Fuel Management in CANDU reactors: Daniel Rozon's Contribution

D. Rozon, E. Varin* and R. Chambon
École Polytechnique de Montréal, Montréal, Québec, CANADA H3C 3A7

ABSTRACT

The CANDU fuel management optimization problem is in many ways different from LWRs fuel management, because of the on-line refueling and the complete 3-D geometry problem. Daniel Rozon was an outstanding leader in the understanding and resolution of this optimization problem and remained during his entire career. Daniel Rozon and his students have used the generalized adjoint formalism implemented in standard mathematical programming methods to solve the optimization of the exit burnup in the reactor as well as the optimization of control rod worth or fuel enrichment. We have summarized here the theoretical basis of fuel management and resolution methods, the latest approaches of optimization and results as obtained using the OPTEX code.

1. INTRODUCTION

The fuel management of Canadian deuterium uranium (CANDU) reactors differs completely from that in LWRS. The CANDU reactors, moderated and cooled by heavy water, are fueled with natural uranium inserted in pressure tubes running horizontally through the core. The on-power refueling of the pressurized fuel channels with short fuel bundles (50 cm) is a continuing function of reactor operations and leads rapidly to equilibrium core conditions, refueling rate, and fuel burnup. Another characteristic of CANDU reactors is that the reactor core is controlled under normal operations using adjuster rods that are perpendicular to the fuel channels, forming a complete 3D geometry problem. These refueling characteristics, the exclusive use of natural uranium and the requirements to simulate the complete 3D reactor, make LWR optimization approaches impractical for CANDU fuel management studies.

One major objective of fuel management studies is to determine the time-averaged power distribution in the reactor under equilibrium refueling, which will minimize the fuel cost or total reactor feed rate (maximizing the average burnup) while meeting a number of operating constraints as channel and bundle powers a constraint on maximum power and on excess reactivity [1]. The optimization problem is a non-linear problem as power distribution depends on flux, that depends on exit burnup.

*Corresponding author: elisabeth.varin@polymtl.ca, currently in AECL Montréal

Professor Daniel Rozon's approach is based on the application of generalized perturbation theory (GPT) and mathematical programming to determine an optimal exit average burnup distribution in a multizone reactor containing control rods [2]. The perturbation theory developed in the early years of numerical modeling of nuclear reactors [3] have been shown to allow the determination of characteristics functional and their gradients. Deriving the functions and its derivatives, mathematical programming can then be employed to linearize and optimize the problem. The OPTEX code was born [4].
The numerical determination of such characteristics functional for a CANDU reactor requires very powerful 3-D code that can solve diffusion equations and generalized gradients. Daniel Rozon was successfully associated with Alain Hébert and his TRIVAC diffusion code [5] since the 80s. Supported by the diffusion solver, the OPTEX code evolved to optimize the fuel burnup under a variety of constraints: the cobalt content of the adjuster rods or the size of the stainless-steel adjuster rods to overcome xenon poisoning of the reactor under short shutdown conditions, the minimization of the channel power peaking factor (CPPF) [6] and the fuel enrichment. In 2005, OPTEX was further developed to improve the original mathematical programming that requires gradient calculations [7], and also to completely replace them by advanced optimization methods based on tabu search [8].
This paper describes the main fuel management problem in CANDU reactors and the GPT use to optimize the average exit burnup calculations. The goal of solving this optimization problem has driven the development of outstanding reactor physics codes, still unique in the CANDU industry. The steps to achieve such numerical calculations and the reactor physics codes used are explained. The following sections are devoted to present the fuel management problem itself and the mathematical programming development in OPTEX. The evolution of the gradient methods to optimize the fuel management problem towards meta-heuristic methods is described. Some results are finally presented on CANDU reactors from Natural Uranium reactor to ACR reactors.

2. CANDU REACTOR FUEL MANAGEMENT

Because CANDU reactors are on-line refueled, the design of the reactor is established using the time-average model where the reactor power distribution represents the reactor powers average over time. This is not a practical definition of the time-average model. The model is whereas defined as a standard static diffusion equation in which the cross sections are averaged locally over the refueling times of each channel. The cross sections Σ_{jk} on bundle k in channel j are computed as:

$$\Sigma_{jk} = \frac{1}{T_j} \int_0^{T_j} \Sigma[B_{jk}(t)] dt \tag{1}$$

where T_j is the refueling time interval of channel j, and B_{jk} the burnup of bundle k in channel j. The standard static diffusion equation to solve is then

$$M[X, L]\phi = \lambda F[X, L]\phi \tag{2}$$

where ϕ is the neutron flux distribution, M is the neutron removal operator, containing the diffusion and the removal operator, F is the neutron production operator, containing the scattering and the fission operator and λ the eigenvalue of the system. The determination of the operators M and

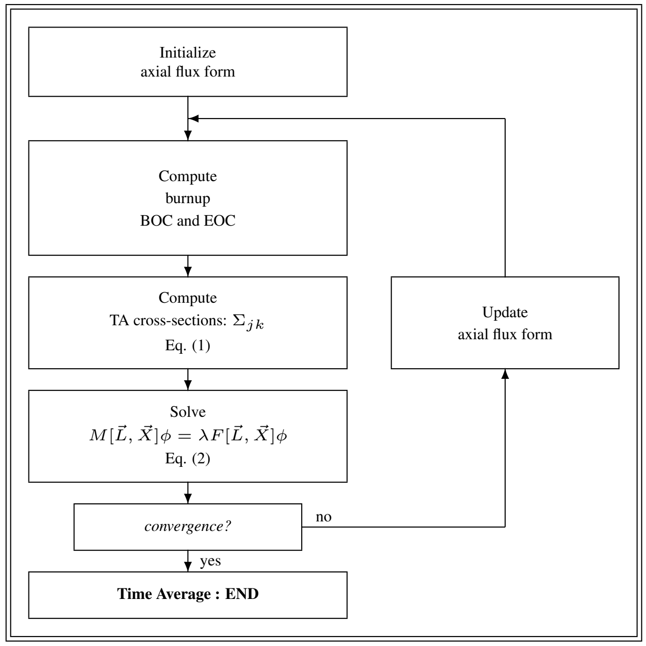

Figure 1: General algorithm of the time average method.

F is function of the global and local parameters L in the reactor, i.e. fuel temperature, coolant density, and function of the actual refueling strategy X, i.e. the refueling scheme, the burnup, the new bundle types. The 3-D geometry of a CANDU reactor dictates that the above equation has to be solved on a 3-D cartesian domain.

A large amount of input information (local parameter dependent cross sections, refueling strategy information, core geometry description) and strong numerical methods are required to achieve the convergence of this equation, from which the outputs quantities are the flux distribution and the domain eigenvalue. Figure 1 shows the main algorithm to obtain the flux/power distribution over the reactor. To optimize the average exit burnup over the reactor, this is the elementary problem to obtain the neutron flux ϕ.

The main codes require to obtain the flux distribution following this basic algorithm: a lattice code to generate the cross sections depending on a range of local parameters to cover the reactor conditions and a reactor code to reproduce a 3-D cartesian geometry domain and to solve the diffusion equation using the time-average model for the cross sections. At the Institut de Génie Nucléaire, two main contributing codes have been developed the lattice code DRAGON [9] and the diffusion code TRIVAC [5].

2.1. Description of non-linear functionals

Any optimization problem is defined by:

- a set of control variables to be optimized,
- the objective function which gives a way to rate different sets of control variables,

- and finally the constraints, either simply the range value of the variables, or representing limitation of the physical problem which is simulated.

Let X be the I-component vector of the control variables. The system characteristics are linear or bilinear ratios of the form:

$$P(\phi; X) = \frac{\langle \Sigma_1(X, \vec{r}), \phi(\vec{r}) \rangle}{\langle \Sigma_1(X, \vec{r}), \phi(\vec{r}) \rangle} \tag{3}$$

or

$$R(\phi^*, \phi, X) = \frac{\langle \phi^*(\vec{r}), F(X, \vec{r})\phi(\vec{r}) \rangle}{\langle \phi^*(\vec{r}), G(X, \vec{r})\phi(\vec{r}) \rangle} \tag{4}$$

where ϕ and ϕ^* are n-component vectors of continuous functions, positive over the spatial domain of $\vec{r}$, and representing the n-energy group time-averaged flux and adjoint flux distributions. The vectors Σ_1 and Σ_2 and the $n \times n$ operator matrices F and G describe the system matrices over the reactor. The brackets $\langle, \rangle$ stand for the integration with $\vec{r}$ over the entire domain and the summation over the n components. The system characteristics can define the functional to optimize or its constraints.

3. DESCRIPTION OF THE CANDU FUEL MANAGEMENT

The CANDU fuel management problem is based on the time-average core definition used to design the reactor. The fuel management problem is also dependent on other parameters such as the maximum allowed channel power, the reactivity reserve to recover from short shutdown. To optimize the design of the efficient CANDU reactors, a tool has been developed by the IGN at Ecole Polytechnique: OPTEX. Before we illustrate the different studies OPTEX has been used for, we present in this section the theory and the formulas involved in the fuel management of CANDU reactor in OPTEX.

3.1. Definition of the fuel management problem

Here we present the fuel management problem. Note that the space dependency ($\vec{r}$) is dropped to simplify the notations. First, the decision vector, or control variables, X includes usually the average exit burnup distribution B. Several parameters have been optimized in previous studies: the adjuster rod geometry α or the fuel enrichment ϵ. Other parameters could also be optimized with OPTEX such as the fuel poison concentration or the number of bundle shift during refueling. Thus the decision vector X can be expressed as

$$X = [B, \alpha, \epsilon, ...] \tag{5}$$

As for the objective function F_C, the main goal is to optimize the equilibrium fuelling costs per unit energy in dollars per megawatt, which can be written as

$$F_C = \frac{\frac{CH}{B}, \phi}{\langle H, \phi \rangle} \tag{6}$$

where C represents the fuel cost per unit of mass, H is the energy released per unit of mass, B the average exit burnup and ϕ is the neutron flux in the reactor.
Finally, the constraints are the following:

- The reactor must be critical, on a time-average basis

$$k_{eff}(X, \phi, \phi^*) = 1.0 \tag{7}$$

- The power distribution in the reactor must be such that limits on the fuel are not exceeded for safety reasons. The approach used in OPTEX to verify the power constraints is to monitor the average power in a limited number of control zones J. Thus, we will impose:

$$q_j(X, \phi) \leq Q_j \quad (j = 1, ..., J) \tag{8}$$

where Q_j is the fixed limit and q_j is the power form factor in zone j. The zonal form factors are defined by

$$q_j(X, \phi) = Z_j(X) \times f_j(X, \phi) \quad (j = 1, ..., J) \tag{9}$$

where f_j is the time-average zonal power fraction, i.e., the ratio of the volume averaged power in zone j (volume V_j) to the volume averaged power in the core (volume V), as obtained from the time-average flux distribution at equilibrium refueling:

$$f_j(X, \phi) = \frac{V}{V_j} \frac{\langle H, \phi) \rangle_{V_j}}{\langle H, \phi) \rangle_V}$$

The control zones are arbitrary and may contain any combination of fuel bundles and/or channels. The term Z_j is the zonal power peaking factor expressing the variation of the instantaneous peak power around the time-average value. The choice of the subvolumes V_j is at most limited to be a single channel or a single bundle, then we have:

$$Z_j = \begin{cases} CPPF(X_j) & \text{if } V_j \text{ is a fuel channel} \\ BPPF(X_j) & \text{if } V_j \text{ is a fuel bundle} \end{cases}$$

where CPPF is the channel power peaking factor and BPPF is the bundle power peaking factor. These factors are the ratio of the instantaneous channel or bundle power over the time-average channel or bundle power.

Thus, a total of J additional constraints are imposed to satisfy the limits on the power form factors q_j. We note that the form factors are homogeneous functionals of the time-average neutron flux.

- Depending of the optimization problem, we may have additional constraints. One of them is the adjuster rods minimum reactivity worth Δk_{adj} to meet design requirements.

$$k_{eff}(X_0, \phi_0, \phi_0^*) - k_{eff}(X, \phi, \phi^*) \geq \Delta k_{adj} \tag{10}$$

where the subscript 0 indicates the absence of adjusters in the reactor.

- An other additional constraint is the core void reactivity ρ_V defined by

$$\frac{1}{k_{eff}(X, \phi, \phi^*)} - \frac{1}{k_{eff}(X_V, \phi_V, \phi_V^*)} \leq \rho_V \tag{11}$$

where the subscript V indicates the absence of coolant (i.e. void) in the fuel channels.

3.2. Generalized adjoint calculation

This optimization problem is a non-linear problem. To solve it by using the GPT, one have to linearize the problem defined by Eq. (6), (7), (9) and other optional constraints Eq. (10) and (11). This leads to the following linear optimization problem:

$$\min_{\Delta X} \nabla f^k.\Delta X \text{ with } \left\{ \begin{array}{ccc} \nabla h_i^k.\Delta X & = & b_i - h_i\left(X^k\right) \\ \nabla g_j^k.\Delta X & \leq & c_j - g_j\left(X^k\right) \\ X_n^{INF} - X_n^k \leq & \Delta X_n & \leq X_n^{SUP} - X_n^k \end{array} \right. \tag{12}$$

where f represents the objective function F_C, h_i is the neutron multiplication factor k_{eff}, g_j represents the power distribution constraints q_j or other $\leq$ constraints (Δk_{adj} or ρ_V). The index k represents the outer iteration of the optimization algorithm. Then, X^k stands for the current value of X at iteration k.
The generalized adjoints are used to compute the gradients required in the simplex related methods. Let us consider, for example, the zonal power form factors q_j in Eq. (9). We note c_{ij}, the i'th component of the gradient of q_j. It accounts for the perturbations of the decision variable ΔX_i, e.g. in a given burnup or any other decision variable. Because of the neutronic coupling between regions in the reactor, it is obvious that the component of the gradient in a control zone V_j outside of the perturbated region V_i will not disappear. Formal differentiation of Eq. (9) yields

$$c_{ij} = \underbrace{\frac{\partial q_i}{\partial X_i}}_{direct} + \underbrace{\langle \frac{\partial q_j}{\partial \phi}, \frac{\partial \phi}{\partial \phi} \partial X_i \rangle}_{flux\ induced} = \frac{\partial q_i}{\partial X_i} + \langle S_j^*, \Gamma_i \rangle \tag{13}$$

The function Γ_i can be obtained by direct differentiation:

$$(M - \lambda F)\Gamma_i = -(\frac{\partial M}{\partial X_i} - \lambda \frac{\partial F}{\partial X_i} - \frac{\partial \lambda}{\partial X_i} F)\phi = S_i \quad (i = 1, ..., I) \tag{14}$$

where M, F and λ are defined in Eq. (2).
The more traditional GPT approach, called "implicit" approach, is possible as

$$c_{ij} = \frac{\partial q_j}{\partial X_i} + \langle \Gamma_j^*, S_i \rangle \tag{15}$$

where $\Gamma *_j$ is the generalized adjoint, solution to the adjoint fixed source eigenvalue problem:

$$(M^* - \lambda F^*)\Gamma_j^* = \frac{\partial q_j}{\partial \phi} = S_j^* \quad (j = 1, ..., J) \tag{16}$$

This approach is labeled implicit in the sense that the flux effect of the perturbations is implicitly accounted for by the generalized adjoints, which act as an importance function for the j'th characteristic functional. We note that

$$\langle S_j^*, \Gamma_i \rangle = \langle \Gamma_j^*, S_i \rangle \quad (i = 1, ..., I)\ ,\ (j = 1, ..., J) \tag{17}$$

Both implicit and explicit approaches can be used in OPTEX. (See companion paper for more details on adjoints and generalized adjoints [10])

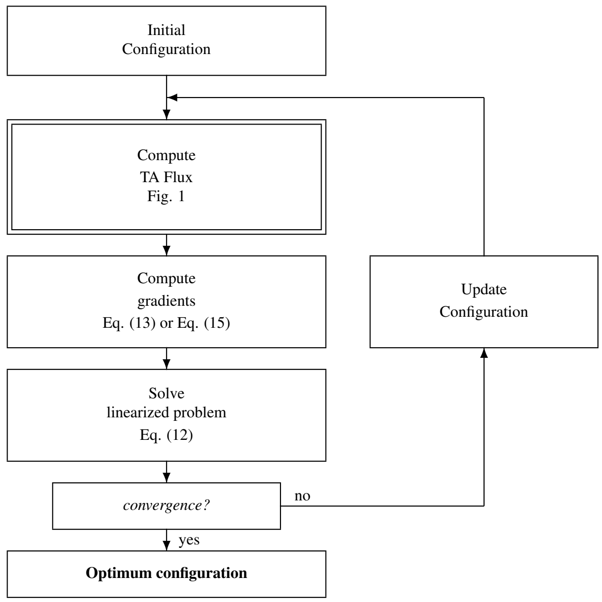

Figure 2: General algorithm of OPTEX.

4. MATHEMATICAL PROGRAMMING OR OTHER APPROACHES IN OPTEX

4.1. Gradient based method

In the previous section, we have presented, together with its definition, how the fuel management problem can be linearized, and how gradients are computed. Here we explain the different methods used to solve the fuel management optimization problem in more details.

The gradient-based methods are implemented in the code OPTEX. It has been built on using the TRIVAC code as a diffusion solver and relying on the DRAGON code to provide fuel and reflector cross sections and control reactivity device cross sections, resulting of 3-D transport calculation in DRAGON. These two codes allow the research on fuel management to be developed in Ecole Polytechnique independently of any other code or any other company. The TRIVAC code also provides a very important feature to OPTEX: **the generalized gradient solver** as defined in Equation (16).

In a mathematical programming point of view, using a gradient-based method to solve an optimization problem is a basic method that poses no difficulties. However in the context of the CANDU reactor fuel management, the evaluations of the objective function and its derivatives over the problem constraints are driven by large effort in solving the diffusion equation (objective function $f(\phi, X)$) and solving the generalized gradients (derivatives).

We first describe the general algorithm of the gradient-based methods. Figure 2 shows that the process is iterative. Starting from a guessed set of variables X^0, one computes the flux and then the gradients. This gives the linearized optimization problem (Eq. 12), which is solved and leads to a new point to start with at the next iteration X^{k+1}.

The first method developed in OPTEX is the well known, so called *Simplex method*. This method

is designed to solve linear problems with constraints of the type equality '='. However in our case, all constraints of the type inequality $\leq$ or $\geq$ have to be replaced by constrains of the type $=$ with the introduction of dummy variables ($x_i \geq 0$) added to the decision vector. The *Simplex* algorithm consists in following the border of the polytope formed by the constraints, from one summit to another, until the optimum is found. This particular method introduces a bias in the optimization path. Indeed, the fact that the new point is always on a summit of the polytope may deviate the current solution from the general direction towards the optimum. To remove this limitation, the Method of Approximate Programming (MAP) developed by Griffits [11] was introduced as one of the available methods in OPTEX. It is mainly based on the *Simplex* method except that a quadratic constraint is added to limit the advance step (ΔX) [2].
Another method available in OPTEX is the so-called *Lemke* method [12]. Based on the fact that the optimum is either on the border of the domain or at a point where all gradients are zero, Lemke introduced a new formulation combining the primal and dual forms of the optimization problem. Solving this new problem leads to the optimum of the original linearized optimization problem.
In general these three basic methods (simplex, MAP and lemke) are very efficient and fast, but they have a major drawback: the starting point. They require a starting point in the feasible domain, i.e. decision variables X which respect all the constraints. Two problems arise directly from this limitation. First, with the general iterative algorithm, the new point (after optimization search) may not respect the constraints if the advance step is too large and if the fuel management problem is highly non-linear. The solution is straightforward by reducing the advance step ΔX, but it increases computation requirements.
The second problem is to find a first point to start the iterative process. A realistic core power distribution can be easily found when only a few surveillance zones are selected in the reactor. For larger number of zones or for a brand new reactor design, we introduced the augmented lagrangian method and the penalty method in OPTEX [7]. Both methods consist in including the constraints within the objective function with an appropriate weight. A new general optimization problem without constraints is then solved, leading to the same results as long as the weights of the constraints are properly managed. The resulting algorithm involves several levels of iterations as it is illustrated in Figure 3. The constraint weights are updated in the outer iteration of the optimization algorithm.

4.2. Metaheuristic methods

The general rule of gradient-based methods is to follow the slope (gradient) toward a better point. They cannot go backwards. Their major drawback is that the optimization algorithm can get trapped into local optimum. To solve this problem, a new method was programmed in OPTEX: the TABU search [8]. Its metaheuristic-type algorithm is presented on Figure 4. The general idea of those methods is to try at each iteration some random configurations of the decision variables (X), and learn from them in which direction to look for. None of the tryouts may be better than the reference value though, but it can still replace the current estimate. This option called 'exploration' helps to get out of a local minimum. It is the major advantage of those methods. However, there is no limit in the TABU method during the exploration phase. It also represents the major drawback of the meta-heuristic method: to have a realistic convergence criterion.
In a mathematical point of view, metaheuristic methods are also very simple to implement. No

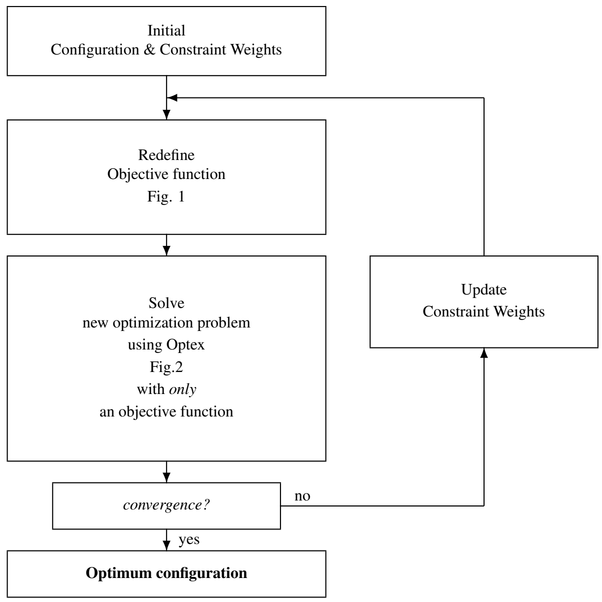

Figure 3: General algorithm of Augmented Lagrangian Method

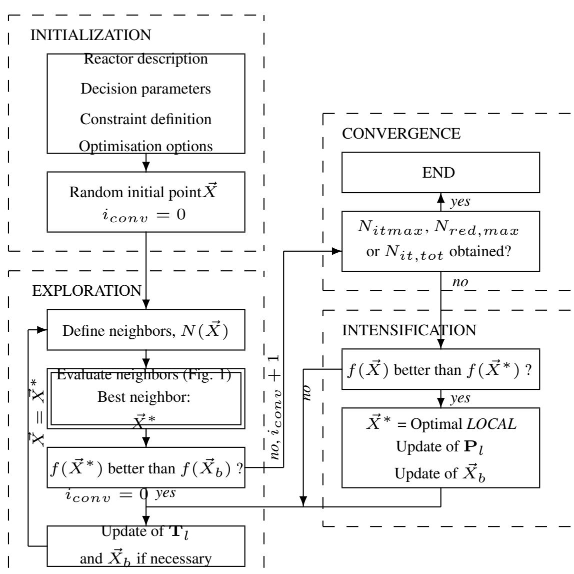

Figure 4: General algorithm for Tabu Search optimization method.

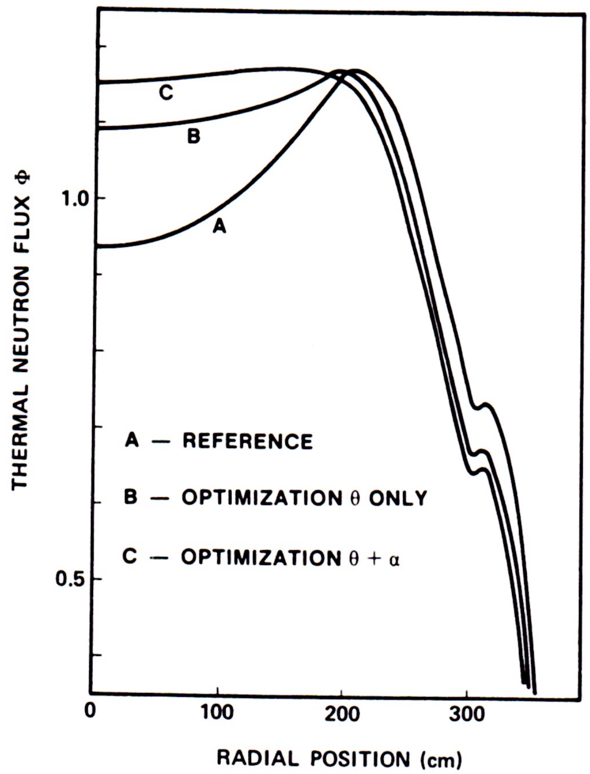

Figure 5: Effect of the burnup optimization on radial flux shape

gradient calculation is needed; only objective function evaluations are required. However, those methods cannot handle optimization problem with constraints, thus similarly to the augmented lagrangian method, the constraints are included in the objective function. The difference though with the augmented lagrangian method is that the weights are constant in the metaheuristic method.

5. CANDU DESIGN STUDIES

5.1. Original CANDU results

OPTEX code was developped first on a 1-D slab reactor to test on the mathematical programming [1]. The optimization algorithm was applied to find the optimum burnup values in a simplified reactor, with a control absorber in the centre of the core. The number of burnup zones was increased to show that it affects positively the problem by increasing the average exit burnup of the reactor. The increase is up to 1.9% when only the burnups are optimized, and up to 7.8% when adjusters zones are introduced and optimized together with the burnup zones. An illustration of the thermal flux profile corresponding to the optimized configurations is compared to the reference flux on Figure 5. On this figure, θ stands for burnup, α for the adjuster zones.
As previously explained, the refueling characteristics of CANDU reactors require to use a complete 3D simulation of those reactors. That was obviously the case in the work of Rozon and Beaudet [2], where the adjuster sizes themselves were optimized. The results of Table III in [2] are too long to

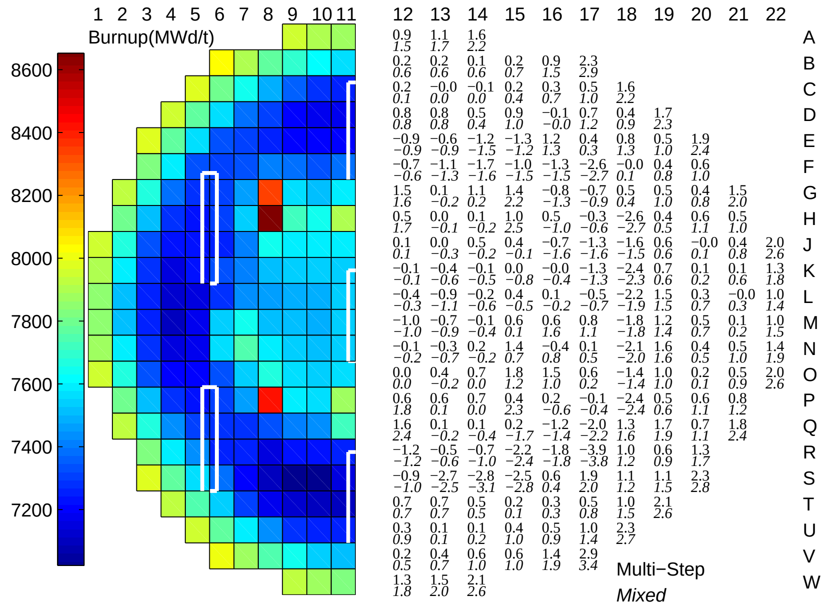

Figure 6: Distribution of the burnup in the 190-zone case

be reproduced here. However, the important fact to notice was that within approximately 10 outer iterations (see algorithm in Figure 2), the optimum was found for an optimization problem with 3 burnup zones and 16 different sizes of adjusters. The OPTEX code is really fast and converges on very similar configurations regardless of the initial guess in this case of average complexity.

Starting in 2005, more advanced mathematical programming method were introduced in OPTEX [7, 13]. The penalty and augmented lagragien methods were included in different general algorithms and approaches to optimize the fuel management problem. Indeed, each mathematical programming method has pros and cons. Several combinations of these methods were presented: the quasilinear programming (QLP), the multistep method (MS) and the mixed method (MM).

Very complex optimization case with 190 burnup zones on a CANDU-6 reactor was considered. In this case, each burnup zone is composed of the two left/right symmetrical channels. Thus they are all composed of 2 channels, no mathematical bias between the zones is introduced. All advanced methods above-mentioned have been tested for this complex case. Figures 6 and 7 illustrate the optimized burnup and channel power distributions which were obtained. For more details on all the results, refer to [7]. On the left hand-side of those figures, the results for the QLP are presented; the right hand-side gives the difference in percent between MS method and MM method compared to QLP results. We just want to stress out that with such a fine and detailed case in terms of burnup zones, one can achieve a very flat power distribution. Moreover, the 3 methods give very similar results. The computations requirements are roughly about the same for all approaches: between 50 to 120 time-average flux calculations and a similar number of gradient set calculations.

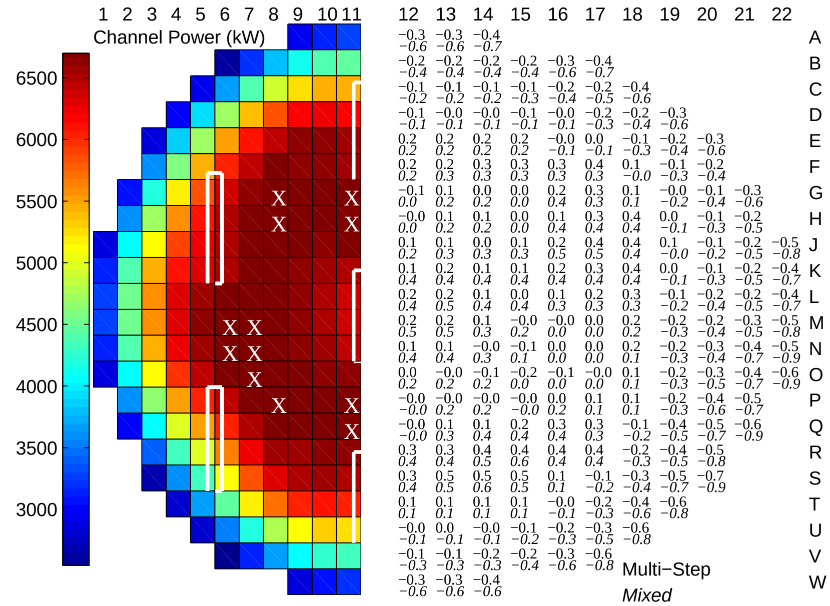

Figure 7: Distribution of the channel power in the 190-zone case

5.2. Applications to the ACR700

In the ACR700[a] reactor, the 22 cm wide fuel lattice cell is composed of a 43-pin bundle, as illustrated on Figure 8a, with sligthly enriched uranium fuel, except for the central pin which is composed of natural uranium and dysprosium to guarantee a negative void reactivity. A front view of the simulated core is illustrated on Figure 8b, including 300 fuel channels, control rods (50% inserted, grey boxes), reflector (approximative round shape line) and the labels of the fuel channels.

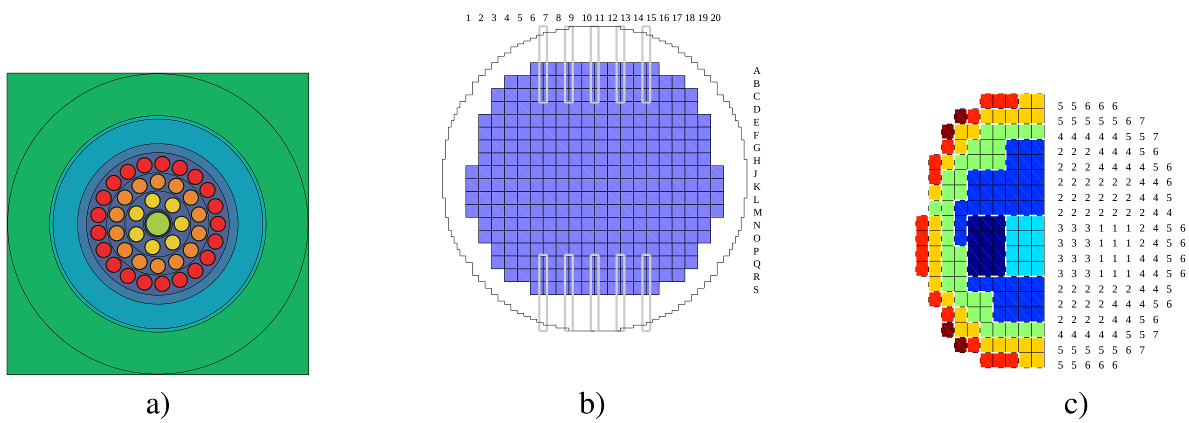

Figure 8: ACR700 model for optimization.

The two gradient-based approaches (MS and MM) have been applied successfully to the fuel man-

[a] ACR^{TM}: Advanced CANDU reactor is trademark of AECL

agement problem of the ACR700 for 7 and 150 burnup zone cases [14]. The ACR700 case is introduced to illustrate an application of the TABU search in the fuel management.
The tabu search method has been used to optimize the average exit burnup distribution for the ACR700 in the case of 7 burnup zones. A case where the fuel enrichment was considered as an additional parameter has also been studied. The fuel enrichment is fixed after the first optimization study. The burnup zones are represented on Figure 8c.
The implemented tabu search method was tested using several analytical functions, that were replacing the double-line box in Figure 4. This gave us confidence and experience in our algorithm, especially experience on the automatic adjustments of the exploration and intensification radii (see [14]). Evaluation of the objective function in these calculations is done within few basic numerical operations, and the total optimization process is around one minute. Each analytical optimization problem resolution were repeated 100 times to give statistics on the reliability of the algorithm [14].
For the CANDU fuel management optimization problem, we have also proceeded with a statistical approach. Unfortunately, the 3D flux distribution in the reactor core (see figure 1) is not an analytic function, and it takes from several seconds to minutes with the time-average model to evaluate the objective function depending on the required discretization. As shown in the last columns of Tables 1 and 2, the number of flux calculations is relatively high. A few hours to one day is needed to perform one optimization using the tabu search on a real ACR reactor for 7 burnup zones. It is therefore not an option to perform 100 optimization resolutions just to get a statistical point of view, especially when we wanted to test different optimization parameters. The number of trials has been limited to 3. We cannot say that a statistics can be obtained from these results, but if the results are consistent between the 3 tests, one can say that a good tendency is obtained. The results of the 3 tests are given in Tables 1 and 2 where B_i represents the average exit burnup in zone i given in GWd/t.
In Tables 1 and 2, N_{imx} represents the number of successive iterations without amelioration of the best estimate before convergence. N_{npdx} is the number of successive iterations without finding a promising area before exploration radii are automatically adjusted. The last important option $N_{red,max}$ is the maximum number reduction of the radii before they get small enough to reach convergence. The burnup distributions can be found in reference [8]. The two significant observations are that the 3 trials do not always end up giving the same results, and that the number of TA flux calculations (column ϕ) is huge, a few thousands flux calculations. The number of flux calculations, i.e. the number of objective function evaluation is actually similar for the fuel management problem and for the analytical problems with the same level of difficulty.

Table 1: Average exit burnup optimization with N_{imx}=50

N_{npdx}=5								
B_1	B_2	B_3	B_4	B_5	B_6	B_7	F_C	ϕ
26.88	26.97	27.76	24.41	15.93	17.61	18.31	4.4793	4449
26.98	27.12	27.57	24.16	14.43	19.85	26.95	4.5140	4684
26.96	27.23	27.33	23.96	14.91	21.25	15.85	4.4964	4954
N_{npdx}=14								
B_1	B_2	B_3	B_4	B_5	B_6	B_7	F_C	ϕ
27.43	26.99	27.41	24.18	17.76	14.66	14.69	4.5151	4837
27.44	27.03	27.40	24.16	16.77	15.75	19.93	4.4870	4677
27.31	27.17	27.28	24.01	15.46	18.58	21.99	4.4759	4693

Table 2: Average exit burnup optimization with N_{imx}=200

N_{npdx}=5								
B_1	B_2	B_3	B_4	B_5	B_6	B_7	F_C	ϕ
27.02	27.14	27.44	24.05	15.44	19.59	17.78	4.4723	3935
26.69	26.95	28.02	24.36	15.31	19.45	16.36	4.4907	4915
27.23	27.18	27.30	24.00	15.25	19.18	21.65	4.4781	6058
N_{npdx}=14								
B_1	B_2	B_3	B_4	B_5	B_6	B_7	F_C	ϕ
27.40	27.13	27.31	24.07	15.43	17.71	25.94	4.4914	8460
27.17	27.14	27.35	24.04	15.92	18.59	16.86	4.4687	7855
27.52	27.10	27.28	24.14	16.31	16.09	23.19	4.4908	10165

6. CONCLUSIONS

The CANDU fuel management optimization problem was efficiently solved by Daniel Rozon and his students using the generalized adjoint formalism implemented in standard mathematical programming methods. We have summarized here the theoretical basis of fuel management and resolution methods, the latest approaches of optimization and results as obtained using the OPTEX code. Daniel Rozon's imagination and intelligence always challenge the reactor physics codes and forces OPTEX to always exceed his expectations.

The CANDU fuel management optimization problem is an evolving problem as the CANDU reactor design changes and the constraints requirements become more stringent on the problem as the coolant void reactivity constraint. Daniel Rozon has always looked to further optimize fuel management and CANDU operation as it directly impacts on the reactor design and on the fuel cost. We can just hope that his work will be pursued in Canada.

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