EFFECTS OF TWO-PHASE MIXING AND VOID DRIFT MODELS ON SUBCHANNEL VOID FRACTION PREDICTIONS IN VERTICAL BUNDLES

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Abstract

The evaluation of the subchannel code ASSERT against the OECD/NEA BFBT benchmark data demonstrated that at low pressures, the void fraction in the corner and side subchannels of a vertical bundle was over-predicted. Preliminary results suggest that this was due to the use of Carlucci's empirical correlation for void drift beyond its applicable range of pressure. Further examination indicates that the choice of the mixing and void drift models has a negligible effect on the error of the subchannel void fraction predictions. A single, isolated subchannel was simulated and results suggest that the root cause behind the over-prediction is inadequate mixing at the sides and corners of the bundle. Increasing the magnitude of the void drift coefficients in Carlucci's model at low pressure was found to improve the overall accuracy of the predictions. A simple correlation relating Ω to the outlet pressure was found to increase the number of points falling within experimental error by 1.0%.

1. Introduction

Boiling Water Reactor (BWR) fuel bundles consist of an 8 × 8 array of vertical fuel rods each 12.3 mm in diameter and 3.708 m in length. These fuel rods are arranged in a square grid with a pitch of 16.2 mm, and the spaces in between the rods are known as subchannels. Fission reactions taking place inside the fuel rods create a large amount of heat which must be removed by a light water coolant that flows upwards through the subchannels. The increase in the enthalpy of the coolant will cause it to undergo vapourization, creating vapour bubbles. Subsequently, the proportion of a given volume occupied by vapour is known as the void fraction or α. The ability to accurately predict this parameter is desirable since the heat transfer characteristics of the coolant are closely linked to the void fraction.

Thermalhydraulic behaviour in fuel bundles is often modelled using specialized subchannel codes. The Advanced Solution of Subchannel Equations in Reactor Thermalhydraulics (ASSERT) code is one such example, and was developed to evaluate the behaviour of the coolant in the calandria tubes of pressurized heavy water reactors. Leung and Novog have demonstrated that the robust nature of ASSERT allows it to also accurately predict the steady-state void fraction distribution in vertical BWR style bundles [1]. However, they noted that the code had difficulty predicting the void fraction in the corner and side subchannels of the bundle under conditions at low pressure / low power, and suspected that the issue was linked to the subchannel mixing models [1]. Hwang et. al. came to similar conclusions in their evaluation of the subchannel code COBRA-IV-I against data from the GE and Ispra partial bundle experiments [2].

Unequal heating, geometry and mass fluxes will cause differences in the enthalpy, void and local pressure of adjacent subchannels. This leads to mass, momentum and energy interactions across the inter-subchannel junctions as the mixture tries to reorganize itself into an "equilibrium state". Lahey and Moody describe several driving mechanisms behind subchannel mixing and they are listed in Table 1 [3].

Table 1 - Driving Mechanisms of Subchannel Mixing

Mechanism	Cause
Diversion Cross Flow	A difference in the local pressure of two adjacent subchannels will cause a lateral driving force across the common junction.
Turbulent Mixing	Mixing which is caused by chaotic and random fluctuations in the subchannel pressure and flow, or by obstacles such as spacer grids.
Void Diffusion	The tendency for the void to migrate from subchannels which are small or have low flow rates to those which are larger or have higher flow.
Buoyancy Drift	Void migration caused gravitational forces. Only of significance in horizontal channels where the void tends to migrate to the top of the bundles [4].

In ASSERT, the magnitude of the subchannel mixing is determined using empirical correlations such as Carlucci or Rogers-Rosenhart [5,6]. These correlations relate the rate at which fluid mixes with adjacent subchannels to parameters such as the local void fraction, mass flux and hydraulic diameter. The conservation of momentum is upheld in the code by separate sets of transport equations for the axial and lateral momentum [5]. The aim of this paper is to determine the cause of the void fraction errors in the low pressure cases, and to examine the performance of the different mixing models. Specifically, the influence of the turbulent mixing models and the void diffusion correlations on the code accuracy will be examined.

2. Methodology

2.1. Facility and Test Description

The code results in this paper were compared against high resolution experimental void fraction data taken by the NUclear Engineering Power Corporation (NUPEC). The experiments were conducted at full scale with electrically heated bundles, and void measurements were taken using X-ray Computed Tomography (CT) with an estimated fractional uncertainty of ±0.03. Table 2 lists the characteristics of the facility and the range of experimental conditions.

Table 2 – NUPEC Test Facility Details (left) and Experimental Conditions (right) [7].

Parameter	Quantity
Maximum Power (MW)	12
Maximum Mass Flux (kg / m^2-s)	2130
Maximum Pressure (MPa)	10.3
Number of Fuel Rods	62
Rod Pitch (mm)	16.2
Fuel Rod Diameter (mm)	12.3
Number of Water Rods	2
Water Rod Diameter (mm)	15
Heated Length (mm)	3708

Quantity	Test Range
Power (MW)	0.23 – 6.48
Mass Flow (kg/s)	2.78 – 19.34
Pressure (MPa)	0.95 – 8.65
Inlet Subcooling (kJ/kg)	44.3 - 128.4
Outlet Mass Quality (%)	2 - 25

The bundle of interest in this study is comprised 2 unheated water rods and 62 ‘hot’ fuel rods with a relative power profile as illustrated on the left side of Figure 1. Subchannels of interest are illustrated on the right side of Figure 1, while the relative axial power profile is illustrated in Figure 2.

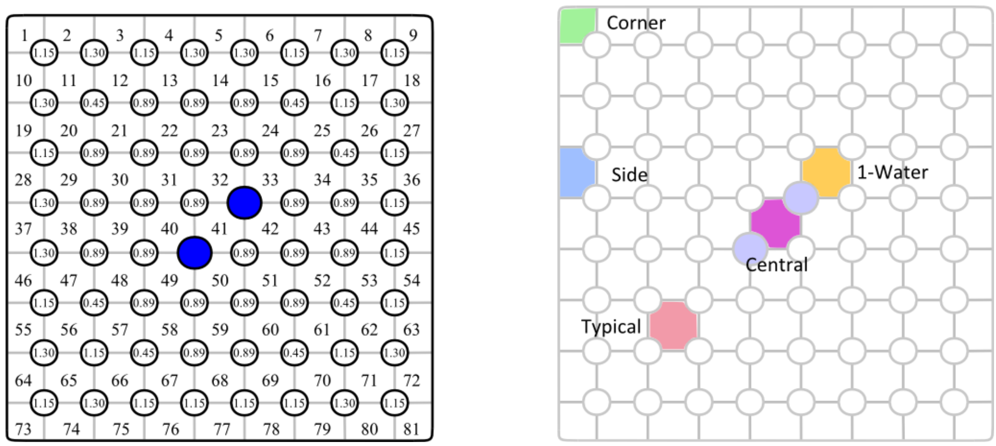

Figure 1 - Bundle cross section with relative rod power and subchannel indices listed (left) and subchannel types of interest (right).

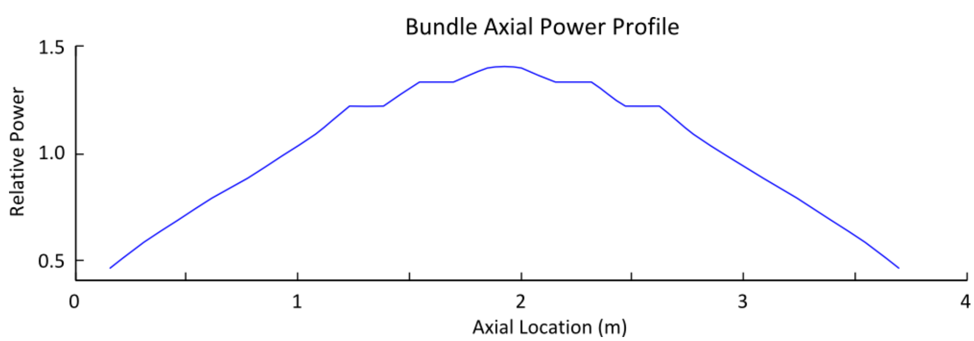

Figure 2 - Relative Axial Power Profile

2.2. ASSERT Model

For the nodalization, parameters such as the flow area, hydraulic diameter, heated and wetted perimeters, subchannel centroid and gap widths are all calculated and implemented into the same model used in [1]. All 81 of the subchannels are modeled using 20 axial nodes, as are the 62 fuel rods in the bundle. The relevant closure relationships used along with their justification are provided inTable 3. The default or recommended values were used for any parameters required by the correlations, and steady state simulations were conducted for each set of corresponding experimental conditions available.

Table 3 – Empirical Correlations Selected

Parameter	Correlation	Justification
Single Phase Friction Factor	Colebrook-White	Valid for flows in the turbulent regime. The Reynolds Number in subchannels is > 40,000.
Two-Phase Friction Multiplier	Friedel	Collier & Thome recommend the use of this correlation for two phase flows where $\mu_f / \mu_g < 1000$ [8].
Single Phase Heat Transfer Model	Dittus-Boelter	Valid for turbulent flows and Prandtl numbers between 0.7 and 120. The Prandtl number for this case is between 0.8 and 1.8.
Two-Phase Heat Transfer Model	Ahmad	Valid for steam-water mixtures under BWR and PWR pressures and mass fluxes [5].

2.3. Mixing & Void Diffusion Models

Although the ASSERT code has both the Carlucci and Rogers-Rosehart correlations for two phase turbulent mixing, only the former is examined in detail in this study. Carlucci proposes in equations (1) and (2) that the mixture rate for both the liquid and vapour phases comprise of a homogeneous component, $w_{l,hom}$ and an incremental component $w_{l,inc}$ [6]. In the two equations, Δx is the lateral spacing between the two subchannels, and f is the friction factor.

$$W_l = \left(w_{l,\text{hom}} + w_{l,inc}\right)(\Delta x)(f) \quad (1)$$

$$W_v = \left(w_{v,\text{hom}} + w_{v,inc}\right)(\Delta x)(f) \quad (2)$$

The homogeneous components represent the effects of obstructions such as grid spacers if the flow were in single phase, while the incremental components account for the additional mixing observed under two phase conditions. The exact derivation of the four terms is extensive, and is omitted for the purposes of brevity.

The tendency of the vapour to move to its 'equilibrium' distribution is represented using a void drift model. Three void drift models are available in ASSERT: Carlucci, Rowe, and a modified version of Shoukri [5,6]. Carlucci's correlation stated in equation (3) proposes that the void drift, ε_α is comprised of a homogeneous and incremental term. It is essential to note that Ω is an 'adjustment factor' deliberately left by Carlucci et. al. in anticipation of efforts to fine tune the correlation [6]. Based on the data which the correlation was based off of, the authors suggest an appropriate value of Ω to be 3. The other parameters in the equation are the density ρ, centroid to centroid distance between the two subchannels Δy, and the gap distance, S.

$$\varepsilon_\alpha = \left[\left(\frac{w_{v,\text{hom}} + w_{l,\text{hom}}}{\rho}\right) + \Omega\left(\frac{w_{v,inc} + w_{l,inc}}{\rho}\right)\right]\left[\frac{\Delta y}{S}\right] \quad (3)$$

Rowe proposes a simple correlation given in equation (4) which relates the void drift to the void fraction, the mass flux G, axial velocity U, and hydraulic diameter D_k.

$$\varepsilon_\alpha = 0.026\left[3\exp\left(-1.1G/1356\right)\right]\left[4\alpha\left(1-\alpha\right)\right]\left[UD_h\right] \tag{4}$$

The modified correlation of Shoukri (5) suggests that void drift is a function of the Ohkawa-Lahey multiplier f_{OL}, void fraction, axial velocity and hydraulic diameter. The Ohkawa-Lahey multiplier is provided in equation (6) and is dependent on only the local void fraction and mass quality of the flow. The thermalhydraulic conditions of the test data which the correlations are based off of are listed in Table 4.

$$\varepsilon_\alpha = 0.05 f_{OL}\alpha^{0.1}UD_h \tag{5}$$

$$f_{OL} = \left\{ \begin{array}{ll} \left[1-\left(\dfrac{\alpha - X}{1-X}\right)^2\right]^{1.5} & ; \quad \alpha > X \\ 1 & ; \quad \text{otherwise} \end{array} \right\} \tag{6}$$

Table 4 - Void Drift Correlation Test Conditions

Parameter	Carlucci	Rowe	Shoukri
Pressure [MPa]	6.9	2.8 - 5.1	0.13
Mass Flux [kg/m²s]	0.72 - 1.46	1.4 - 4.1	0.95 - 1.56

3. Results

A reference case was run in order to demonstrate the nature of the over-prediction of the void fraction. Figure 3 illustrates the predicted void fraction plotted against the measured void fraction for an experiment run with an outlet pressure of 0.969 MPa, bundle power of 0.60 MW, a flow rate of 8.33 kg/s and an inlet subcooling of 45.7 kJ/kg. All correlations and input flags in ASSERT were set to default of recommended values.

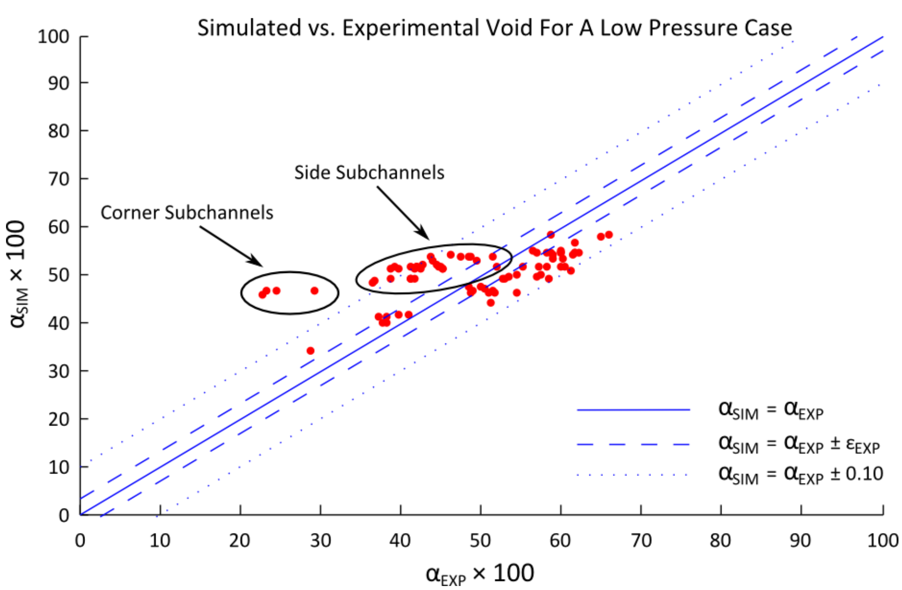

Figure 3 - Predicted vs. Measured Void Fraction for the Reference Case

In terms of the bundle averaged void fraction, the code over-predicted the experimentally observed results by 0.0143, which is well within experimental uncertainty of ±0.03. However, it is clear from Figure 3 that the majority of the subchannels near the center of the bundle are under-predicted, whereas the void fraction along the side and corners is over-predicted. In the corner subchannels, the predicted void is almost twice the experimentally measured value. This behaviour is attributed to an issue with the mixing and void drift model, and is examined in depth in the subsequent sections.

3.1. Comparison with Single Subchannel Results

A single, isolated subchannel was simulated in ASSERT in order to determine whether the mixing was too strong or too weak. The results in Figure 4 indicate that a model consisting of a single subchannel and a single fuel rod would predict a void fraction higher than a model of the full bundle. This implies that mixing correlation is causing void to be advected out of the side and corner channels, and into the central channels. This is consistent with Lahey and Moody's theory that the "equilibrium void distribution" calls for the void to move towards larger subchannels and those with higher flow rates.

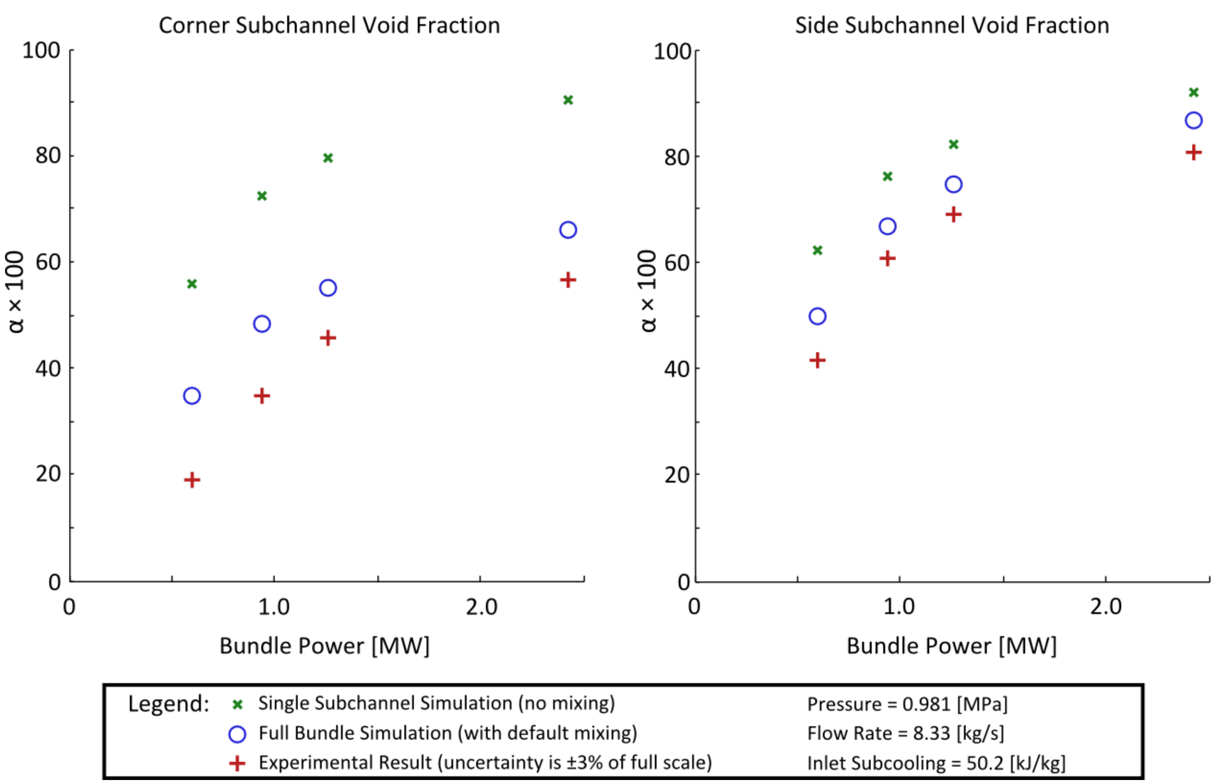

Figure 4 - Corner and Side Subchannel Averaged Void Fraction as a Function of Bundle Power at Low Pressure

The data in Figure 4 indicates that the problem with the simulations appears to be that the mixing rates in the corner and side subchannels at low pressure is too low. The plot in Figure 5 illustrates the effect of increasing the bundle to 7.159 MPa - typical of what would be found in a BWR under normal operations. With the increased pressure, the isolated subchannel still over-predicts the experimental void fraction. However in every case, the void fraction in the subchannels of the full bundle simulation is within the experimental uncertainty of the measured value. This demonstrates that the mixing correlation is correct when used at high pressure, but does not adequately reflect the experimental results when used at low pressure. Since the Carlucci mixing and void drift correlations are primarily functions of mass flow rate, local void fraction and viscosity, the model is not particularly sensitive to pressure, despite the current evidence that there should be a pressure dependency.

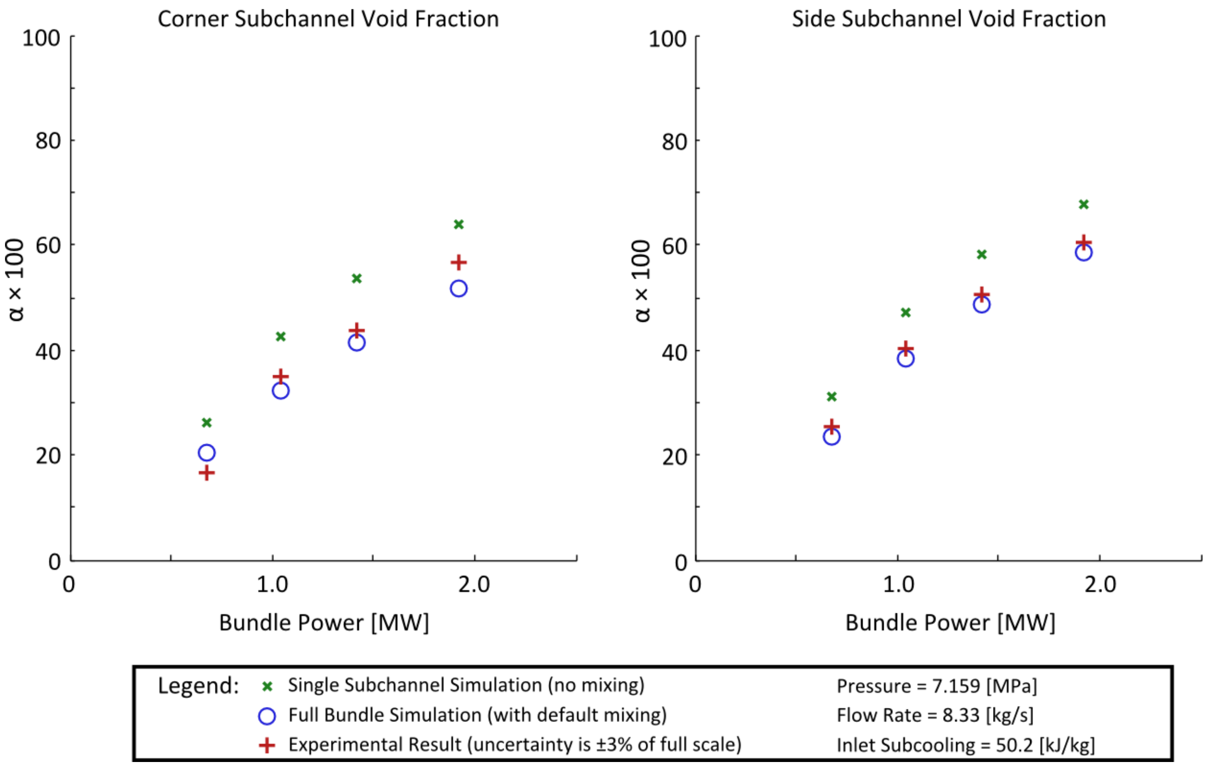

Figure 5 - Corner and Side Subchannel Averaged Void Fraction as a Function of Bundle Power at High Pressure

Table 5 - Effects of the Mixing & Void Drift Model on Accuracy at Low Pressure

Mixing Model	**Carlucci**	**Carlucci**	**Carlucci**	**Rogers-Rosehart**	N_{Points}
Void Drift Model	**Carlucci**	**Rowe**	**Shoukri**	**Shoukri**	
Average Corner Subchannel Error	.1755	.1888	.1850	.1856	44
Average Side Subchannel Error	.0702	.0734	.0697	.0700	308
All Other Subchannels	-.0063	-.0082	-.0058	-.0060	539
Bundle Average Error	.0291	.0297	.0297	.0297	891

In Table 5, 11 experimental cases at a pressure of 0.981 MPa, with varying power, flow rate and subcooling, were analyzed in order to examine the effects of model selection on the accuracy of the void fraction predictions. The data presented in the table represents the average error over the cases run for the particular subchannel being examined. At first glance, it appears the Carlucci-Carlucci model is the most accurate, however it is noted that the experimental uncertainty is ±0.03, and so the differences are insignificant. Although the bundle averaged error is small, and within experimental uncertainty, the fact that the error in the corner subchannels is about 6 times the experimental uncertainty indicates that the mixing correlation at low pressures could be improved.

3.2. Influence of the Incremental Mixing Factor in Carlucci

In order to prove that the void fraction is being under-mixed at low pressures, Figures 6 and 7 demonstrate the effects of increasing the incremental mixing coefficient, Ω, from equation 3.

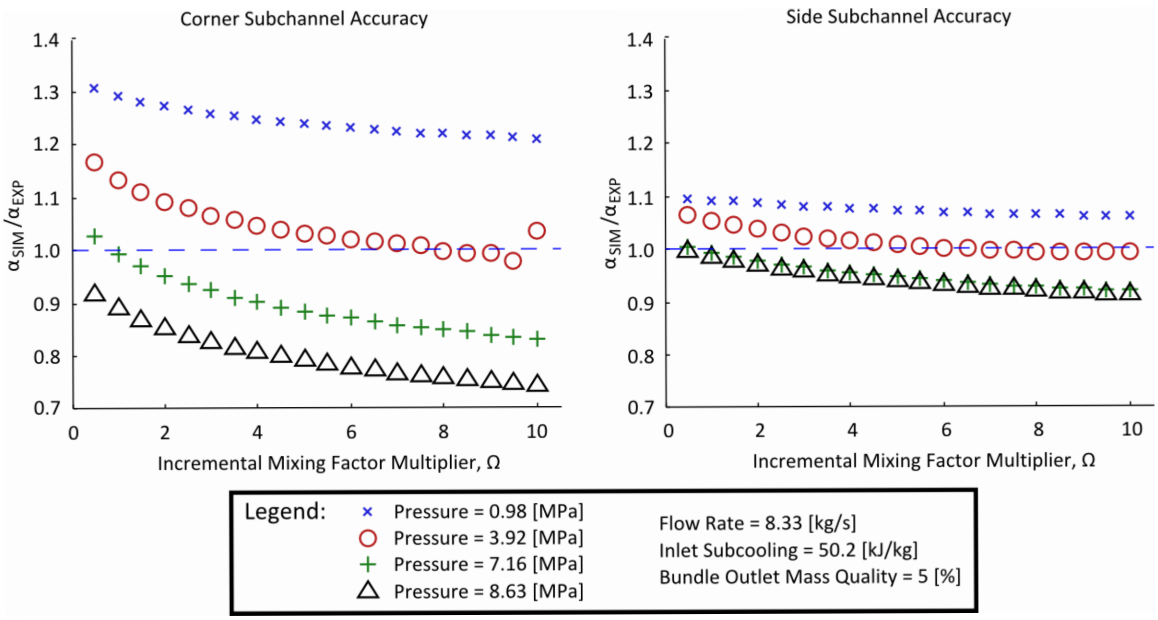

Figure 6 - Corner (left) and Side (right) Subchannel Accuracy

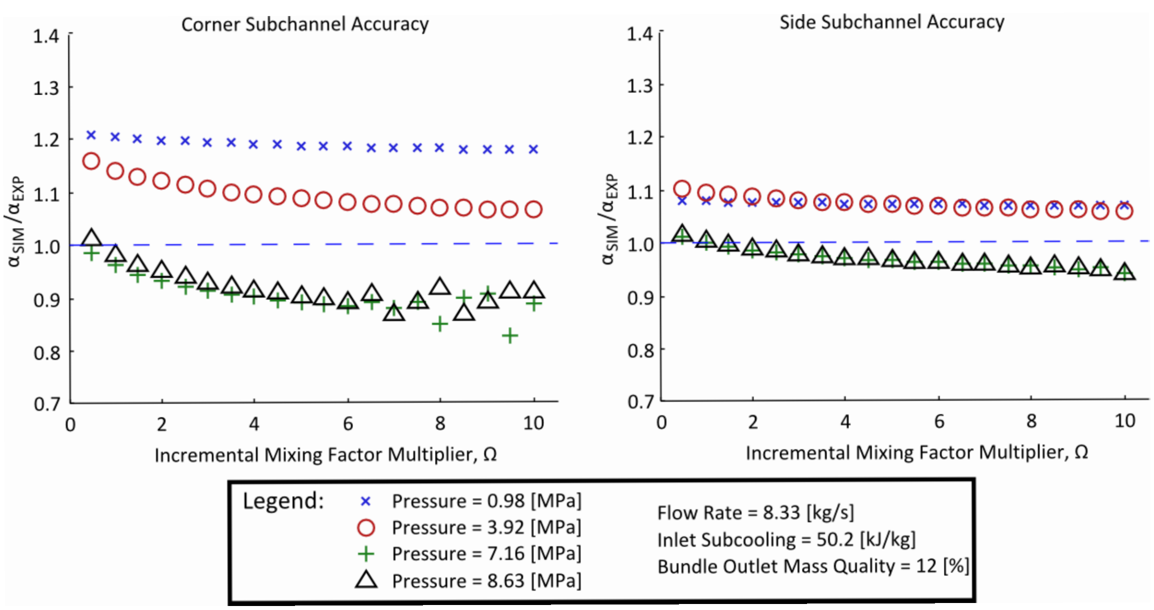

Figure 7 - Corner (left) and Side (right) Subchannel Accuracy

Cases in the same figure have the same flow rate, inlet subcooling and outlet mass quality, and only the outlet pressure is varied. In each instance, increasing the incremental mixing coefficient decreases the void in the corner and side subchannels, and at low pressures, drives the simulated to measured void fraction ratio closer to unity. At higher pressures, the void fraction in these channels seems to be under-predicted, and the "optimal" value for the incremental mixing coefficient should be set close to 0.

From a theoretical point of view, the net mixing term is dominated by the homogeneous component, which is justifiable since one would expect that in a vertical bundle, grid spacers and other obstructions would be the primary source of diversionary flows. For the 0.98 MPa case examined in Figure 6, the mixing rate at the top node of the corner subchannel (junction 1-2 according to Figure 1) is plotted against the incremental mixing factor in Figure 8. Increasing Ω from the default value of 3 to 8 causes a 5.8% increase in the actual mixing rate of this node and corresponds to a 3.9% increase in accuracy.

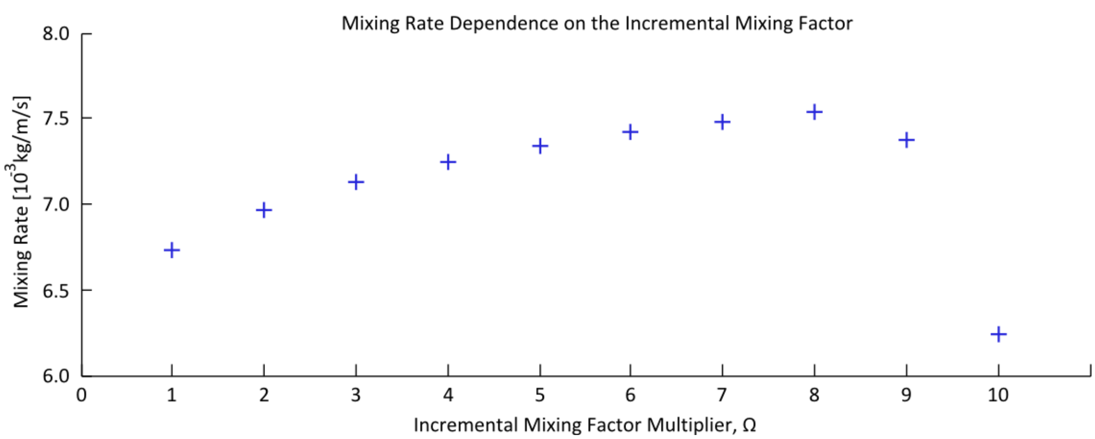

Figure 8 - Mixing Rate Dependence on the Incremental Mixing Factor Multiplier

According to Carlucci's presentation of the experimental data obtained by Rowe and Angle, the two-phase mixing rate in a junction 2.1 mm in size, at 2.8 MPa pressure and 1356 kg/m^2s mass flux should be between 0.02 – 0.08 kg/m/s [6]. Since the subchannel mixing rate increases with an increasing junction size, and the size of the corner junction in the test bundle being examined is 3.4 mm, it would be reasonable to expect simulated mixing rate to be higher than that reported by Rowe and Angle. However, this is not the case, as even with using Ω = 9, the simulated results predict a total mixing rate one order of magnitude lower than expected, strengthening the claim that the mixing in the simulated bundle is under-predicted.

3.3. Preliminary Proposed Correction Factor

In Figure 9, the average bundle void fraction error for 81 different test cases is plotted against pressure with a trend line in the form $\varepsilon = a \ln(P) + b$, where ε is the average bundle void error, a and b are coefficients, and P is the pressure in MPa. Assuming that the average error with respect to pressure does in fact follow this trend, a rudimentary "correction factor" for Ω may be formed so that the incremental mixing factor multiplier is highest at low pressure, and decreases as pressure is increased.

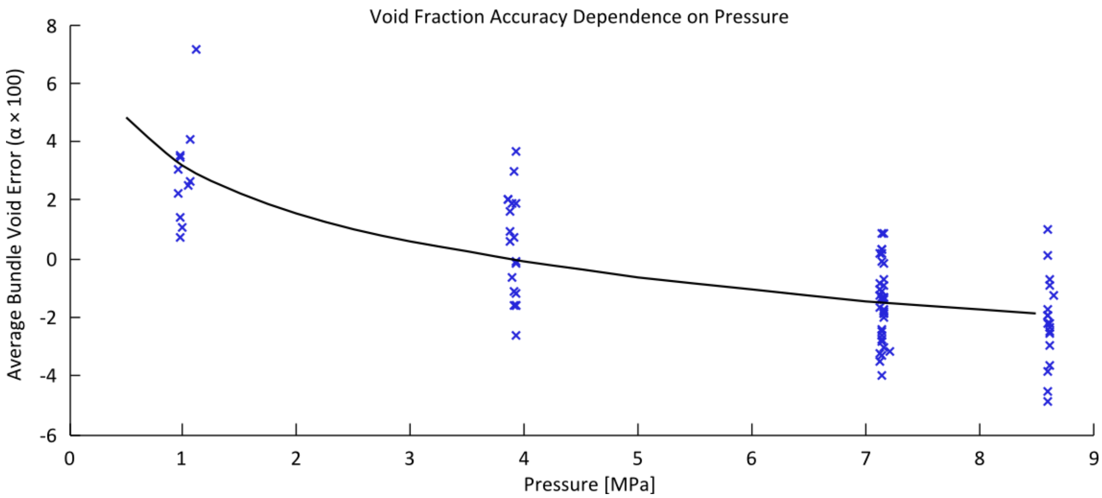

Figure 9 - Void Fraction Accuracy as a Function of Pressure

A preliminary analysis was performed using equation 7 to calculate Ω as a function of pressure.

$$\Omega = -3.0\ln\left(P\right) + 8.0 \qquad (7)$$

Running the entire test matrix using the newly calculated value for Ω yields the results listed in Table 6, where δ is defined as α_{SIM} - α_{EXP}. On an aggregate level, linking Ω to the outlet pressure increased the number of points falling within experimental error by 66, or about 1.0%. If only the corner and side subchannels are considered, the corrected Ω increases the number of points falling within experimental uncertainty by 2.8% and 2.5% respectively. In the analysis of this data, the sample standard deviation of the error should be highlighted, as significant decreases occurred in the corner and side subchannels as a result of applying the correction, suggesting that the preliminary work is influencing the predicted results to converge with the experimental results.

Table 6 - Subchannel Void Fraction Accuracy Comparison Between Uncorrected and Corrected Ω Values

Subchannel Type	$\Omega = 3.0$				$\Omega = -3.0\ln\left(P\right) + 8.0$				N_{Points}
	# of points where δ ≤ 0.03	# of points where δ ≤ 0.10	$\bar{\delta} \times 100$	$\sigma_\delta \times 100$	# of points where δ ≤ 0.03	# of points where δ ≤ 0.10	$\bar{\delta} \times 100$	$\sigma_\delta \times 100$	
Corner	101 (31.6%)	230 (71.9%)	-0.1353	8.7231	110 (34.4%)	243 (75.9%)	0.1820	7.8764	320
Side	961 (42.9%)	2120 (94.6%)	0.2142	5.2621	1017 (45.4%)	2142 (95.6%)	0.3588	4.9856	2240
All Others	1892 (48.3%)	3863 (98.5%)	-1.4184	4.2796	1893 (48.3%)	3849 (98.2%)	-1.4582	4.3061	3920
All Subchannels	2954 (45.6%)	6213 (95.9%)	-0.7907	5.0017	3020 (46.6%)	6234 (96.2%)	-0.7491	4.8625	6480

4. Conclusions & Extensions

To summarize, this paper has determined that:

- ASSERT can accurately predict the void fraction in most of the subchannels in a vertical, BWR style bundle, however it has problems with the side and corners at low pressures.
- The void fraction over-prediction was determined to be caused by the mixing in these subchannels being under-predicted.
- Changing the turbulent mixing and void drift correlations had negligible effect on accuracy at these conditions.
- Increasing the Ω reduced the void in the corner and side subchannels, and when under the low pressure conditions, increased the accuracy of the solution.
- A preliminary attempt at relating Ω to the outlet pressure yielded a crude, but simple relationship which when applied, increased the number of corner and side subchannels falling within experimental error by 2.8% and 2.5% respectively.

The most obvious extension of the work would be to incorporate the bundle outlet quality and the mass flow into the correction factor, as both these parameters do influence the simulation accuracy.

5. Acknowledgements

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6. References

[1] Leung, K. H. and Novog, D. R., Evaluation of ASSERT-PV V3R1 void fraction predictions against the OECD/NEA BFBT benchmark data, *submitted to the 30th Annual Conference of the Canadian Nuclear Society*, Calgary, Canada, 2009.

[2] Hwang, D.H., et. al., Assessment of the interchannel mixing model with a subchannel analysis code for BWR and PWR conditions, *Nuc. Eng. Des.*, 199, pp. 257-272, 2000.

[3] Lahey, R. T. and Moody, J. F., The Thermal-hydraulics of a Boiling Water Nuclear Reactor, American Nuclear Society, New York, United States of America, 1979.

[4] Tye, P., Teyssedou, A., Tapucu, A., An investigation of the constitutive relations for intersubchannel transfer mechanisms in horizontal flows as applied in the ASSERT-4 subchannel code, *Nuc. Eng. Des.*, 149, pp. 207-220, 1994.

[5] ASSERT-PV V3R1 Theory Manual. Atomic Energy of Canada Limited, Mississauga, Canada, 2007.

[6] Carlucci, L. N., Hammouda, N., and Rowe, D. S., Two-phase turbulent mixing and buoyancy drift in rod bundles, *Nuc. Eng. Des.*, 227, pp. 65-84, 2004.

[7] B. Neykov, F. Aydogan, L. Hochreiter, K. Ivanov, H. Utsuno, K. Fumio, *NUPEC BWR Full-Size Fine-Mesh Bundle Test (BFBT) Benchmark. Volume 1: Specifications*, NEA/NSC/DOC(2005)5.

[8] Collier, J. G., and Thome, J. R., Convective Boiling and Condensation, Oxford Science Publications, New York, United States of America, 2001.