value, the root mean square of the deviations, will hence fourth be referred to as non-flatness.

According to the third prediction, where the axial component of velocity is predominant, as non-flatness of the flow velocity profile increases, the deviation from U should also increase. The graph in figure 14 plots the deviation of the average measured velocity from U against the non-flatness of the flow velocity profile. Each point on the plot is generated by data from one of the locations along the pipe where the flow velocity profile was calculated. With the exception of one point, the results show a clear linear relation confirming the third prediction.

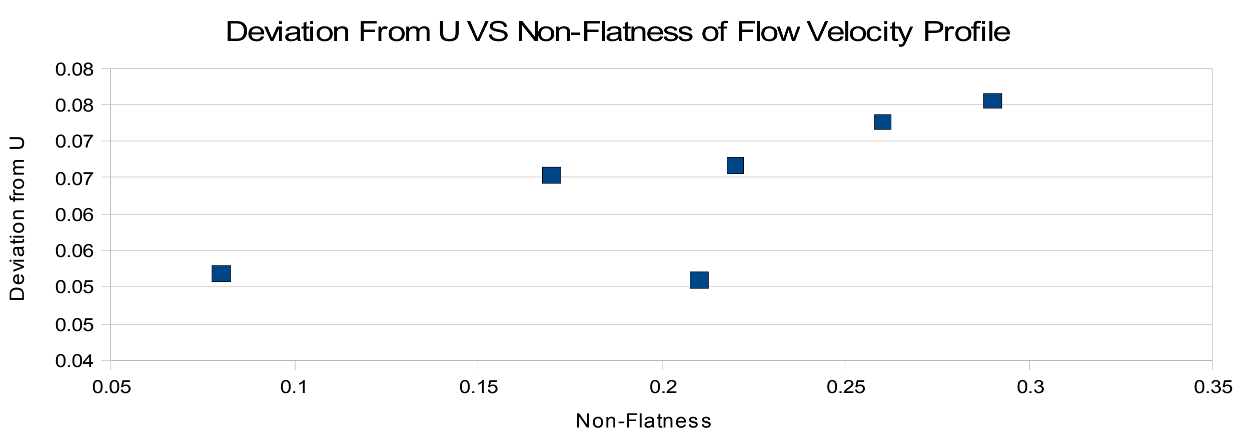

Figure 14 Relation between deviation of average measured velocity from U and non-flatness of flow velocity profile.

The point that is the exception, the third point from the left, is generated by data from 6 diameters from the upstream elbow. This is expected, because close to the upstream elbow, the radial component of the velocity is comparable to the axial component, and hence, the axial component is not predominant, and the necessary conditions for the third prediction are not present.

6. Conclusion

Qualitative results predicted by the developing theory have been confirmed with both experimental results and computational fluid simulation. Farther development of the theory would involve moving beyond the two dimensional approximation, and making quantitative, as well as qualitative, predictions to test against experimental and computationally simulated results.

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