DESIGN OF A DECOUPLING ALGORITHM FOR THE REACTOR NODAL CORE MODEL OF A LARGE PHWR

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Abstract

This paper proposes a new decoupling algorithm for the reactor nodal core model of a large 540 MWe PHWR. The purpose of decoupling the nodal model is to reduce its size, so as to facilitate controller design. This decoupling method is based on the minimization of an objective function, which maximizes the mean sensitivity and minimizes the uniformity. The decoupled system is composed of 3 partitions with 20, 26, and 26 states in each partition. Distributed controllers can then be designed for these partitions to form a Distributed Control System (DCS).

1. Introduction

Nodal methods are an accurate way of describing the behavior of a large nuclear reactor, like a large PHWR (Pressurized Heavy Water Reactor). A variety of nodal methods exist, all of which have the common goal of solving the diffusion equation for averaged fluxes in homogenized zones [1]. The nodal model is based on the concept of coupled-core kinetics [2]. The reactor core is divided into divisions or nodes where the neutron flux and material composition are considered to be homogenous. These zones can then be considered as small cores and coupled through neutron diffusion. This model allows for spatial control for a large nuclear reactor.

There have been various methods of model reduction or decoupling algorithms developed for different cases. For example, in fuzzy logic there exists fuzzy clustering which takes a data set and divides it into clusters or groups. Data in the same cluster is as similar as possible and data in different clusters are as dissimilar as possible. There exists several types of fuzzy clustering methods, the most popular being fuzzy c-means clustering [3]. This method, however, was not suitable for this case because of its discrete nature and the use of states. Other reduction methods such as Krylov based model methods [4] and SVD (Singular Value Decomposition) methods were not applicable for various factors of inapplicability.

A reactor nodal core model of a 540 MWe PHWR based on the model used by Talange, Bandyopadhyay, and Tiwari [5] was used. The reactor core is comprised of 14 zones, 7 zones per axial half, each zone representing one node in the model. Each zone includes 5 state equations per zone plus the fuel and coolant temperature thus making it a 72^{nd} order system. It is challenging to deal with higher order systems in control design. Therefore, a reduced model is more manageable. The present work introduces an innovative approach to reducing

the model by using a new decoupling algorithm. This method decouples the system by dividing the states into partitions by having the most dependent states in the same partition. These partitions can be used for the design of sub-controllers thus creating distributed controllers. This creates a decoupled reactor control system using distributed controllers that will communicate with each other in a DCS network architecture.

The large PHWR is a pressurized heavy water reactor that uses natural uranium oxide as fuel and heavy water as the moderator and coolant. Its power outputs are 1800 MW thermal power and 540 MW electrical power. The core dimensions are 800 cm diameter and 600 cm length; because of its vastness in comparison to the neutron migration length, there is a need for reactivity devices distributed spatially and flux detecting mechanisms. To be able to control and observe the neutron flux distribution, the core has been divided into 14 zones. Each zone contains a Liquid Zone Controller (LZC) compartment which is used as the primary method of short term reactivity control by varying the light water levels. The higher the water level, the more reactivity will decrease in that particular zone and surrounding area and vice versa. The in-core vertical flux detectors measure the neutron flux at various points in the core for the estimation of power distribution and total power. The reactor flux is the controlled variable whereby its proportional electronic signal will be fed to the control computer and the adjustment to the flow control valve in the light water supply line will be made accordingly making the light water level the manipulated variable.

The rest of the paper is organized as follows, Section 2 describes the nodal core model of the 540 MWe PHWR along with its linearization, Section 3 explains the decoupling algorithm proposed, and Section 4 gives and discusses the simulation results of the partitions.

2. Reactor core model

2.1 Nodal core model of the 540 MWe PHWR

The 14 zones in the reactor are considered as small cores coupled through neutron diffusion. With the various neutron interactions like neutron production and absorption in each zone and the leakage of neutrons among different zones, the rate of change of power in a zone can be given as [5]:

$$\frac{dP_i}{dt} = \frac{\left(\rho_{exi} + \rho_{fi} + \rho_{ci}\right) - \beta - \frac{\bar{\sigma}_{xi} X_i}{\Sigma_{ai}}}{l} P_i + \sum_{g=1}^{m_d} \left[\left(\lambda_g C_{ig}\right) + \frac{1}{l} \sum_{j=1}^{N} \left(\alpha_{ji} P_j - \right] \alpha_{ij} P_i \right) \quad (1)$$

$$\bar{\sigma}_{xi} = \frac{\sigma_{xi}}{E_{eff} \Sigma_{fi} V'_i} \quad (i = 1, 2, \ldots, N) \quad (2)$$

The accuracy of the nodal model depends highly on the coupling coefficients which are described as:

$$\alpha_{ij} = \frac{D v l \Psi_{ij}}{d_{ij} V'_i} \quad ; \qquad i \neq j \tag{3}$$

where,

i and j = subscripts to denote zones,
N = number of zones in the reactor,
m_d = number of delayed neutron precursor groups,
P = power level,
$\rho_{exi}, \rho_{fi}, \rho_{ci}$ = reactivity related to the external control mechanism, feedback due to fuel temperature and feedback due to primary coolant temperature,
β = total delayed neutron fractional yield,
λ_g = decay constant for gth group of delayed neutron precursors,
X = xenon concentration,
Σ_a and Σ_f = thermal neutron absorption and fission cross sections respectively,
l = prompt neutron lifetime,
E_{eff} = energy liberated per fission,
V' = volume,
σ_x = xenon microscopic thermal neutron absorption cross sections,
α_{ij} = coupling coefficient,
D = diffusion coefficient,
v = thermal neutron speed,
Ψ_{ij} = area of interface between ith and jth zones,
d_{ij} = distance between ith and jth zones.

The delayed neutron precursors' concentrations in different zones are given by:

$$\frac{dC_{ig}}{dt} = \frac{\beta_g}{l} P_i - \lambda_g C_{ig} \,, \tag{4}$$
$$i = 1, 2, \ldots N, g = 1, 2, \ldots, m_d \,.$$

Only one group of delayed neutrons is considered for simplicity, i.e. m_d =1. In the ith zone, the iodine and xenon concentrations vary and are represented as:

$$\frac{dI_i}{dt} = \gamma_I \Sigma_{fi} \mathbf{P}_i - \lambda_I I_i \tag{5}$$

$$\frac{dX_i}{dt} = \gamma_X \Sigma_{fi} \mathbf{P}_i + \lambda_I I_i - (\lambda_X + \overline{\sigma}_{xi} P_i) X_i \tag{6}$$

where,

I = iodine concentration,
γ_x and γ_I = xenon and iodine yield per fission,
λ_x and λ_I = xenon and iodine decay constants.

The changes in fuel and coolant temperatures are defined as:

$$\frac{dT_f}{dt} = k_a P_g - k_b \left(T_f - T_c\right) \quad (7)$$

$$\frac{dT_c}{dt} = k_c \left(T_f - T_c\right) - k_d (T_c - T_1) \quad (8)$$

where,

T_f = fuel temperature at any point within the fuel volume,
T_c = coolant temperature,
T_1 = coolant inlet temperature,
P_g = global power which is the sum of all the zonal powers,
k_a, k_b, k_c, k_d = constants that depend on the thermal capacity and conductivity of the fuel and coolant.

The change in water level as a function of input signals to control valves is described as:

$$\frac{dh_i}{dt} = -m_i q_i \quad (9)$$

where,

h_i = instantenous water level in the ith zone control compartment,
m_i = constant,
q_i = voltage signal given to the control valve of the ith zone.

The change in reactivity associated with the fuel and coolant temperature are defined as:

$$\rho_{fi} = \mu_{fi} \left(T_f - T_{f0}\right) = \mu_{fi} \delta T_f \quad (10)$$

$$\rho_{ci} = \mu_{ci} (T_c - T_{c0}) = \mu_{ci} \delta T_c \quad (11)$$

where,

μ_f and μ_c = fuel and coolant reactivity coefficients,
T_{f0} and T_{f0} = steady state values of the fuel and coolant temperatures.

The reactivity due to the control mechanism of the LZC that is directly proportional to the water level in the ZCC in its respected zone is defined as:

$$\rho_{exi} = -K_i^{'}(h_i - h_{i0}), (i = 1,2, \ldots N). \quad (12)$$

2.2 Linearization

A set of linear equations are needed in order to describe the behavior of the reactor in the area of the steady-state operating point due to any minor change in power, delayed

precursor's concentration, iodine, xenon concentration, liquid zone water levels, fuel and coolant temperatures:

$$\frac{d}{dt}\left(\frac{\delta P_i}{P_{i0}}\right) = -\frac{1}{l}\left(\beta + \sum_{j=1}^{N}\alpha_{ij}\frac{P_{j0}}{P_{i0}}\right)\frac{\delta P_i}{P_{i0}} + \frac{1}{l}\sum_{j=1}^{N}\alpha_{ij}\frac{P_{j0}}{P_{i0}}\frac{\delta P_i}{P_{i0}} + \frac{\beta}{l}\frac{\delta C_{ik}}{C_{ik0}} - \frac{\bar{\sigma}_{xi}X_{i0}}{l\Sigma_{ai}}\frac{\delta X_i}{X_{i0}} - \frac{K'_i}{l}\delta h_i + \frac{\mu_{fi}\delta T_f}{l} + \frac{\mu_{ci}\delta T_c}{l} \quad (13)$$

$$\frac{d}{dt}\left(\frac{\delta C_{ik}}{C_{ik0}}\right) = \lambda_k\frac{\delta P_i}{P_{i0}} - \lambda_k\frac{\delta C_{ik}}{C_{ik0}} \quad (14)$$

$$\frac{d}{dt}\left(\frac{\delta I_i}{I_{i0}}\right) = \lambda_I\frac{\delta P_i}{P_{i0}} - \lambda_k\frac{\delta I_i}{I_{i0}} \quad (15)$$

$$\frac{d}{dt}\left(\frac{\delta X_i}{X_{i0}}\right) = \left(\lambda_X - \lambda_I\frac{I_{i0}}{X_{i0}}\right)\frac{\delta P_i}{P_{i0}} + \lambda_I\frac{I_{i0}}{X_{i0}}\frac{\delta I_i}{I_{i0}} - \frac{(\lambda_x + \bar{\sigma}_{xi}P_{i0})(\delta X_i)}{X_{i0}} \quad (16)$$

$$\frac{d\delta h_i}{dt} = -m_i\delta q_i \quad (17)$$

$$\frac{d\left(\delta T_f\right)}{dt} = k_a\sum_{i=1}^{N}\delta P_i - k_b\delta T_f + k_b\delta T_c \quad (18)$$

$$\frac{d(\delta T_c)}{dt} = k_c\delta T_f - (k_c + k_d)\delta T_c \quad (19)$$

The state-space representation of the system is given as:

$$\dot{\boldsymbol{z}} = A\boldsymbol{z} + B\boldsymbol{u} \quad (20)$$

$$\boldsymbol{y} = C\boldsymbol{z} \quad (21)$$

The state, control, and outputs vectors are defined as:

$$\boldsymbol{z} = \left[z_I^T z_X^T z_C^T z_H^T z_P^T z_{T_f}^T z_{T_c}^T\right]^T \quad (22)$$

$$\boldsymbol{u} = [\delta q_1 \delta q_2 \delta q_3 \dots \delta q_N]^T \quad (23)$$

$$\boldsymbol{y} = z_p \quad (24)$$

Therefore, the state variables can be defined as:

$$z_I = \left[\frac{\delta I_1}{I_{10}}\frac{\delta I_2}{I_{20}}\frac{\delta I_3}{I_{30}}\dots\frac{\delta I_N}{I_{N0}}\right]^T \quad (25)$$

$$z_X = \left[\frac{\delta X_1}{X_{10}}\frac{\delta X_2}{X_{20}}\frac{\delta X_3}{X_{30}}\dots\frac{\delta X_N}{X_{N0}}\right]^T \quad (26)$$

$$z_C = \left[\frac{\delta C_1}{C_{10}} \frac{\delta C_2}{C_{20}} \frac{\delta C_3}{C_{30}} \dots \frac{\delta C_N}{C_{N0}}\right]^T \quad (27)$$

$$z_H = [\delta h_1 \delta h_2 \delta h_3 \dots \delta h_N]^T \quad (28)$$

$$z_P = \left[\frac{\delta P_1}{P_{10}} \frac{\delta P_2}{P_{20}} \frac{\delta P_3}{P_{30}} \dots \frac{\delta P_N}{P_{N0}}\right]^T \quad (29)$$

$$z_{T_f} = \delta T_f \quad (30)$$

$$z_{T_c} = \delta T_c \quad (31)$$

3. Decoupling algorithm

The complexity of such a large system makes it challenging for controller design which gives rise to model reduction. A convenient way of reducing a model without losing any of the important features of the system can be done by decoupling. This groups the most dependent states together and thus, partitions will be created containing a reduced number of states and can be used to design efficient sub-controllers. The following is a systematic decoupling algorithm that can be applied to the reactor nodal core model of a large PHWR for distributed controller design.

This method is based on the notion of sensitivity that can measure the dependency of each state to another by slightly changing a state from equilibrium and observing the behaviour of the other states accordingly. In general, the sensitivity in system engineering can be defined as,

Definition (Sensitivity): Sensitivity of a parameter X of a system with respect to parameter Y at equilibrium is the rate of change of X when parameter Y is disturbed by a small change, namely $\Delta > 0$. In an LTI (linear time invariant) system, the sensitivity of state z_i with respect to z_j while z_j is perturbed by δ z_j is calculated after τ seconds,

$$S_\tau(z_i, z_j) = \frac{\dot{z}_i(\tau)}{\delta z_j} \quad (32)$$

In order to calculate $\dot{z}_i(\tau)$, the following system of ordinary differential equation needs to be solved:

$$\dot{z}(t) = Az(t) \quad (33)$$

$$\dot{z}(0) = z_0^j = (0, 0, \dots, \delta z_j, 0, \dots, 0)^T \quad (34)$$

where z_0^j is the state at the initial point when being perturbed by δz_j .

The solution of $\mathbf{z}$ at time t is:

$$\mathbf{z}(t) = e^{A\tau}\mathbf{z_0}^j \tag{35}$$

and based on (33), $\dot{\mathbf{z}}$ at time τ is:

$$\dot{\mathbf{z}}(\tau) = Ae^{A\tau}\mathbf{z_0}^j . \tag{36}$$

Therefore, sensitivity at time τ is defined as:

$$S_\tau\left(z_i, z_j\right) = \frac{\dot{z}_i(\tau)}{\delta z_j} = row_i[A] \times column_j\left[e^{A\tau}\right] . \tag{37}$$

This metric is going to be utilized as a dependency measure of states to put all coupled states in a partition to be controlled separately. The larger the sensitivity is, the more z_i is dependent to z_j. In this formulation, all of the states are non-dimensionalized with respect to the equilibrium point.

In order to divide the system into a few sub-systems which are mostly coupled, an objective function is minimized that is defined based on two main criteria:

a. The weighted average of sensitivity of the partitions (*mean sensitivity*)
b. The variance of the number of states in the partitions (*uniformity*)

The former denotes the coupling degree of the states over all of the partitions and the latter checks the distribution of the states in partitions.

The mean sensitivity is the weighted mean square of sensitivity of the states in a partition with respect to the corresponding partition center and is defined as:

$$M = \sqrt{\frac{1}{\sum_{i=1}^{m}\sum_{k=1}^{n_i} w_{ik}} \sum_{i=1}^{m}\sum_{k=1}^{n_i} w_{ik} S^2\left(z_k^i, z_c^i\right)} \tag{38}$$

where,

m = number of partitions,
n_i = number of states in the ith partition,
z_k^i = the kth state in the ith partition,
z_c^i = the center state in the ith partition,
w_{ik} = weight of z_k^i belonging to the ith partition.

The uniformity is the variance of the number of elements in the partitions with respect to the average number of elements which is defined as:

$$U = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(n_i - \frac{n}{m}\right)^2} \tag{39}$$

Therefore, based on the above definitions, the mean sensitivity should be maximized while the uniformity is minimized to achieve an optimal partitioning of the states. Accordingly, an objective function that incorporates both aspects can be defined as:

$$J = \frac{1}{n}U - M \tag{40}$$

where n is the total number of states and the factor $\frac{1}{n}$ normalizes the uniformity.

In this partitioning method the number of partitions should be known in *a priori*. A state is placed arbitrarily in each empty partition as the center of partition. Sensitivity of all other states to the centers is calculated and they are transferred to the partition with the maximum sensitivity with the respect to its center. In this way, *(m,n)* number of different partitionings is determined out of which the best can be selected by minimizing the objective function.

4. Results and discussion

The system was modeled and simulated using MATLAB. The sensitivity was taken at $\tau = 0.05$ seconds. Three partitions were chosen and the simulation yielded the first having 20 states, the second having 26 states and the third having 26 states. The partitions are given in Table 1,

Zone	Partition 1	Partition 2	Partition 3
1	$z_1, z_{15}, z_{29}, z_{43}, z_{57}$		
2	$z_2, z_{16}, z_{30}, z_{44}, z_{58}$		
3		$z_3, z_{17}, z_{31}, z_{45}, z_{59}$	
4	$z_4, z_{18}, z_{32}, z_{46}, z_{60}$		
5	$z_5, z_{19}, z_{33}, z_{47}, z_{61}$		
6		$z_6, z_{20}, z_{34}, z_{48}, z_{62}$	
7		$z_7, z_{21}, z_{35}, z_{49}, z_{63}$	
8			$z_8, z_{22}, z_{36}, z_{50}, z_{64}$
9			$z_9, z_{23}, z_{37}, z_{51},$

			z_{65}
10			$z_{10}, z_{24}, z_{38}, z_{52}, z_{66}$
11			$z_{11}, z_{25}, z_{39}, z_{53}, z_{67}$
12			$z_{12}, z_{26}, z_{40}, z_{54}, z_{68}$
13		$z_{13}, z_{27}, z_{41}, z_{55}, z_{69}$	
14		$z_{14}, z_{28}, z_{42}, z_{56}, z_{70}$	
None		z_{72}	z_{71}

Table 1: Simulation Results of Partitions

where states 1-14 are the corresponding zonal iodine concentrations, 15-28 the xenon concentrations, 29-42 the delayed neutron concentration, 43-56 the water level, 57-70 the power, 71 the fuel temperature, and 72 the coolant temperature. The center of each partition that was randomly chosen was state 15, xenon concentration in zone 1, for partition 1, state 23, xenon concentration in zone 9, for partition 2, and state 27, xenon concentration for zone 13.

The corresponding zonal water levels were logically placed in the partition that had the most states for its zone since they depend only on the input to the system and would yield zeros for sensitivity thus being grouped together.

From the results, it can be seen that the states were divided into the 3 partitions according to which states they were most coupled with. These states correspond to the most coupled zones which were defined through the coupling coefficients. The coupling coefficients between non-neighbouring zones and its own zone are assumed to be zero. For neighbouring zones, the coupling coefficients were calculated based on the area of interface and the distance between the *i*th and *j*th zones. Through these relationships, the model was decoupled. Therefore, an optimal distribution of states were acquired that can be used for controller design.

5. Conclusion

A new decoupling algorithm was designed and proposed for the reactor nodal core model of a large 540 MWe PHWR. This algorithm produced a model that created 3 partitions of 20, 26 and 26 states each. Its objective function was based on two main criteria, mean sensitivity and uniformity. Minimizing this function ensured optimal partitioning of the nodal model which will be utilized for the design of the distributed controllers to make up a DCS.

6. References

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