METHODS FOR CHARACTERIZING UNCERTAINTY EFFECTS IN KEY INPUT PARAMETERS USED IN THE 2-DIMENSIONAL MONTE-CARLO SIMULATION OF DRYOUT POWER

D. Quach[1], P. Sermer[1], O. Nainer[2], and B. Phan[3]
[1] Nuclear Safety Solutions Limited, Toronto ON. Canada
[2] Bruce Power LP, Toronto ON. Canada
[3] Ontario Power Generation Inc., Toronto ON. Canada

Abstract

A 2-dimensional Monte-Carlo simulation of dryout power has been developed. A requisite of the probabilistic analysis is a clear discrimination of the two fundamental types of uncertainty effects that impacts on the interpretation and estimation of the model predictions of dryout powers. These uncertainty effects are aleatory, which is a property of the system itself, and epistemic uncertainty, which describes the lack of knowledge of the system. In this paper, we present the development of statistical error models and methods used to characterize the aleatory and epistemic uncertainties in the key input parameters used in predicting dryout power.

1. Introduction

During a postulated power excursion, such as a Loss of Regulation (LOR) event, the reactor power may increase sufficiently to induce an unstable dry patch on the fuel sheath in a high power channel. This condition is commonly known as *dryout*. Although the onset of fuel sheath dryout does not necessarily lead to fuel or fuel channel failures, elevated fuel temperatures can result in fuel element deformations and possibly, fuel centre-line melting and eventually pressure tube failure. Thus, the prevention of the onset of intermittent dryout has been used by the safety analysis industry as a conservative criterion for preventing fuel failures leading to radiological releases.

Advances in the safety analysis industry includes the development of TUF (Two-Unequal Fluids), which is a thermal hydraulic code capable of simulating the Heat Transport System, Auxiliaries and Secondary-Side steady-state, and transient responses to accident conditions [1]. Other developments include the establishment of statistical frame works for uncertainty analysis and the development of Monte-Carlo methods for the probabilistic modeling and uncertainty analysis of dryout power [2, 3]. These advances have made it possible to apply statistical theory to make probabilistic statements of the properties in the model output distribution in light of uncertainty. This information is a requisite for decision-makers with regard to the acceptance criteria.

The uncertainty in the computed results (i.e., dryout power) is a consequence of the uncertainty in the input parameters, and the initial and boundary conditions of the problem. These types of uncertainty are aleatory uncertainty, which is a property of the system itself, and computational/measurement uncertainty, which describes the lack of knowledge about the system. In this paper, we implement the statistical basis developed in [2] to establish methods for characterizing both the epistemic and aleatory uncertainties in the input parameters. The development of statistical error models is a requisite for Monte-Carlo methods in the probabilistic modeling and uncertainty analysis of dryout power [3].

2. Background

2.1 Reactor Design

The probabilistic modeling of dryout power is implemented based on the configuration and operating conditions of the Bruce B nuclear generating station (NGS). The Bruce B Heat Transport System (HTS) is a single closed loop. The core is divided into two hydraulic zones. Outer zone channels are supplied with D_2O directly from the four circulating primary pumps, which draw coolant from the steam generator outlets. Inner zone channels are fed by coolant, which passes from the pumps through four separate pre-heaters, where heat is transferred from the coolant to the feedwater. A simplified schematic of the Bruce B NGS heat transport system and reactor core face is given in Figures [F-1] and [F-2], respectively.

2.2 Computation of the Dryout Power Under an Aged Core Configuration

The prediction of the required power within each channel to induce dryout is determined iteratively by a series of steady-state thermalhydraulic calculations. The header condition and bundle power distribution inputs are used to calculate the steady-state channel flow and thermalhydraulic conditions along the channel. Based on the local thermalhydraulic conditions, the critical heat flux (CHF) at each axial node is determined and compared against the axial heat flux. The computed channel power is increased until the critical heat flux profile becomes tangential to the axial heat flux profile. The channel power corresponding to this condition is the Critical Channel Power (CCP) (i.e., dryout power).

There are two relevant aging mechanisms applicable for the analysis and probabilistic modeling of dryout power. The first is the magnetite dissolution and selective depositions which primarily result in an increase in the reactor inlet header temperatures and a reduction in the core coolant flow. The other aging mechanism is the irradiation induced effects that result in non-uniform pressure tube diametral expansion (i.e., creep).

The impact of HTS aging are explicitly modeled in the probabilistic analysis of dryout power. For example, empirically-based correlations are developed to account for the changes in the thermal hydraulics and fuel cooling behavior due to crept pressure tube diameters (e.g., CHF correlation, Pressure Tube Diametral Creep (PTDC) correlation, etc.). In addition, operational data trending are completed to predict future operating conditions in the thermal hydraulic headers to account for aging effects.

2.3 The 2-Dimensional Monte Carlo Analysis

A 2-dimensional Monte-Carlo algorithm for the simulation and uncertainty analysis of dryout power using the TUF thermal hydraulic code has been developed and discussed in [3]. The development of the algorithm provides a means of evaluating the individual consequences of aleatory and epistemic uncertainties on the computed dryout power estimates. Other applications of the methodology include the ability to evaluate the effects of epistemic uncertainty around specific levels of aleatory uncertainty (or vice versa). These single and multi-dimensional distributions provide a better tool to assess the impacts of epistemic uncertainty in light of aleatory uncertainty (or vice versa).

The first step in the algorithm is to identify and segregate errors in the input parameters affected by uncertainty effects. The input parameters used in computing dryout powers are either measured or computed using complex models or codes. Input parameters subject to epistemic uncertainties are defined as a vector variable $\mathbf{X}=[X_1, X_2,\ldots, X_M]$ where $\mathbf{X}$ is a best-estimate of the true values with corresponding errors $\boldsymbol{\varepsilon} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_M]$. That is:

$$\mathbf{X} = \mathbf{x} + \boldsymbol{\varepsilon} \qquad (1)$$

Aleatory uncertainties also arise in a system or in nature, which result in the desired variable to be random. When no random sample is available, one may estimate the desired variable to be fixed [2]. The aleatory uncertainty describes the deviations from the fixed, non-random estimates of the true value as follows:

$$\mathbf{x} = \mathbf{x_o} + \boldsymbol{\vartheta} \qquad (2)$$

where $\mathbf{x}$ is a vector valued random variable $[x_1, x_2, \ldots, x_N]$ subject to aleatory uncertainty: $\boldsymbol{\vartheta}= [\vartheta_1, \vartheta_2, \ldots, \vartheta_N]$. Probability or frequency distributions for the errors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\vartheta}$ are utilized in a Monte-Carlo sampling method with the thermal hydraulic code TUF, to compute dryout powers as follows:

$$\mathbf{CCP} = \boldsymbol{f}(\mathbf{X}) = \boldsymbol{f}(\mathbf{x} + \boldsymbol{\varepsilon}) \qquad (3)$$

$$\mathbf{ccp} = \boldsymbol{f}(\mathbf{x}) = \boldsymbol{f}(\mathbf{x_o} + \boldsymbol{\vartheta}) \qquad (4)$$

where $\boldsymbol{f}$ represents the relationship in the physical model (i.e., dryout power). A 2-dimensional Monte-Carlo sampling technique can be used to model the interactions between the two uncertainty effects as described in [3]. That is, a vector ***CCP*** of sample size *m* from *M* input frequency distributions and vector ***ccp*** of sample size *n* from *N* input probability distributions are generated. The 2-dimensional simulation is facilitated through a repetitive evaluation of the ***CCP*** for each of the *n* estimates of the rank in ***ccp***. The model output is a set of sample values for the joint distribution of all the random variable inputs to the model. This results in a set of sample values which can be treated statistically as if they were an experimentally or empirically observed set of data.

The objective of this paper is to describe the statistical methods required to estimate the probability or frequency distributions for the errors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\vartheta}$, which can then be propagated in the calculation of the CCP to generate the single or multidimensional distributions required for decision-making [3].

3. Uncertainty Analysis in the Input Variables used to Compute Dryout Powers

3.1 Epistemic Uncertainty Analysis

Epistemic uncertainties arise from our basic lack of knowledge of the true value that may be due to limited availability of empirical information, as well as imperfections in the instruments, models, or techniques used to develop representations of complex physical processes. The methods used in characterizing the epistemic uncertainties are described in this section.

3.1.1 Epistemic Uncertainty in the Modeling of the Heat Transport System Parameters

As discussed in Section 2.2, the Reactor Inlet Header Temperature (RIHT), Reactor Outlet Header Pressure (ROHP), and Header-to-Header Pressure Drop (HHPD) are required inputs in establishing the header conditions for computing dryout power. These operational parameters are affected by the system variations and random in nature. Thus, the true values for any of these operational parameters are random and denoted by *p* (where *p* is any of RIHT, ROHP, HHPD). Our ability to estimate *p* is subject to epistemic uncertainty ε, that arise as a result of the underlying computational model or measuring instrument as follows[1] [1]:

$$P = p + \varepsilon \tag{6}$$

In our probabilistic analysis, we predict dryout powers for each channel under an aged core configuration represented by static operational conditions. That is, we do not estimate the values of p, but rather p_o, which corresponds to static operational conditions as follows:

$$P_o = p_o + \varepsilon_o \tag{7}$$

Note that the difference between p and p_o is the aleatory uncertainty, as discussed in Section 3.2. A method for estimating the epistemic uncertainty ε_o, of an instrument is a study of the random errors in the elemental components of the instrument. For example, the elemental components for a pressure data acquisition system is provided in Figure [F-3] (e.g., power supply, calibration instrument, environmental effects, etc.). The deviations in the elemental components can be related to the deviations in the estimates of p_o by a first-order approximation method. The epistemic uncertainty in each operational parameter is characterized by a normal probability distribution with a zero mean and $\sigma^2_{\varepsilon_{P_o}}$, as shown in Figure [F-4]. Visual tests for normalcy can also be completed as shown in Figure [F-5].

3.1.2 Epistemic Uncertainty in Empirical Correlations

Epistemic uncertainties also arise in our empirical correlations. Statistical error analysis is required in identifying and quantifying the contributions of the epistemic uncertainties in both the dependent and independent variables from the regression analysis as follows:

$$\sigma^2_{\varepsilon_P} = \left(\sigma_{\varepsilon_x} \frac{\partial P}{\partial x}\right)^2 + \sigma^2_{\varepsilon_{P_{inst}}} \tag{8}$$

where $\sigma_{\varepsilon_{P_{inst}}}$ and σ_{ε_x} correspond to uncertainty in the dependent and independent variables, respectively. For example, a best-estimate model for predicting pressure tube diameters for all channels in the core was developed to explicitly incorporate aging-effects in the estimation of dryout power. Pressure tube diameters are measured using *Channel Inspection and Gauging Apparatus for Reactors* (CIGAR). The PTDC correlation is developed based on knowledge that the pressure tube diametral creep is affected predominantly by two variables, namely fluence ψ, and lifetime average temperature T (^{o}C), above some nominal reference value. Thus, the following general functional form can be used in predicting the axial profile of the PT diameters:

$$S^{meas}_{ij} = a + b\,\psi_{ij} + c\omega_{ij} + e_{ij} \tag{9}$$

where:

i = 1,2, … M = bundle position in a fuel channel

j = 1,2, … J = fuel channel

$S^{meas}_{ij} = \dfrac{D^m_{ij}}{D_o} - 1$ = the measured strain at bundle i, channel j

D^m_{ij} = Pressure tube diameter at bundle position i and channel j [mm]

D_o = Nominal pressure tube diameter

[1] Note that the epistemic uncertainty is independent of the values observed in the operational parameter and hence the associated error distribution is applicable over the whole range of possible operational values for the parameter of interest.

ψ_{ij} = Fluence at bundle position i and channel j [n/(m^2 s)]
$\omega_{ij} = \Im(T_{ij}, T_o)$
T_{ij} = Average pressure tube bundle temperature at bundle position i and channel j
T_o = Reference temperature
e_{ij}^{PT} = error at bundle i and channel j
a, b, c = model coefficients

Efforts to decompose the overall uncertainty into epistemic (γ_{ij}^{PT}) and aleatory (ϑ_{ij}^{PT}) uncertainties were completed in [4] using the following error model:

$$e_{ij}^{PT} = \vartheta_{ij}^{PT} + \gamma_{ij}^{PT} \tag{10}$$

The epistemic uncertainty in the correlation was validated using statistical error modeling methods in the development of the Fixed-effects Model (FeM), which preserves the axial functional dependence of the pressure tube diameter. A comparison of the results of the FeM to the measured data demonstrated excellent agreement as shown in Figures [F-6] and [F-7] with the only remaining residual corresponding precisely to epistemic uncertainties[2] (γ_{ij}^{PT}). From [4], the epistemic uncertainty was assessed as a random variable affecting the PTDC predictions at every bundle and every channel, and appropriately characterized by a normal probability distribution as follows:

$$\gamma_{ij}^{PT} \sim N\left(0, \sigma_{\gamma^{PT}}^2\right) \tag{11}$$

3.2 Aleatory Uncertainty Analysis

Some quantities are inherently subjected to temporal, spatial, or inter-individual variability, rather than from any individual event or component. This variability is referred to in a number of ways, such as stochastic uncertainty, Type A uncertainty, or aleatory uncertainty [5, 6]. Stochastic or aleatory uncertainties are types of variability that may not be explainable using mechanistic theory, as it is an inherent property of the system. In this section, we study and capture the effects of aleatory uncertainty using statistical error models.

3.2.1 Aleatory Uncertainty in Modeling the Heat Transport System Parameters

As described in Sections 2.2 and 3.1, we define the RIHT, ROHP, and HHPD as a set of required inputs in establishing the header conditions for the dryout power calculations, which are random variables subject to system variations. Time-averaging methods are used to project the core to future aged conditions and subject our estimation of the operational parameters to aleatory uncertainties as follows:

$$p = p_o + \vartheta_o \tag{12}$$

where p is any of RIHT, ROHP, or HHPD, and ϑ_o is in the form of an error and referred to as the aleatory uncertainty. Through analysis of the operational data, estimates of the random system variations are estimated within defined regions of operationally steady states. Using a dataset of size [8760 x 91] of measured data, Principal Component Analysis (PCA) is used to statistically analyze the multivariate relationships in the data set to differentiate operationally steady states from states under transition [8, 9]. We observed operational steady states that may occur within high power regions that may be driven due

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[2] The FeM assumes that data is available for all channels under consideration. Thus, it can not be used for prediction for channels that do not have available measurements but used in this case for validation of the statistical error model.

to changes in one or two conditions. As an example, Figure [F-8] illustrates four distinct operational steady states at high reactor power following a decomposition of the multivariate data into a lower dimensional subspace. The time series for the operational parameters which correspond to the four steady-state regions are shown in Figures [F-9] and [F-10]. The probability distributions for each parameter and each state can be inferred from empirical data. The results are probabilistically modeled within the Monte-Carlo analysis of dryout power to capture any covariance effects within each K operational states defined using PCA, and recognizing that the variability are not necessarily equal for each operational state. Random sampling for the three operational parameters (i.e., RIHT, ROHP, and HHPD where j= 1 to 3) can be completed using the following algorithm:

(1) Define K operationally steady states using PCA analysis
(2) Define probability distributions for each of the three operational parameters and for all K states
(3) Randomly select a region k
(4) In the selected region k, and for every parameter j, randomly select from ϑ_o to obtain $p = p_o + \vartheta_o$

3.2.2 Aleatory Uncertainty in Empirical Correlations

Empirical correlations may also be affected by aleatory uncertainties. This is evident in the development of the best-estimate PTDC correlation in which a limited number of fuel channels are used in the development of a correlation (that is neither bundle- nor channel-specific) for predicting the pressure tube diameter profile of all channels in the core. Variability arises due to limitations in the correlation to capture the spatial and inter-individual effects that include differing rates of diametral creeps as a result of spatial position, irradiation rates, or due to differences in material properties of the pressure tube. The PTDC correlation's inability to capture the spatial variability in the core from channel-to-channel, and also at each bundle position, is evident in the pressure tube diameter profile comparisons with measured data and the FeM predictions as shown in Figure [F-7].

As discussed in Section 3.1, efforts to decompose the overall residual into epistemic and aleatory uncertainties were completed in [4]. A statistical error model that captures the bundle and channel variabilities was derived as follows:

$$\vartheta_{ij}^{PT} = \tau_i^{PT} + \delta_j^{PT} \tag{13}$$

where τ_i^{PT} captures the bundle-to-bundle variabilities and δ_j^{PT} characterizes the channel-to-channel variabilities. The error components for the PTDC model were determined to be appropriately characterized by a normal probability distribution, where the estimates of the variances were derived using the Maximum Likelihood Estimation (MLE) method [4]. The statistical error model for the PTDC correlation can be summarized as given in Figure [F-11].

4. Conclusions

Successful efforts in error analysis require an understanding of both the physical phenomena of the system of interest, and a statistical basis for the development of error models to capture the aleatory and epistemic uncertainties. In this paper, the statistical framework discussed in [2] was implemented in the context of probabilistic modeling and uncertainty analysis of dryout power. Methods for developing statistical error models to

characterize the epistemic and aleatory uncertainties in the input parameters of dryout power were presented. The error models are in a form suitable for evaluating the individual consequences of aleatory and epistemic uncertainties on the model outputs (i.e., dryout power) which is a requisite for decision-makers.
Using the results presented in this paper, a probabilistic model and uncertainty analysis of dryout power can be completed using a 2-dimensional Monte-Carlo analysis as discussed further in [3].

5. Acknowledgements

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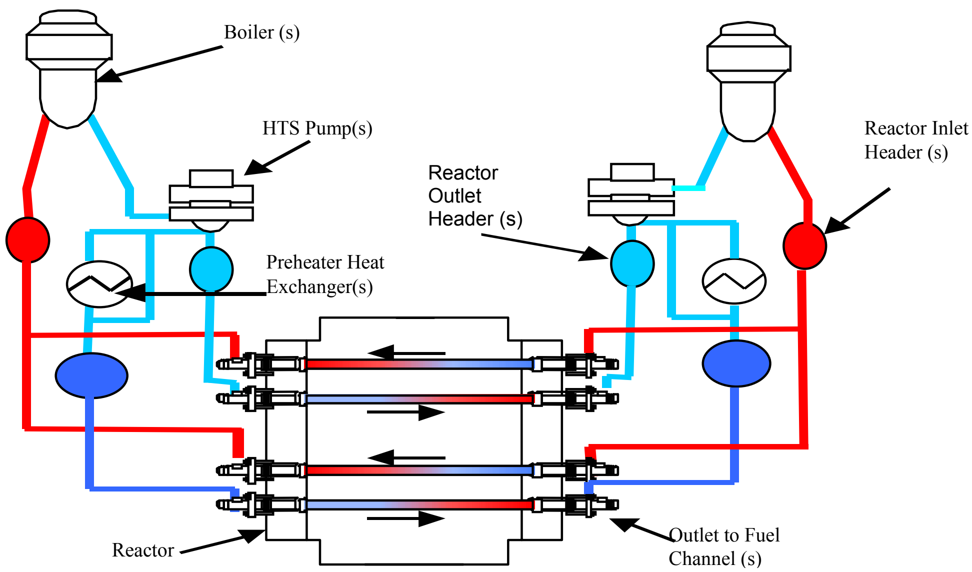

Figure [F-1] – Simplified Bruce B NGS design: Heat Transport System (HTS)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
A								OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ								A
B						OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ						B
C					OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ					C
D				OZ	OZ	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	OZ	OZ				D
E			OZ	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	OZ			E
F		OZ	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	OZ		F
G		OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ		G
H		OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ		H
J	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	J
K	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	K
L	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	L
M	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	M
N	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	N
O	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	O
P	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	P
Q	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	Q
R		OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ		R
S		OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ		S
T		OZ	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	OZ		T
U			OZ	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	OZ			U
V				OZ	OZ	OZ	OZ	OZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	IZ	OZ	OZ	OZ	OZ	OZ				V
W					OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ					W
X						OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ						X
Y								OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ	OZ								Y
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	

Figure [F-2] – The reactor core face which identifies the location of Inner Zone (IZ) and Outer Zone (OZ) channels. Note that the Inner and Outer Zone fuel channels have separate process loops (i.e., there is an IZ and OZ Reactor Inlet Header and an IZ and OZ Reactor Outlet Header)

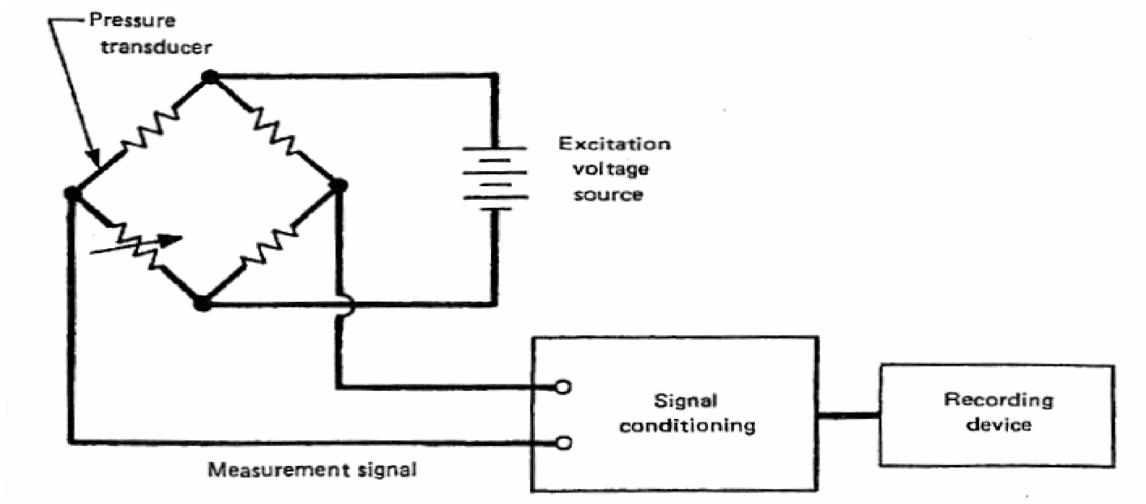

Figure [F-3] – An instrument loop for a pressure data acquisition system. The elemental random errors for the instrument loop include uncertainties associated with the calibrating instrument, power supply, signal conditioning process, and environmental effects, etc. (Figure taken from Reference [7]).

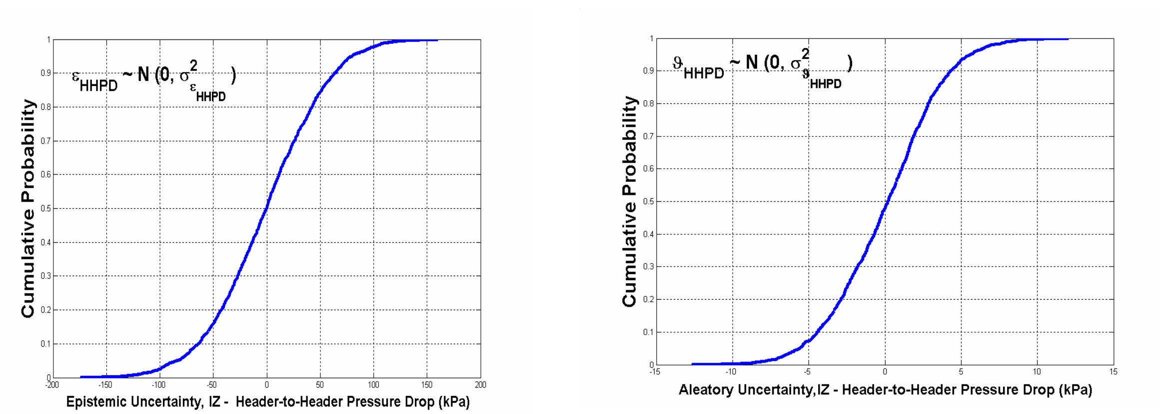

Figure [F-4] – The cumulative distribution functions used in characterizing the epistemic (RIGHT) and aleatory (LEFT) uncertainties in the reactor header-header pressure drop under aged conditions.

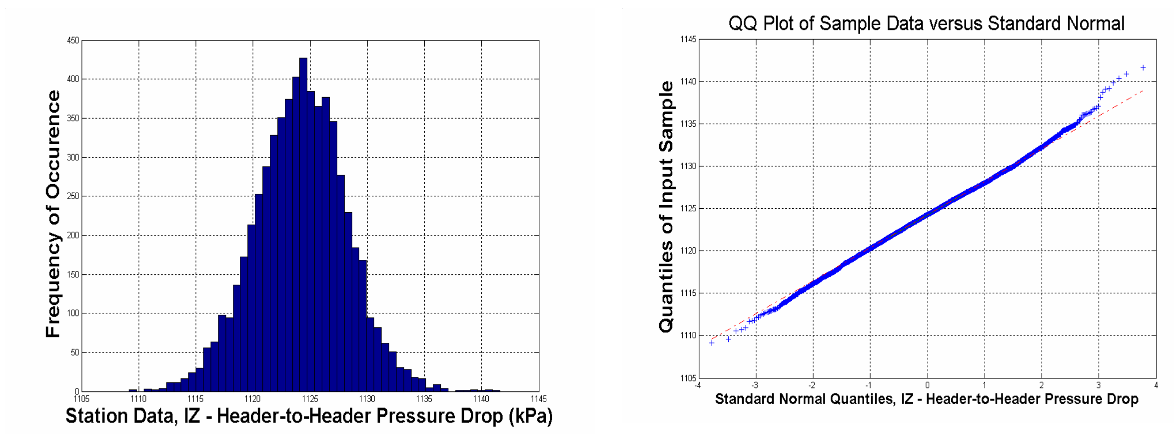

Figure [F-5] – Tests for normalcy using operational data using a histogram (LEFT) and a QQ-plot (RIGHT).

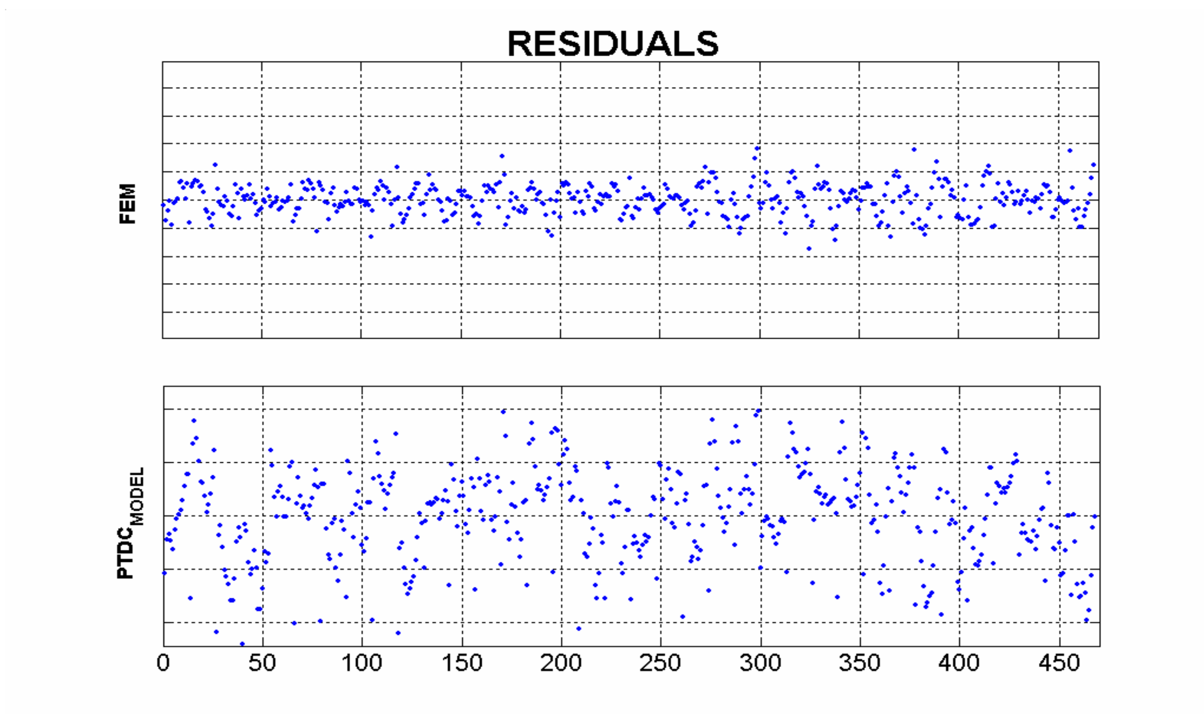

Figure [F-6] – A comparison of the residuals for the Fixed-effects Model (TOP) and the pressure tube diametral creep model (BOTTOM) (Reference [4]).

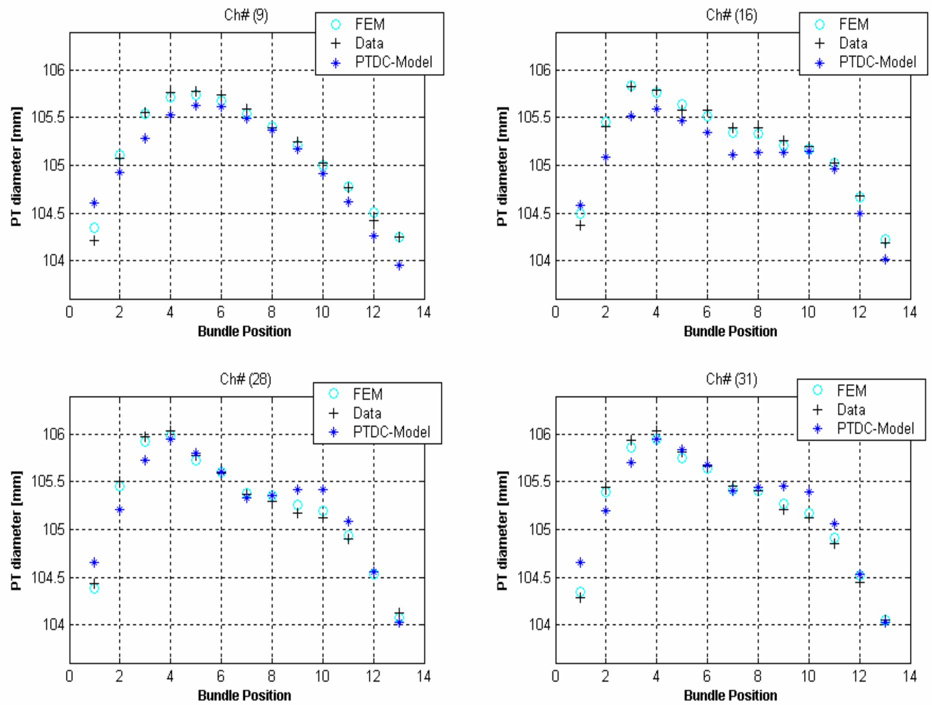

Figure [F-7] – A comparison of the Pressure Tube Diametral Creep (PTDC) correlation with measured data and the Fixed-Effects Model (FeM) (Reference [4]).

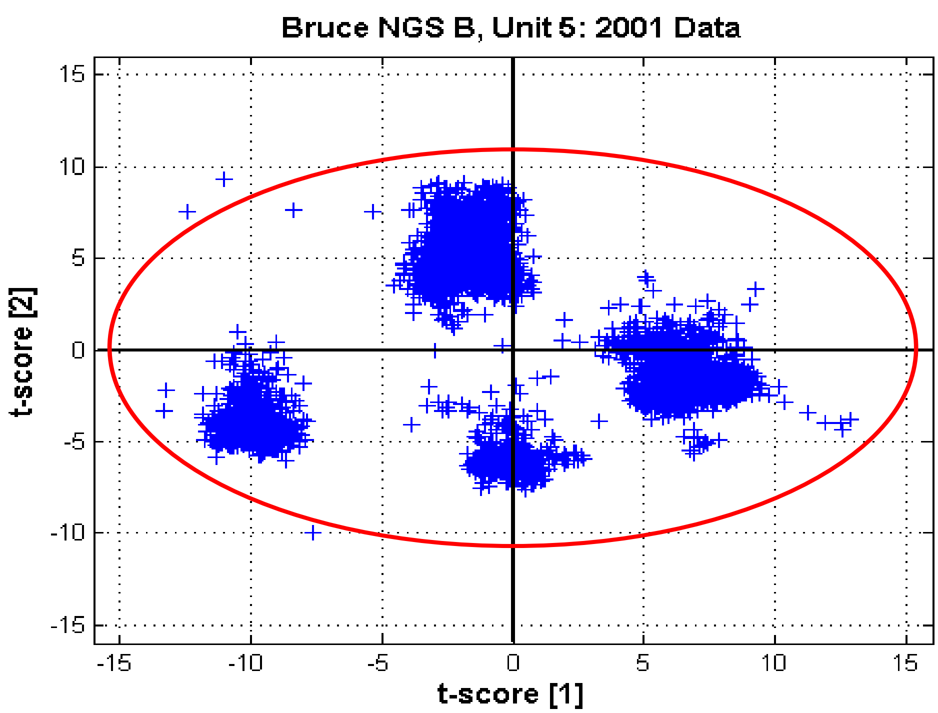

Figure [F-8] – Defining operationally-steady state data using Principal Component Analysis.

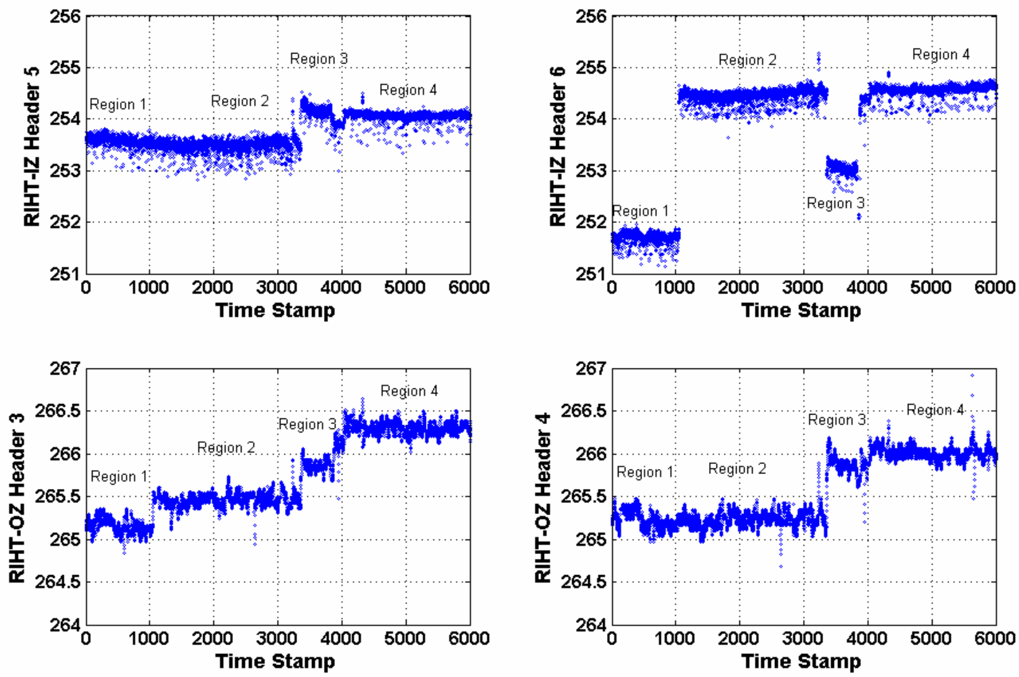

Figure [F-9] – The time-series for the reactor inlet header temperatures for the inner and outer flow zone.

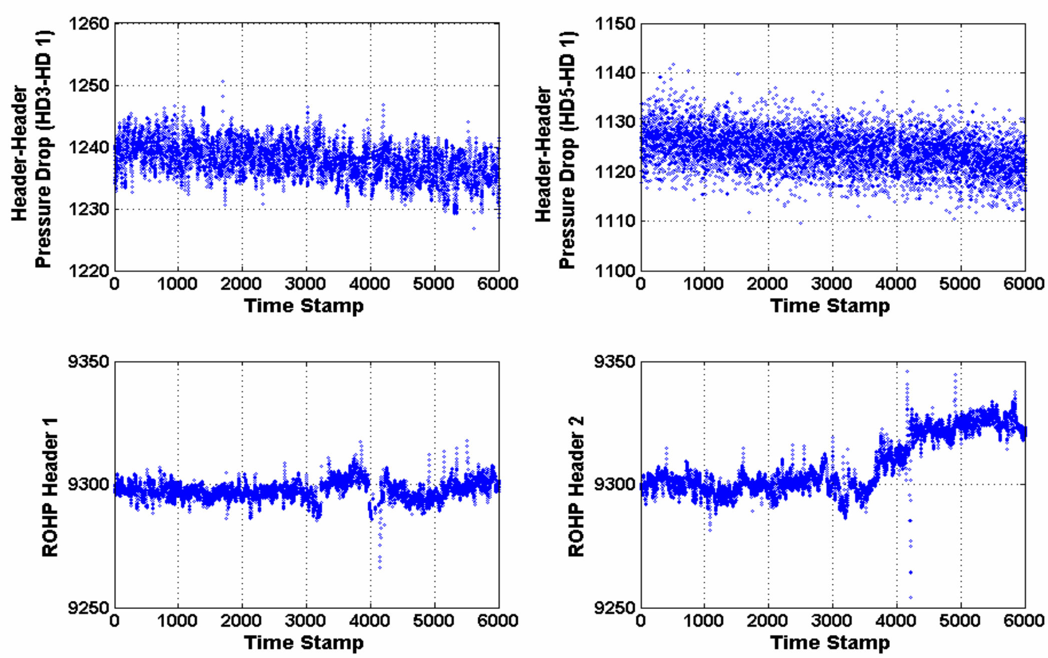

Figure [F-10] – The time-series for the reactor header-to-header pressure drop and reactor outlet header pressure.

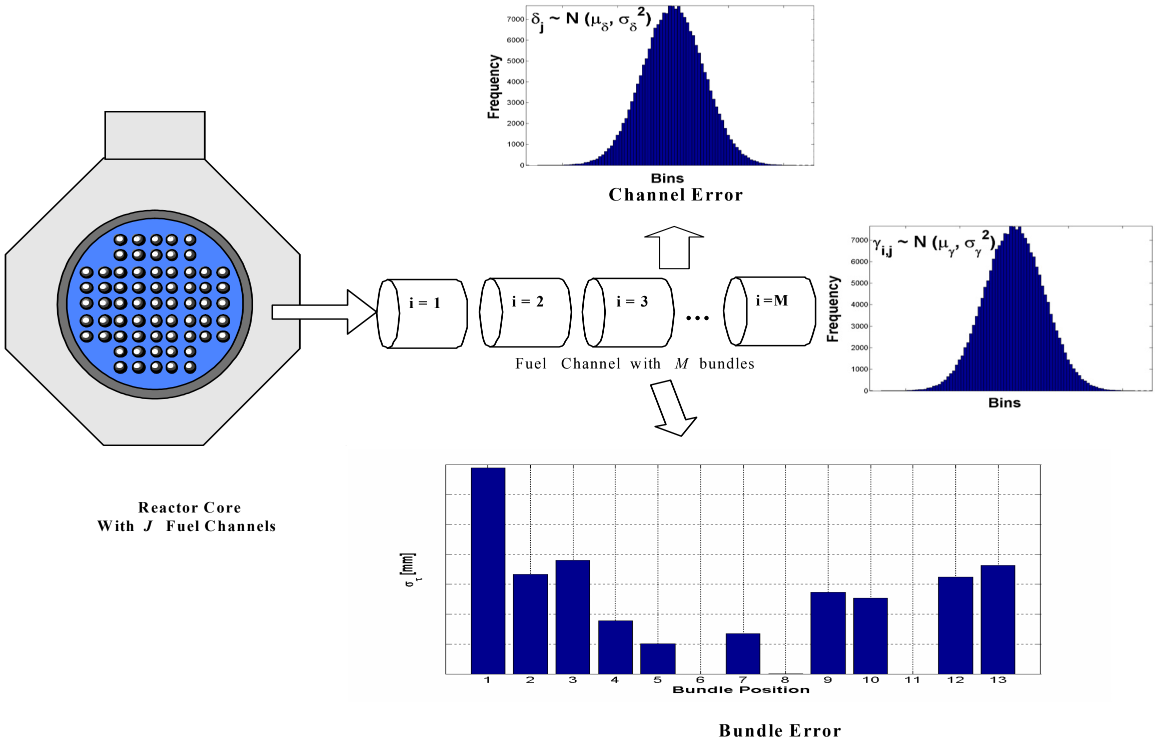

Figure [F-11] – The statistical error model for the PTDC correlation. The overall residual in the predictive model is decomposed into the following form: $e_{ij} = \tau_i + \delta_j + \gamma_{ij}$ *. Note that there is a frequency distribution for each bundle location for the bundle error,* τ_i *which is characterized my a normal distribution with mean,* μ_τ and *standard deviation,* σ_τ.

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