Diametral Creep Prediction of Pressure Tube using Statistical Regression Methods

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Abstract

Diametral creep prediction of pressure tube in CANDU reactor is an important factor for ROPT calculation. In this study, pressure tube diametral creep prediction models were developed using statistical regression method such as linear mixed model for longitudinal data analysis. Inspection and operating condition data of Wolsong unit 1 and 2 reactors were used. Serial correlation model and random coefficient model were developed for pressure tube diameter prediction. Random coefficient model provided more accurate results than serial correlation model.

1. Introduction

Pressure tube in CANada Deuterium Uranium-Pressurized Heavy Water (CNADU-PHW) reactors occur dimensional change by fast flux, high temperature, and stress during operating condition. Especially, fast neutron irradiation enhances the dimensional change. As a result of irradiation-induced creep, diametral expansion causes coolant by-pass around the fuel and decrease cooling efficiency of coolant. Decreased cooling efficiency reduces the margin of Critical Heat Flux (CHF). CHF is a key parameter determining the Critical Channel Power (CCP), which is used to calculate the Regional Overpower Protection Trip (ROPT) set point. Therefore, accurate prediction of pressure tube diametral creep is important for ROPT set point.

Existing creep prediction model, RC-1980, is based on creep mechanism which consider complex interaction among operation condition, crystallographic texture, and anisotropic microstructure and consist of additive components such as thermal creep, irradiation creep, and irradiation growth [1]. However, this model has a large prediction uncertainty and need a lot of experiment constant. Recently, statistical error modelling was developed using pressure tube inspection data from Bruce B Nuclear Generating Station (NGS) [2].

In this study, we predict the diametral creep of pressure tube using inspection and operating condition data of Wolsong CANDU reactors by statistical regression method.

2. General linear mixed model for longitudinal data

2.1 Longitudinal data analysis

Pressure tube diametral creep data are repeated measure data over time within subject. Longitudinal data are potentially correlated on the same subject. These correlations of longitudinal data produce a complicated covariance structure which was used to estimate the regression coefficient. However, estimated regression coefficients using liner model are no longer valid because two assumptions that are independent observation and equal variance are deviated. Therefore, another regression method is needed. General linear mixed model is available to dealing longitudinal data. General liner mixed model use the block-diagonal matrix which not only can examine the differences between subjects, but also examine the change within subjects. Also, subjects are independent each other and the measurements within each subject are correlated in block-diagonal matrix.

2.2 General linear mixed model for longitudinal data

The general liner mixed model extends the general linear model by the addition of random effect parameters and by allowing a more flexible specification of the covariance matrix of the random errors. The general form of linear mixed model is as flows;

$$y = X\beta + Z\gamma + \varepsilon \tag{1}$$

where y is the vector of observed responses, X is the design matrix of independent variables, β is the vector of fixed-effect parameters, Z is the design matrix of random variables, γ is the vector of random-effect parameters, and **ε** is the vector of random errors. Linear mixed model basically assume that, first random effects and error terms are normally distributed with mean zero and second, random effects and error terms are independent of each other and finally, the relationship between response variable and independent variable is linear.

Random-effects (subject-specific) parameters are allowed to vary over subjects and represent the natural heterogeneity in population. Fixed-effects (population-specific) parameters, β, represent the population average. Random error term, ε, have a constant variance and be decomposed as

$$\varepsilon_i = \varepsilon_{i,1} + \varepsilon_{i,2} \tag{2}$$

$\varepsilon_{i,1}$ is measurement error and $\varepsilon_{i,2}$ is error associated with the serial correlation. ε_i represents within subject variation and has a covariance matrix that is block diagonal corresponding to a subject.

Linear mixed model estimate not only the parameter of fixed effect as in the general linear model but also these of random effect, the covariance structure of random effects and random errors, respectively. The solution to parameters in linear mixed model is generalized least square (GLS) estimates. The generalized least squares take into account the covariance matrices G and R which are the covariance structure of random effects and random errors, respectively. The GLS solution of estimated fixed effects, $\hat{\beta}$, is as flows;

$$\hat{\beta} = (X'\hat{V}^{-1}X)^{-1}X'\hat{V}^{-1}y \tag{3}$$

The GLS solution of estimated random effects, $\hat{\gamma}$, is as flows;

$$\hat{\gamma} = \hat{G}Z'\hat{V}^{-1}(y - X\hat{\beta}) \tag{4}$$

where $\hat{V} = Z\hat{G}Z' + \hat{R}$.

To estimate the $\hat{\beta}$ and $\hat{\gamma}$, maximum likelihood estimation method is used to find the parameters. The maximum likelihood estimation method finds the parameters that are most likely to occur given data. The estimated parameters are derived by maximizing the likelihood function which is a mathematical expression that describes the joint probability of obtaining the data expresses as a function of the parameters. The log likelihood functions are as follows;

$$ML: l(G,R) = -\tfrac{1}{2}\log\left|V\right| - \tfrac{1}{2} r'V^{-1}r - \tfrac{n}{2}\log 2\pi \tag{5}$$

where $r = y - X(X'V^{-1}X)^{-1}X'V^{-1}y$ and $p = rank(X)$.

2.2.1 Serial correlation model (population-specific model)

Inferences of linear mixed model for longitudinal data are obtained by testing the fixed effects against the appropriate background variability, which is modelled by the covariance structure. This background variability may consist of several errors such as measurement, serial correlation, and random effects error.

The within subject variability is represented by serial correlation error. A common assumption is that the serial effect is a population phenomenon independent of the subject. Therefore, serial correlation model is able to explain both population average which mean fixed effects and variance within subject which was not explained by fixed effects. Also serial correlation model omit the random effect because assumption that all variability between subject is attributed to the fixed-effect variable. Thus, serial correlation model is as follows;

$$y = X\beta + \varepsilon_{i,1} + \varepsilon_{i,2} \tag{6}$$

2.2.2 Random coefficient model (subject-specific model)

In random coefficient model, the fixed effect parameters represent the population average of intercept and slopes. The random effect for intercept and slope represent the difference between population average and each subject.

$$y = \beta_0 + \beta_1 x_i + a_i + b_i x_i + \varepsilon_i \tag{7}$$

where β_0, β_0 is population intercept and slope and a_i, b_i is subject specific deviation of intercept and slope.

3. Applications

In this study, pressure tube creep predictions were developed. Inspection and operating condition data of Wolsong unit 1 and 2 were used. Pressure tube diameters were measured by CIGAR (Channel Inspection and Gauging Apparatus for Reactors) at 2136, 2967, 3383, 5726, 6577, and 7336 EFPDs (Effective Full Power Days) from Wolsong unit 1. In case of Wolsong unit 2, these were measured at 1501.04, 1943.71, and 3255.53 EFPDs. Operating condition data such as fast flux, temperature, and pressure data at each channel and bundle position were obtained by Time-averaged model considering refuelling history.

Creep predictions were separated between Wolsong unit1 and unit2 because they had different impurity concentration in which double melted Zr-2.5Nb alloy ingot were used in Wolsong unit 1 and quadruple melted Zr-2.5Nb ingot were used in Wolsong unit 2.

To find the relationship between dependant variable and independent variable exploratory data analysis were performed. Figure 1 show that dependant variable, diameter strain has positive correlation with fluence and temperature and negative correlation with pressure. Also interaction between fluence and pressure and fluence and temperature affect the diametral creep rate.

Selection of reasonable covariance structure is important because these were used to estimate the regression parameters. Kenward-Roger degrees of freedom calculation was used to find the reasonable block-diagonal covariance matrix. Unstructured covariance structure, compound symmetry structure, spatial power structure, spatial gaussian structure, spatial exponential structure, spatial linear structure, and spatial spherical were used, and then calculate the model statistics such as AIC (Akaike's Information Criterion), AICC (AIC corrected), and (BIC (Bayes' Information Criterion). Comparing the AIC, AICC, BIC, reasonable block-diagonal covariance structure in model were selected.

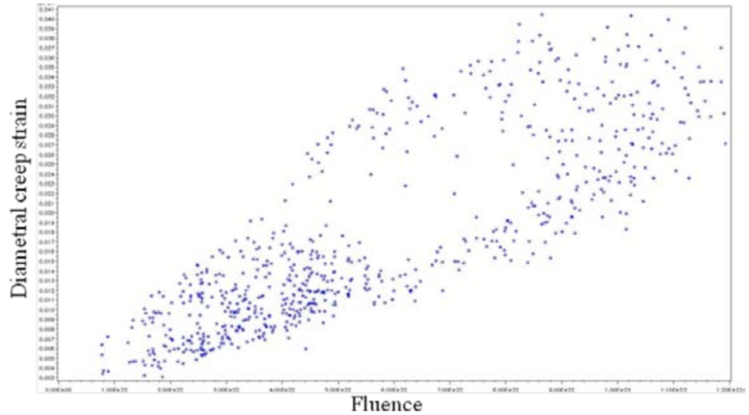
After an appropriate covariance structure is selected, appropriate mean model select using ML estimation method which mean that eliminate the least significant variable. Then refit the final model using REML method because REML estimators are superior.

$$REML: l_R(G,R) = -\tfrac{1}{2}\log\left|V\right| - \tfrac{1}{2}\log\left|X'V^{-1}X\right| - \frac{1}{2}r'V^{-1}r - \tfrac{n-p}{2}\log 2\pi \qquad (8)$$

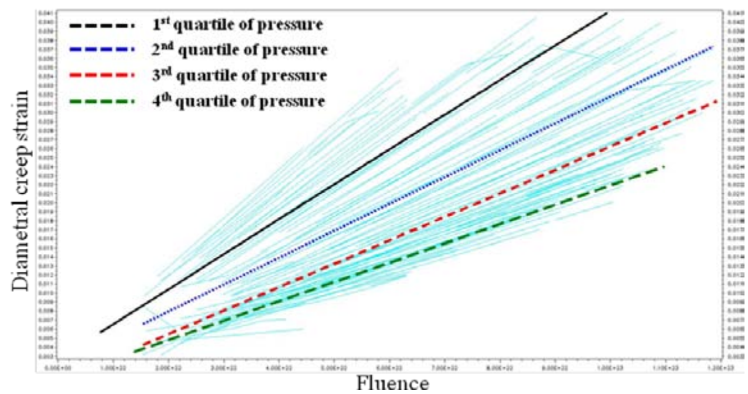
where $r = y - X(X'V^{-1}X)^{-1}X'V^{-1}y$ and $p = rank(X)$.

Finally, serial correlation model and random coefficient model were used for pressure tube diameter prediction. However, the model including between serial correlation and random effects are not developed because they has estimation and convergence problems.

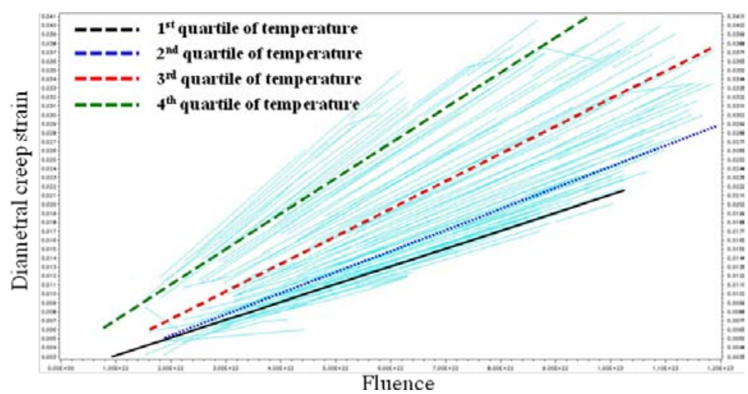
Figure 2 and 3 show the estimated residuals of Wolsong unit 1 and 2 which is measured diameter minus predicted diameter of pressure tube as a function of fluence. As you can see the figure 2 and 3 random coefficient model is superior to serial correlation model. Also, the model used Wolsong unit 2 is more precise than Wolsong unit 1. The RMS error of pressure tube diameter is 0.191549945mm and 0.022491846mm for Wolsong unit 1 data in serial correlation model and random coefficient model, respectively. In case of Wolsong unit 2, RMS error is 0.15909059mm and 0.002385766mm in serial correlation model and random coefficient model, respectively. Table 1 summarise the prediction ability of each liner mixed models.



(a)

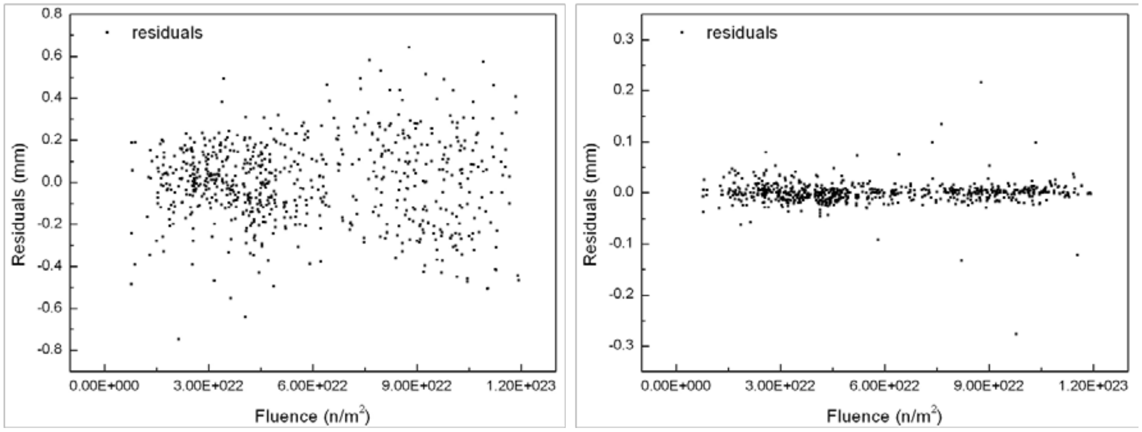


(b)



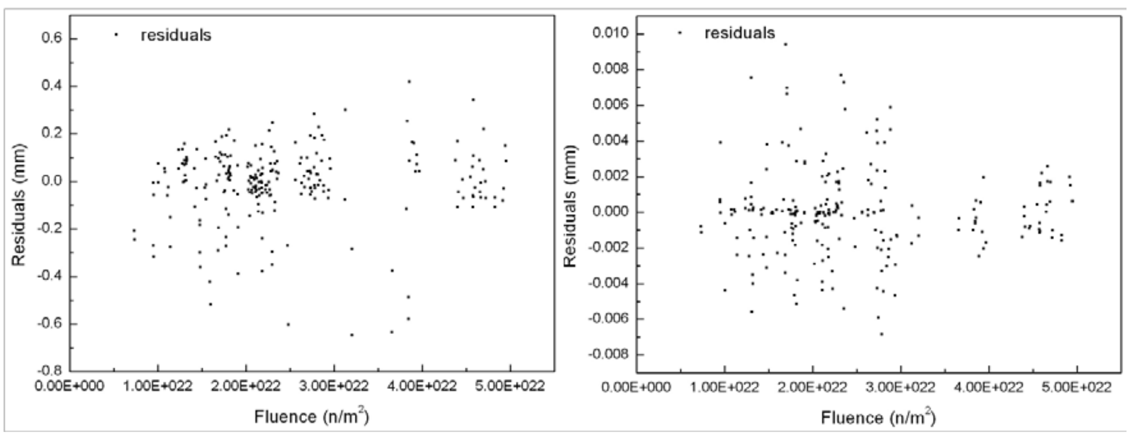
(c)

Figure 1 Diametral creep ration of pressure tube as a function of (a) fluence, (b) fluence and pressure (c) fluence and temperature



(a) (b)

Figure 2 Estimated residuals as a function of fluence for (a) serial correlation, (b) random coefficient model using Wolsong unit 1



(a) (b)

Figure 3 Estimated residuals as a function of fluence for (a) serial correlation, (b) random coefficient model using Wolsong unit 2

Table 2 RMS error of pressure tube creep

Data base	Wolsong unit 1		Wolsong unit 2	
Model	Serial correlation	Random coefficient	Serial correlation	Random coefficient
RMS error (mm)	0.191549945	0.022491846	0.15909059	0.002385766

4. Conclusion

In this study, pressure tube diametral creep prediction models for Wolsung 1 and 2 were developed by statistical regression method using inspection and operating condition data. Diameter strain showed positive correlation with fluence and temperature and negative correlation with pressure. Also interaction between fluence and pressure and fluence and temperature affected the diametral creep rate. Serial correlation model and random coefficient model were used for creep prediction. Random coefficient model provide more accurate results than serial correlation model.

5. References

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