Reliability Analysis of Nuclear Piping System using Semi-Markov Process Model

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Abstract

The paper presents a general model for evaluating the rupture frequencies and reliability of the piping system in nuclear power plant based on the theory of semi-Markov process. The proposed model is able to incorporate the effect of aging related degradation of pipes. Time dependent rupture frequencies are computed and compared against those obtained from the homogeneous Markov process model.

1 Introduction

Piping systems in nuclear power plants are susceptible to aging mechanisms such as corrosion, cracking and fatigue. Since data regarding pipe ruptures in the nuclear plant are rare, different modelling approaches have been developed in the literature to estimate the rupture frequency, which serves as a useful input of the frequency of an initiating event in probabilistic safety analysis (PSA). The Markov process model has been applied to analyze reliability of the piping system [1]. This method identifies various states of degradation, and requires input regarding the transition rates and average time taken to recover from one state to another. Based on this input, the Markov model is able to predict the rupture frequency in a future operating interval.

In the context of modelling of pipe failure, the Markov model consists of three main states or events other than the normal state of the pipe. They are flaw initiation, leakage and rupture. The Markov process model assumes constant transition rate, which means that the transition time follows an exponential distribution. In case of an aging piping system, this assumption is problematic. For example, flaw initiation rate in degrading pipes is likely to change with the age of the pipe. The exponential distribution with constant hazard rate cannot capture this aspect of aging. Typically, the Weibull distribution with time-dependent hazard rate is used for modelling the aging effects. In summary, the homogeneous Markov process model is not adequate for modelling the aging effects contributing to the pipe rupture.

The objective of this paper is to present a more advanced semi-Markov process (SMP) model for the evaluation of rupture frequencies including the effect of aging related degradation mechanisms.

Section 2 defines the problem of piping reliability analysis, as described in [1]. Section 3 discusses formulation of the piping system reliability using semi-Markov process model.

2 State Space Model for Piping Reliability Analysis

2.1 Formulation

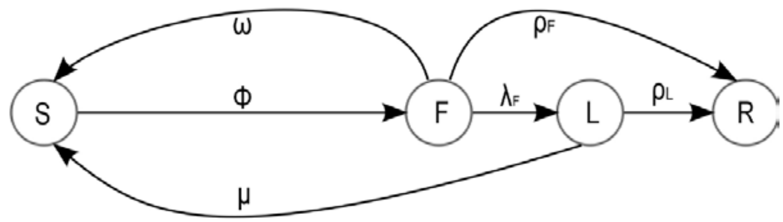

Figure 1. Four-state transition model for nuclear piping system degradation

The Markov process model to predict piping system reliability was proposed in [1]. The model consists of four states as seen in Figure 1. In the first state *S*, the piping system is assumed to be in a normal operational state. Flaws formed in the system grow gradually until they become detectable. At this time, the system moves to the state *F* with a transition rate of ϕ per year. A detectable flaw is either detected and repaired with a repair rate of ω, or further degrades until it becomes a detectable leak, or directly leads to rupture of the piping system. If the flaw is detected and repaired, the system moves back to state *S*, if not, it moves to either state *L* or *R*. The rates to transit from state *F* to *L* and *F* to *R* are λ_F and ρ_F per year respectively. In this model, the transitions $S \rightarrow F$, $F \rightarrow L$, and $L \rightarrow R$ represent gradual degradation processes.

A leak when detected is either repaired with a repair rate of μ or it develops into a rupture with a rate of ρ_L per year. If the leak is repaired, the system moves back to the state *S*, otherwise it transits to the state *R*.

The system is assumed to be non-repairable, fail state once a flaw or leak develops in to a rupture *i.e.*, the state *R* is an absorbing state. This is primarily done to evaluate the reliability of the piping system. It is assumed that all other repairs bring back the system to 'as good as new' condition.

This four state model is applicable to pipe failure mechanisms which are a combination of crack propagation (e.g. thermal fatigue near welds) and wall thinning (e.g. flow accelerated corrosion in pipe base metal) failure mechanisms. Failures due to severe loading such as overpressure are not accounted as observed from the absence of direct transitions $S \rightarrow R$ and $S \rightarrow L$. In other words, leak or a rupture can only occur from the state of an existing flaw [1].

2.2 Homogeneous Markov Process Model

The system state transition matrix [2] for the model in Figure 1 is given by:

$$T = \begin{bmatrix} -\phi & \phi & 0 & 0 \\ \omega & -(\omega + \lambda_F + \rho_F) & \lambda_F & \rho_F \\ \mu & 0 & -(\mu + \rho_L) & \rho_L \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

Let *S(t)*, *F(t)*, *L(t)*,and *R(t)* represent the time-dependent probabilities of being in the states *S,F,L,* and *R* respectively. These state probabilities can be obtained by solving the system of differential equations (2) with the initial condition *S(0)*=1, *F(0)=L(0)=R(0)=0.* The initial condition ensures that the system initially starts operating in state *S.*

$$\begin{aligned} dS(t)/dt &= \omega F(t) + \mu L(t) - \phi S(t) \\ dF(t)/dt &= \phi S(t) - (\omega + \lambda_F + \rho_F)F(t) \\ dL(t)/dt &= \lambda_F F(t) - (\mu + \rho_L)L(t) \\ dR(t)/dt &= \rho_F F(t) + \rho_L L(t) \\ \\ S(t) + F(t) &+ L(t) + R(t) = 1 \end{aligned} \tag{2}$$

This system of equations is based on the fact that the rate of change of probability of being in any state *S* is negatively proportional to the rate at which the transitions occur outward from *S* and positively proportional to the rate at which inward transitions occur from other states [2].

For example, from Figure 1, it is seen that there are two inward transitions in to state *S* originating from states *F* and *L* with transition rates ω and μ respectively. $\omega F(t)$ and $\mu L(t)$ are weighted transition rates added to $dS(t)/dt$. There is one outward transition to state *F* with transition rate ϕ and hence negatively influences $dS(t)/dt$ as seen in the system of equations. The numerical solution to this system yields the state probabilities.

2.3 The Semi-Markov Process Model

2.3.1 Analysis

This paper follows the general formulation of the continuous-time discrete-state semi-Markov process model as described in [3, 4].

Let the model have *N* states. Let $f_{ij}(t)$ and $F_{ij}(t)$ represent the probability density function (*pdf*) and cumulative distribution function (*cdf*) respectively of the event corresponding to the transition from state *i* to state *j* at time *t.*

Let the system be in state *i.* Then the probability that the next state is *j* and not any other state *k* is given by:

$$c_{ij}(t) = f_{ij}(t) \prod_{k \neq j} (1 - F_{ik}(t)) \tag{3}$$

For *N*=2, $c_{ij}(t) = f_{ij}(t)$. The matrix *C(t)*=[$c_{ij}(t)$] is called the kernel or core of the semi-Markov process model and

$$w_i(t) = \sum_{j=0}^{(N-1)} c_{ij}(t) \tag{4}$$

is called the waiting time distribution for the state *i*. It represents the probability that the system waits in state *i* for *t* time units before making a transition. Hence it is an unconditional probability. The probability that the system stays in state *i* without making any transitions is given by:

$$W_i(t) = 1 - \int_0^t w_i(t)dt \tag{5}$$

The objective of the model is to determine the probability $\phi_{ij}(t)$ of being in each state *j* given that the system initially is in a particular state *i*. $\phi_{ij}(t)$ can be determined by solving a system of integral equations called the "Markov renewal equations":

$$\boldsymbol{\phi_{ij}(t)} = \delta_{ij} W_i(t) + \sum_k \int_0^t c_{ik}(\tau)\boldsymbol{\phi_{kj}(t-\tau)}d\tau \tag{6}$$

Where $i=j=k=0,1,2,...N-1$.

The right hand side of Equation 6 describes the following probabilities:

- $i=j$ and second term=0: W_i (t) is the probability that the process does not leave state *i*.
- $i=j$ and second term not 0: process leaves state *i* and returns to *i* by time *t*.
- $i \neq j$ and second term$\neq 0$: process leaves state *i* and reaches state *j* by time *t*.

The system of equations can alternatively be written in a compact form as a matrix:

$$\boldsymbol{\phi(t)} = diag(W(t)) + \int_0^t C(\tau)\boldsymbol{\phi(t-\tau)}d\tau \tag{7}$$

2.3.2 Reliability estimation

Given that the system started its operation in state *i* and that state *j* is the only absorbing state, the failure probability of the system is given by $\boldsymbol{\phi_{ij}(t)}$, which represents the cumulative distribution function (*cdf*) of the time to rupture (failure). The reliability is given from [2]:

$$R_s(t) = 1 - \boldsymbol{\phi_{ij}(t)} \tag{8}$$

The hazard rate $\lambda_s(t)$ of the system is related to the reliability [5]:

$$\lambda_s(t) = -\frac{1}{R_s(t)}\frac{dR_s(t)}{dt} \tag{9}$$

3 Application of SMP to the Rupture Frequency Analysis

3.1 Formulation

The *pdf*s and *cdf*s are denoted by general symbols *f(t)* and *F(t)* respectively, and the subscripts are used to denote the states as shown on in Figure 1.

$$f(t) = \begin{bmatrix} 0 & f_{SF}(t) & 0 & 0 \\ f_{FS}(t) & 0 & f_{FL}(t) & f_{FR}(t) \\ f_{LS}(t) & 0 & 0 & f_{LR}(t) \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad F(t) = \begin{bmatrix} 0 & F_{SF}(t) & 0 & 0 \\ F_{FS}(t) & 0 & F_{FL}(t) & F_{FR}(t) \\ F_{LS}(t) & 0 & 0 & F_{LR}(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The elements $c_{ij}(t)$ of the kernel matrix *C(t)* are found according to Equation 3:

$$\begin{aligned} c_{SF}(t) &= f_{SF}(t) \\ c_{FS}(t) &= f_{FS}(t)[1 - F_{FL}(t)][1 - F_{FR}(t)] \\ c_{FL}(t) &= f_{FL}(t)[1 - F_{FS}(t)][1 - F_{FR}(t)] \\ c_{FR}(t) &= f_{FR}(t)[1 - F_{FS}(t)][1 - F_{FL}(t)] \\ c_{LS}(t) &= f_{LS}(t)[1 - F_{LR}(t)] \\ c_{LR}(t) &= f_{LR}(t)[1 - F_{LS}(t)] \end{aligned}$$

The transition probability matrix and its elements are denoted by $\boldsymbol{\phi(t)}$ and $\boldsymbol{\phi_{ij}(t)}$ respectively as per Howard's [4] notation. These are a function of time and will be written in bold font in this paper. ϕ has been used by Fleming in [1] to denote the rate of flaw growth and is independent of time. This symbol is written in normal font in the present paper. This approach to distinguish the symbols has been done so as to be consistent with the notation of both the authors.

The flaw occurrence rate ϕ is based on the data from results of Non-Destructive Examination (NDE) [1]. In order to obtain parameters of an assumed non-exponential distribution for the time to flaw growth *i.e.*, for the transition *S->F*, it is beneficial to additionally consider the variability associated with the time to flaw growth from the test results. Hence, the parameters affecting the *cov* will be the chemical, material, texture and other properties taken in to consideration in the NDE inspections. For example, let the time to flaw growth until being detectable in the piping system represented by the state transition $S \rightarrow F$ be considered a Weibull distribution with scale λ_ϕ and shape γ_ϕ corresponding to the mean ϕ^{-1} years and coefficient of variation (*cov*) of *c*. Let the rest of the transition times follow exponential distribution. *W(t)* is constructed as per Equation 5, and the details are presented in the Appendix. Then the Markov Renewal given in Equation 7 is formulated as :

$$\boldsymbol{\phi(t)} = \begin{bmatrix} e^{-(\lambda_\phi t)^{\gamma_\phi}} & 0 & 0 & 0 \\ 0 & e^{-(\omega+\lambda_F+\rho_F)t} & 0 & 0 \\ 0 & 0 & e^{-(\mu+\rho_L)t} & 0 \\ 0 & 0 & 0 & S \end{bmatrix} + \int_0^T \begin{bmatrix} 0 & c_{SF}(\tau) & 0 & 0 \\ c_{FS}(\tau) & 0 & c_{FL}(\tau) & c_{FR}(\tau) \\ c_{LS}(\tau) & 0 & 0 & c_{LR}(\tau) \\ 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{\phi(t-\tau)} d\tau$$

By solving the above system using the trapezoidal rule in Equation A.17, the state probabilities $\boldsymbol{\phi(t)}$ can be found. Given that the system initially started in a perfect operating condition (state *S*), the probability of a flaw being detectable (state *F*), a leak being detectable (state *L*) and that of a rupture (state *R*) are given by $\phi_{SF}(t) = \phi_{12}(t)$, $\phi_{SL}(t) = \phi_{13}(t)$, and $\phi_{SR}(t) = \phi_{14}(t)$ respectively.

3.2 Results

The transition rates for the Markov model are taken from [1] and are summarized in Table 1.

Table 1. Constant transition rates for the piping system model [1]

Parameter	Value	
ϕ	Flaw detection rate	4.35 x 10^{-4}/yr
ω	Repair rate of a detected flaw	2.1 x 10^{-2}/yr
λ_F	Leak detection rate	1.79 x 10^{-4}/yr
ρ_F	Rupture occurrence rate from flaw state	9.53 x 10^{-6}/yr
μ	Repair rate of a detected leak	7.92 x 10^{-1}/yr
ρ_L	Rupture occurrence rate from leak state	1.97 x 10^{-2}/yr

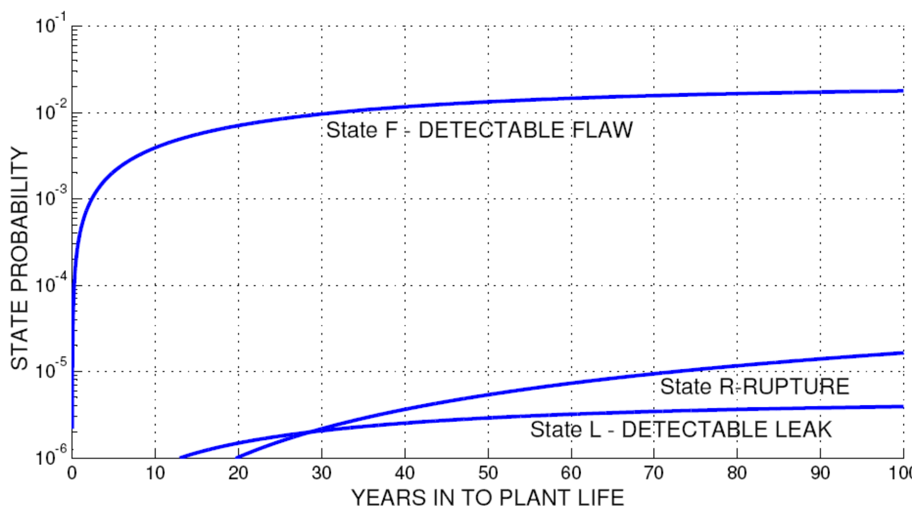

Figure 2. State probabilities. Time to flaw growth: mean =ϕ^{-1} years, *cov* = 1.0

When the coefficient of variation of the flaw initiation time is *c*=1.0, the semi-Markov process model yields the same results as reported in [1] using homogeneous Markov model (Figure 2). It is seen that the state probability of being in state *F* is higher than being in states *L* and *R*. This is due to timely detection and repair of detectable flaws. On repair, the system goes back to state *S* thus reducing the probability of going to state of rupture.

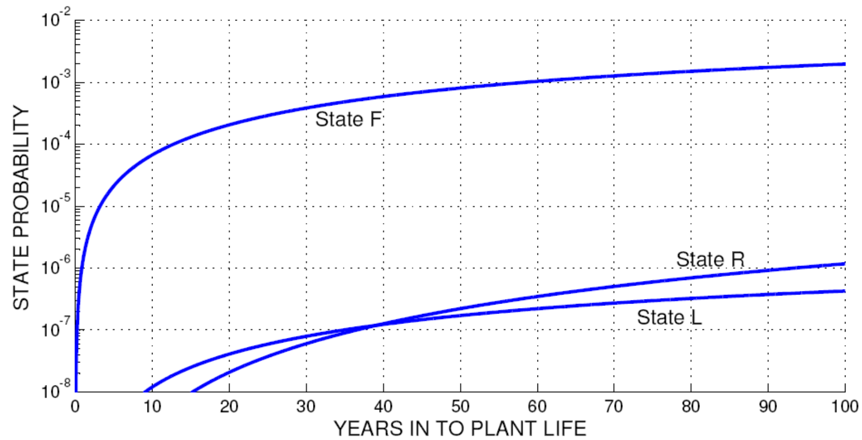

Figure 3. State probabilities. Time to flaw growth: mean =ϕ^{-1} years, *cov*=0.6

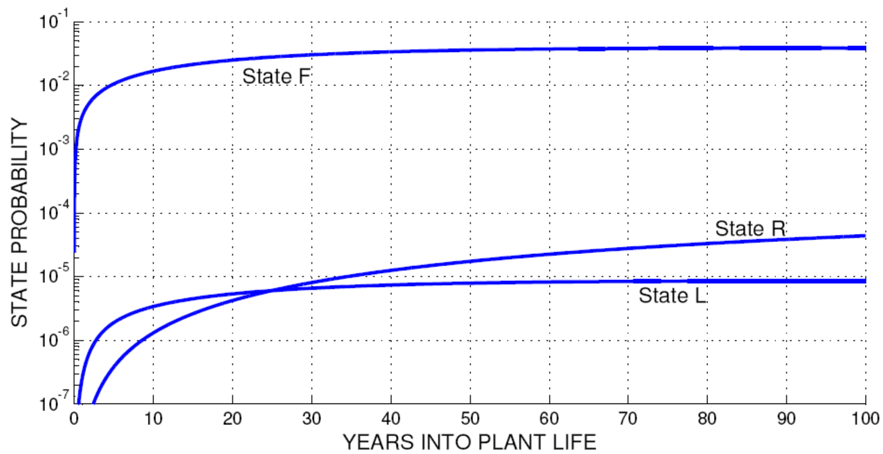

Figure 4. State probabilities. Time to flaw growth: mean =ϕ^{-1} years, *cov*=1.3

Now we consider the cases in which flaw initiation time is modelled by the Weibull distribution and proposed SMP model is used for reliability computation. Figure 3 and Figure 4 show the state probabilities for c=0.6 and c=1.3 respectively. When c < 1, the state probabilities are lesser than that when c = 1. A reduced c implies lesser variability in the time-to-flaw initiation, which leads to smaller state probabilities as compared to the case of c= 1. On the other hand, increased c means that there is large variance in the observed data. Therefore, the flaw initiation rate is higher, which in turn increases the probability of leak and rupture events.

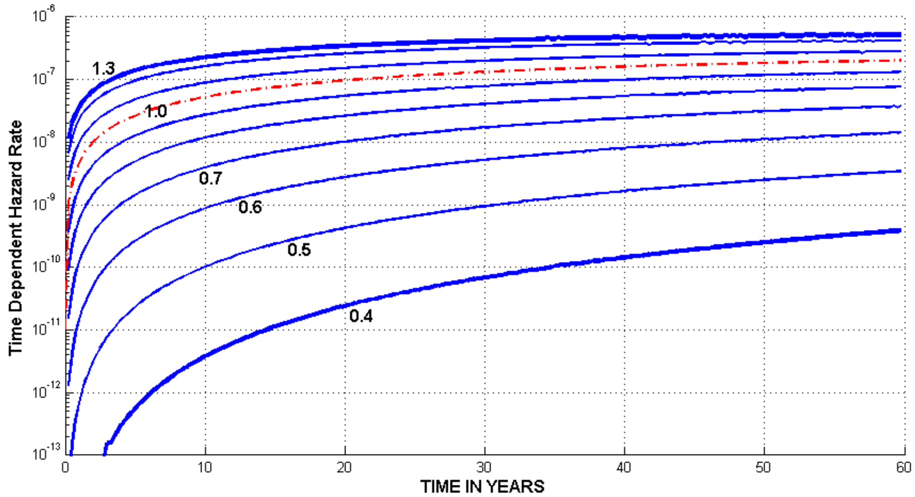

Figure 5. Hazard rate of rupture with repair.
Time to flaw growth: mean ϕ^{-1} years with *cov*=0.4, 0.5, …, 1.3

The rupture frequency increases with increase in variability (or cov) associated with the time to flaw initiation distribution, as shown in Figure 5. In early life time, the transient nature of solution is seen by increasing nature of the hazard rate curve. However, a steady state solution is likely to be achieved at in long term, which may be way beyond the intended life time of the nuclear plant.

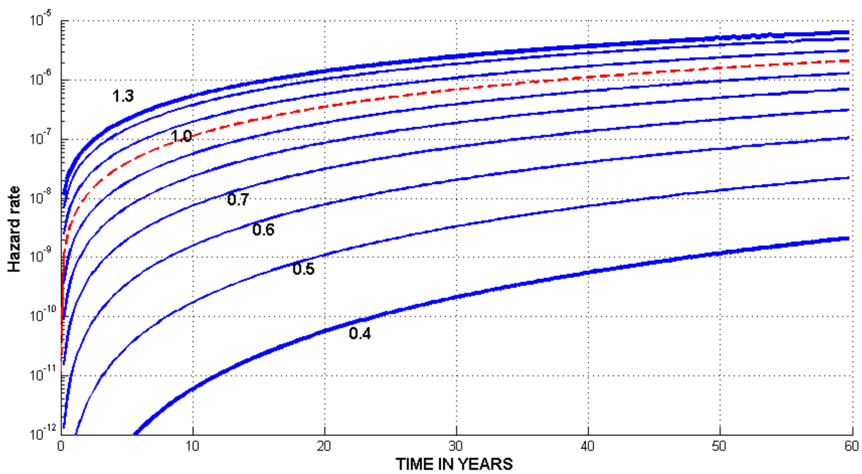

Figure 6. Hazard rate of rupture without repair.
Time to flaw growth: mean ϕ^{-1} years with *cov*=0.4, 0.5,…, 1.3

The rupture hazard rate in the absence of repair is plotted in Figure 6. As expected, in the absence of repair, the rupture rate will increase significantly.

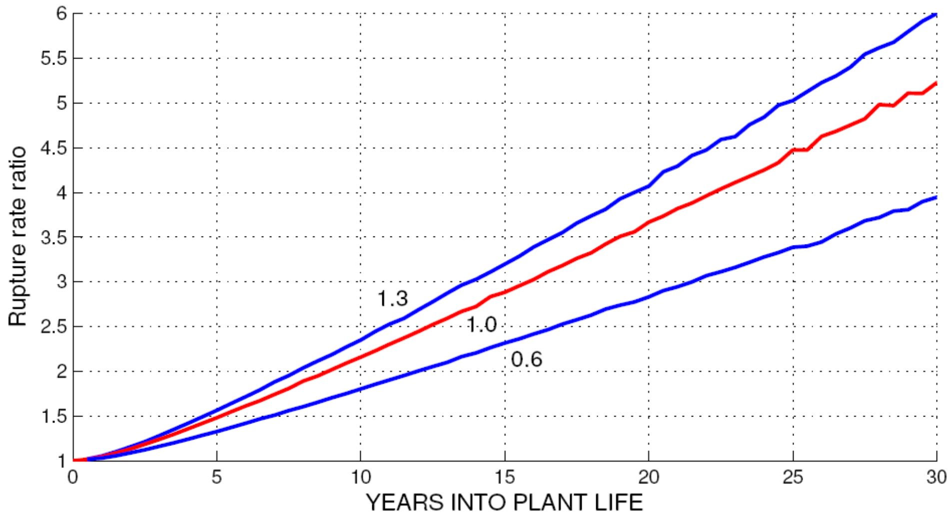

Figure 7. Ratio of rupture rate without repair to rupture rate with repair

The ratio of rupture rate without repair to rupture rate with repair is shown in Figure 7 for three cases, c=0.6, 1 and 1.3. The increasing ratio with time shows that in the absence of repair, rupture rate is larger. Moreover, higher the variability in flaw growth, larger is the ratio. At the end of 30 years with a variation of 0.6 in the time to flaw growth, it is seen that the rupture rate without repair is four times larger than that in the presence of repair. This demonstrates the importance of effective in-service inspection (ISI) programs for timely detection and repair of flaws. Further research involves using the knowledge of these rupture rates in risk informed programs to optimize the inspection intervals.

4 Conclusion

A semi-Markov process model is proposed to analyze reliability of the nuclear piping system. In this model, the flaw initiation is modelled by Weibull distribution, which allows to incorporate the aging effect, i.e., increase in flaw initiation rate with time. It was observed that the pipe rupture rate increases with increase in the variability of time to flaw initiation distribution. Hence, a maintenance program that removes the flaw from piping systems and repair leaks promptly will improve the reliability against rupture event. The proposed model provides a tool set to optimize the pipe inspection and maintenance program over the life cycle of the plant.

5 References

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6 Appendix

6.1 Closed form solution for *$W_i(t)$*

The Weibull distribution has the following cumulative distribution function:

$$F_T(t|\gamma,\lambda) = 1 - e^{-(\lambda t)^\gamma} \tag{A.10}$$

where λ is the scale parameter and γ is the shape parameter.

Consider a row *i* with more than one non-zero entry. Assume that all the failures and repairs are Weibull distributed. Note that repairs can be exponentially distributed by setting shape parameter to 1 in the Weibull distribution. Then for *j*=0,1,..*N*-1, we have

$$\begin{aligned}\int_0^t \sum_j c_{ij}(t)dt &= \int_0^t \sum_j [f_{ij}(t|\gamma_{ij},\lambda_{ij}) \prod_{k\neq j} (1 - F_{ik}(\gamma_{ik},\lambda_{ik}))]dt \\ &= \int_0^t \sum_j [(\lambda_{ij}\gamma_{ij})(\lambda_{ij}t)^{\gamma_{ij}-1} e^{-\lambda_{ij}t^{\gamma_{ij}}} e^{-\sum\limits_{k\neq j}(\lambda_{ik}t)^{\gamma_{ik}}}]dt \\ &= -e^{-\sum\limits_k (\lambda_{ik}t)^{\gamma_{ik}}} + 1 \end{aligned} \tag{A.11}$$

Using Equation 5, *$W_i(t)$* can be obtained as:

$$W_i(t) = 1 - \int_0^t \sum_j c_{ij}(t)dt = e^{-\sum\limits_k (\lambda_{ik}t)^{\gamma_{ik}}} \tag{A.12}$$

6.2 Implementation issues

Equation 7 can be evaluated by direct numerical integration or using Laplace transforms method. Most distributions do not possess a closed form Laplace transform. [6] in a thesis dissertation employed a transform approximation method (TAM) to evaluate the Laplace transform of Weibull distribution and then numerically evaluated its Laplace inversion. In this paper, we resort to the direct numerical integration technique since we assume that we do not know the degradation distribution beforehand.

Equation 5 is computationally expensive if the integral has to be evaluated for each *t*. Instead, it can be computed as a recurrence relation as follows:

$$W_i(t_n) = \begin{cases} 1 - \int_0^{\Delta t} w_i(t_n)dt & n = 1 \\ W_i(t_{n-1}) - \int_{t_{n-1}}^{t_n} w_i(t)dt & n > 1 \end{cases} \tag{A.13}$$

Where, by trapezoidal rule, we have:

$$\int_{t_{n-1}}^{t_n} w_i(t)dt = \frac{\Delta t}{2}\{w_i(t_{n-1}) + w_i(t_n)\} \tag{A.14}$$

However, when all the failure/repair distributions follow Weibull distribution with scale λ_{ik} and shape γ_{ik}, $W_i(t)$ reduces to a closed form:

$$W_i(t) = e^{-\sum_k (\lambda_{ik} t)^{\gamma_{ik}}} \tag{A.15}$$

To solve the system of integral equations, [7] derived the following recurrence relation based on trapezoidal rule by distributing *t* on a set of equally spaced points in the interval [0,*t*]:

$$\boldsymbol{\phi(t_n)} = [I - \frac{\Delta t}{2}C(0)]^{-1}[diag(W(t_n)) + \Delta t \sum_{k=1}^{n} C(t_k)\boldsymbol{\phi(t_n - t_k)} - \frac{\Delta t}{2}C(t_n)\boldsymbol{\phi(0)}] \tag{A.16}$$

Where $\Delta t = t_n - t_{n-1}$. The solution is started with $\boldsymbol{\phi(0)} = W(0) = I$.

The convolution operation in the above equation involves repeated addition and multiplication of matrices thus slowing down the computations as *n* grows. With sufficiently large storage space, the following technique for convolution improves the speed:

$$\sum_{k=1}^{n} C(t_k)\boldsymbol{\phi(t_n - t_k)} = \begin{bmatrix} C(t_1) & C(t_2) & \dots & C(t_n) \end{bmatrix} \begin{bmatrix} \boldsymbol{\phi(t_{n-1})} & \boldsymbol{\phi(t_{n-2})} & \dots & \boldsymbol{\phi(t_0)} = I \end{bmatrix}^T \tag{A.17}$$