UTILIZATION OF SIMULATED ANNEALING ALGORITHM IN THE ROP DETECTOR LAYOUT OPTIMISATION

Doddy Kastanya[a] and Victor Caxaj[a]
[a]Atomic Energy of Canada Limited, Mississauga, Ontario, Canada

Abstract

The Regional Overpower Protection (ROP) systems protect CANDU®[1] reactors against overpower in the fuel that could reduce the safety margin to dryout. The overpower could originate from a localized power peaking within the core or a general increase in the core power level. In the CANDU-6 design, there are two ROP systems in the core, one for each of the two fast-acting shutdown systems. Each ROP system consists of a number of fast-responding self-powered flux detectors suitably distributed throughout the core within vertical and horizontal assemblies. Traditionally, the placement of these detectors was done using a method similar to the detector layout optimization (DLO) module of the ROVER-F code. A new approach is introduced in this paper to optimize the placement of detectors using the simulated annealing stochastic optimization algorithm. Advantages and disadvantages of both methods are examined. Finally, numerical examples are provided to illustrate the capability of the new approach.

1. Introduction

The Regional Overpower Protection (ROP) systems in the Canada Deuterium Uranium (CANDU) reactor protect the reactor against overpower in the fuel. The overpower could originate from a localized power peaking within the core (for example, due to a certain reactivity device configuration) or a general increase in the core power level. This overpower could lead to dryout[2].

In the CANDU-6 design, there are two ROP systems in the core. Each system is connected to a fast-acting shutdown system. The first system is connected to shutdown system 1 (SDS1), which is a rod-based shutdown system, and the second system is connected to shutdown system 2 (SDS2), which is a liquid-poison-based shutdown system. Each system consists of a number of fast-responding, self-powered flux detectors which are appropriately distributed throughout the core. For the CANDU-6 design, the placement of these detectors was done using a method similar to the detector layout optimisation (DLO) module of the ROVER-F code.

[1] CANDU is a registered trade-mark of Atomic Energy of Canada Limited.
[2] Dryout is a condition where the fuel is operating at temperatures higher than desired temperature. The coolant around the fuel sheath surface produces many small bubbles that could eventually coalesce into a vapour film enveloping the fuel element. This reduces the heat transfer from the fuel to the coolant and in turn further elevates the fuel temperature.

The objective of this paper is to introduce a new approach in optimising the placement of the detectors in the core. The optimisation is performed using the simulated annealing stochastic optimisation algorithm. It is done within the deterministic design of the ROP systems.
The paper is organized as follows. In Section 2, the general deterministic design process of the ROP systems is presented. Both the traditional and the new optimisation approaches are discussed. Some numerical examples for both methods will be given in Section 3. Section 4 discusses the ongoing development of this optimisation tool. Finally, some conclusions are provided in the last section.

2. Deterministic Design of the ROP Systems

The deterministic design process can be broken down into three main stages: margin optimization, logical (Boolean) reduction of potential protecting detectors, and detector selection (including placing detector into independent channels).

2.1. Objectives

There are two objectives that should be satisfied during the first stage of the deterministic design process. The first one is the safety objective. To satisfy this objective, the design must meet the requirement that there is a minimum number of detectors (usually tied to the voting logic of the safety channels) for which the required trip setpoint is higher than the base trip setpoint. The second one is the economic objective. The choice of detectors to deterministically cover the design-basis flux shapes is made to minimize unnecessary lowering of reactor power (beyond the power level at which a particular case is simulated) in any of economically significant flux shapes. These flux shapes are called the economic cases. Examples of economic cases are startup or xenon override, adjuster shim, load following, stepback, setback, and full power operation at various average zone-controller level.

2.2. Envelope of Detector Reading

The starting point for the first stage is to bring the reactor power of all economic flux shapes to a value that maintains limits, other than ROP, from:

- Bundle-Power/Channel-Power Compliance
- Limiting Critical Power Ratio – pre-ROP conditions.

It is assumed that the set of reactor power limits above are hard, *i.e.* not based on bundle power or channel power at dryout considerations, and/or cannot be addressed further within ROP.

The reactor power and maximum detector signals for the economic cases is used to calculate the associated desired or optimal trip setpoint (TSP_0) for each detector from the point of view of economic trip margin. It is defined as follows:

$$TSP_{0,i} = M \times \max_{v}\left\{RP_v \times \Phi_{v,i}\right\} \quad (1)$$

where

M is the specified fixed “minimum margin-to-trip” for the economic flux shape set (this is a designer-defined target)

RP_v is the reactor power for the economic case v, and

$\Phi_{v,i}$ is the normalized detector reading for detector i for economic case v.

2.3. Margin-to-Trip and Margin-to-Dryout

ROP deals with two margins for all flux shapes:

a. The margin-to-trip of the protecting detector (for CANDU-6 design, it is the first detector to trip in the third safety channel of the second shutdown system to be actuated), and
b. The margin-to-dryout of the most limiting fuel channel.

The safety margin is defined as the margin-to-dryout divided by the margin-to-trip. The safety margin is the relative power of dryout above trip.

2.4. ROP Coverage Equation

The deterministic ROP trip setpoint or the ROP coverage equation can be written as follows:

$$TSP_i = CPPF \times \Phi_{k,i} \times \frac{CCP_{k,j}}{RipEnv_j \times CP_{k,j}} \times \frac{1}{1+EA} \tag{2}$$

where:

TSP_i	Trip setpoint for detector i
$CPPF$	Channel Power Peaking Factor
$\Phi_{k,i}$	Normalized detector reading for detector i for case k
$RipEnv_j$	Ripple envelope (maximum ripple over all ripple cases) for channel j
EA	Error Allowance
$CCP_{k,j}$	Critical channel power for channel j of case k
$CP_{k,j}$	Channel power for channel j of case k

2.5. Uncertainty Values and Error Allowance

The error allowance (as in Eq. (2)) used in deterministic design is related to the uncertainty values used in the probabilistic assessment by the following formula:

$$EA = 2 \times \sqrt{\sigma_{CH}^2 + \sigma_{CR}^2} - \varepsilon_{bias} \tag{3}$$

where:

σ_{CH} is the channel-random uncertainty,

σ_{CR} is the common-random uncertainty, and

ε_{bias} is the systematic bias.

2.6. Boolean Reduction

Starting with the matrix of design-basis cases by detector number and applying the ROP coverage equation on each case, one can build a Boolean matrix (true or false) indexed by case (*i.e.*, flux shape) and detector number. A true value at location (k,i) indicates that case k is covered by detector i.

This matrix is then reduced in two steps:

a. Any case that is covered by a set of detectors, for which there exists a subset of detectors that are tripping detectors of a second case, is considered redundant and that case is removed. This step is taken because any trip that would occur for the case using the smaller number of detectors would also occur for the case using the larger number of detectors.
b. Any detector that trips for a number of cases that are a subset of the cases that trip for a different detector are considered redundant and that detector is removed. The justification for this step is that the detector covering the larger number of cases makes the other detector redundant. The exception to this rule occurs when the removal of the redundant detector would reduce the number of detectors tripping for any case below the lower limit (*i.e.*, the number of safety channels in the system).

2.7. Detector Selection and Channelization

This is where the new methodology is introduced. In order to make a representative comparison between the two approaches, each methodology is explained in more detail in the following sub-sections. In the following discussion, it is assumed that the ROP system consists of three safety-channels.

2.7.1. <u>Detector Layout Optimization (DLO)</u>

The following are steps for the traditional approach of detector layout optimization:

a. Set a target number of detectors in a single safety channel, M.
b. Look for as many single channel solutions, each with M detectors.
c. For a safety system consisting of N single channels, look for N independent single channel solutions (*i.e.*, N single-channel solutions that do not share any common detectors):
    i. If there is only one solution, then this is the global minimum solution. The deterministic design process is complete.
    ii. If there are more than one system of N single-channel solutions, choose one system as the solution and archive the other ones as backup solutions.
    iii. If there are only $N-\alpha$ independent solutions available, then these solutions could be combined with α independent single-channel solutions, each consisting of $M+1$ detectors. Increase M by 1, then go to step "b".
    iv. If there is no independent single-channel solution; increase M by 1, then proceed to step "b".

2.7.2. Detector Placement using Simulated Annealing (DETPLASA)

There are two important parts to the implementation of the new methodology for determining the detector layout for the ROP system. The first one is related to how detectors are selected and channelized. Given a certain sequence of cases to be considered (note that the initial sequence of cases comes from the Boolean reduction), a certain algorithm is used to select the detectors and place them in appropriate channels. This is discussed in Section 2.7.2.1. The second one is related to how the number of detectors can be minimized using the Simulated Annealing (SA) stochastic optimization algorithm. The detailed algorithm is discussed in Section 2.7.2.2.

2.7.2.1. Algorithm for Detector Selection and Channelization

Given a certain sequence of cases, the following are steps for selecting and channelizing detectors:

a. Set the "channel flag" for all channels to NO.
b. For each case (starting from the first case in the sequence), check from left to right (*i.e.*, first to last) whether a detector already covers other case (*i.e.*, already selected and channelized from evaluating previous cases). If so, set the "channel flag" corresponding to the channel where the detector is located to YES.
c. Fill the remaining safety-channel(s) with available detectors (from left to right). For each detector added to a channel, two steps are performed: (i) set the "channel flag" of that channel to YES; and, (ii) update the list of detectors in that channel.
d. When all channel flags are already set to YES for this particular case, then we are ready to proceed to the next case. If this is the last case, then the total number of detectors will be passed on to the main Simulated Annealing algorithm.
e. If no more detectors are available for this case and there is still some "channel flags" with NO value; then this particular sequence of case fails. The algorithm to deal with a failed sequence of cases is further discussed in the next sub-section.

2.7.2.2. Simulated Annealing Algorithm

There are various optimization problems that become unmanageable using combinatorial methods as the number of parameters involved increases. One of the classic examples of this optimization problem is the traveling salesman problem. One of the techniques used to resolve this type of problem is the simulated annealing technique [1]. It is a stochastic optimization algorithm which was developed using an analogy with the thermodynamics process, specifically with the way that liquids freeze and crystallize or metals cool and anneal (hence the name "simulated annealing").

In the current implementation of the SA algorithm for detector layout optimization, the objective of the optimization process is to minimize the number of detectors in the system. Therefore, the objective function, F_{obj}, is the number of detectors in a configuration. The basic implementation of SA algorithm in DETPLASA can be described as follows:

a. The first step is to initialize the "temperature" parameter to a high value (an analogy to the physical annealing process), initialize the counter for the number of histories (called

ITER) for this particular temperature to 0, initialize the counter for the number of temperatures (called TCOUNT) to 1, and find an initial solution[3] (and subsequently save it as the best solution, *i.e.* $F_{obj}^{\min}$).

b. Load[4] the best solution as the current solution and increase ITER by one.
c. Check if ITER is greater than $ITER_{max}$ (the maximum number of histories allowed for a particular temperature). If it is true, then we need to reduce the temperature (assuming that TCOUNT is less than $TCOUNT_{max}$ – the maximum number of temperature reductions) using an appropriate cooling schedule, initialize ITER to 1, and go to step "d". If it is not true, then we can proceed to step "d".
d. Perform a permutation on the sequence of cases. This is done by interchanging two rows in the current sequence of cases.
e. Find a new solution by performing a detector selection and channelization process.
f. Check if the number of detectors in the new solution is lower than the number of detectors in the current solution. If so, then check whether it is lower than the best solution. If it is lower than the best solution, we need to save[5] the new solution as both the current solution and the best solution (and also update the $F_{obj}^{\min}$ value); otherwise, we only save the new solution as the current solution. If the number of detectors is higher than the current solution, then it can still be accepted (*i.e.*, saved as the current solution for the next iteration) if it satisfies the following criterion:

$$e^{-\frac{\Delta F_{obj}}{T}} \geq R(0,1) \tag{4}$$

where $R(0,1)$ is a random number between 0 and 1 (and is generated every time this evaluation is needed), ΔF_{obj} represents the increase in the number of detectors, and T is the current "temperature".

2.7.3. Advantages and Disadvantages of the Two Methods

Table 1 summarizes the strength and weakness of the two methods. Basically for a small problem (*i.e.,* limited number of detectors and cases), both methods will perform equally well and will be able to find the optimum solution (*i.e.,* the global minimum for the number of detectors). For designing a large systems where there exists thousands of potential detectors and hundreds of cases to be considered, the DETPLASA method is superior.

Table 1 Comparison of the Two Methods

[3] The initial solution is found by performing a detector selection and channelization process on the initial sequence of cases from the Boolean reduction stage. Alternatively, we can perform certain numbers of permutations on the initial sequence of cases prior to performing detector selection and channelization.

[4] Loading the best solution means assigning the lowest number of detectors as the minimum objective function and setting the sequence of cases producing the best solution as the current sequence of cases.

[5] Saving a solution means keeping both the information on the number of detectors in the solution and the sequence of cases that produces this solution.

DLO	DETPLASA
It is likely to find the global minimum, particularly for small problems.	It is not easy to find a global minimum, particularly for large problems.
It is time consuming to find single-channel solutions.	No single-channel solution is needed.
It is time consuming to find a system consisting of N independent single-channel solutions.	Channelization is done at the same time as detector selection.
A family of global minima is available.	For general case, it can only find a family of solutions close to the quasi-optimum solution (which is likely a local minimum).

2.8. Overall Process

The detector layout resulting from the detector selection and channelization step is the final product of the deterministic design process. The trip setpoint corresponding to this detector layout is determined from the probabilistic assessment. If the trip setpoint from this assessment does not provide enough margin for operation at the desired reactor power level, then the design process should be repeated with a higher minimum number of detectors in mind.

3. Numerical Examples

To illustrate the algorithm described in the previous sections, a simple example is provided in the first part of this section to examine more closely the steps in the detector layout optimisation process using both the traditional and the new approaches.

3.1. A Simple Example

The deterministic design process is identical for both approaches up to the end of Boolean reduction. For this exercise, the output from Boolean reduction is presented in Table 2.

Table 2 List of Protecting Detectors

Case	Protecting Detectors							
1	233	240	243					
2	44	13	81	53				
3	160	223	233	60	131			
4	258	75	13	203	53	63		
5	258	13	81	203	53	240	90	
6	196	112	185	35	194	203	200	207

3.1.1 DLO Approach

First we set *M* (the number of detectors in a single safety channel) to 1. Obviously, there is no solution in this case. A similar result is obtained when *M* is 2. So, we increase *M* to 3. This time, we can find a number of single-channel solutions, each having 3 detectors. Some of these solutions are shown in Table 3.

Table 3 Single-Channel Solution for *M* equals 3

203-233-44	13-233-196	13-233-194	53-233-112	53-233-200
203-233-13	13-233-112	13-233-200	53-233-185	53-233-207
203-233-81	13-233-185	13-233-207	53-233-35	
203-233-53	13-233-35	53-233-196	53-233-194	

Unfortunately, from this set of solutions, there are no three independent single-channel solutions found for *M* equals 3. Therefore, *M* is increased to 4. There are significantly more single-channel solutions (each with 4 detectors) available and there are more than one solutions for a system of three independent single channel solutions, for example:

Channel 1	Channel 2	Channel 3
233	240	243
44	13	53
258	223	131
196	203	194

However, since we have some single-channel solutions from *M* equals 3, the code will first use these solutions in conjunction with the solutions from *M* equals 4. One of possible systems is shown below:

Channel 1	Channel 2	Channel 3
233	240	243
53	13	44
196	223	131
	203	194

3.1.2 DETPLASA Approach

It should be noted that the overall SA algorithm cannot be described here. Instead, the detector selection and channelization process for a particular sequence of cases within the SA algorithm will be examined.

As an example, it is assumed that the current sequence of cases is 1-5-3-2-6-4. Table 2 is rearranged reflecting assumption (results are presented in Table 3) and then the detector selection and channelization process is performed.

Table 3 List of Protecting Detectors after a Series of Permutations on Sequence of Cases

Case	Protecting Detectors							
1	233	240	243					
5	258	13	81	203	53	240	90	
3	160	223	233	60	131			
2	44	13	81	53				
6	196	112	185	35	194	203	200	207
4	258	75	13	203	53	63		

The algorithm will proceed from the first case to the last case applying the rules for selecting a detector and placing it in a channel along the way. The system is built as the algorithm proceeds from one case to the next. The progress is monitored in a step-by-step fashion as demonstrated below.

For the first case, the first detector will go to channel 1, the second detector goes to channel 2, and the third detector goes to channel 3.

Channel 1	Channel 2	Channel 3
233	240	243

For the second case, sweeping through the protecting detector list from left to right, one finds that detector 240 is in the list. Therefore, channel 2 already covers this case. Channel 1 and channel 3 are filled with the remaining detectors from left to right.

Channel 1	Channel 2	Channel 3
233	240	243
258		13

For the third case, sweeping through the protecting detector list from left to right, one finds that detector 233 is in the list. Therefore, channel 1 already covers the case. Channel 2 and channel 3 are filled with the remaining detectors from left to right.

Channel 1	Channel 2	Channel 3
233	240	243
258		13
	160	223

For the fourth case, detectors 44 and 81 will be selected and placed in channel 1 and channel 2, respectively, since channel 3 is already covered (by detector 13).

Continuing with this process for the remaining cases, a three-channel system with a total of 13 detectors can be built.

Channel 1	Channel 2	Channel 3
233	240	243
258	160	13
44	81	223
196	112	185
	75	

3.2 Optimization of a Large Problem using DETPLASA

In order to demonstrate the performance of the new methodology, a matrix of protecting detectors which consists of 97 cases and the maximum number of detectors for a case is 194 has been created. Assuming that one sequence of cases will produce a detector layout, we can estimate the maximum number of possible configurations which is 97! or around 10^{150}. Therefore, since it is impossible to evaluate all combinations, the use of a stochastic optimisation technique such as simulated annealing is appropriate.

The DETPLASA approach is used to arrive at a quasi-optimum solution for this problem. The result of the search is presented in Figure 1. There are several important points that should be highlighted from this figure (as labelled appropriately in the plot).

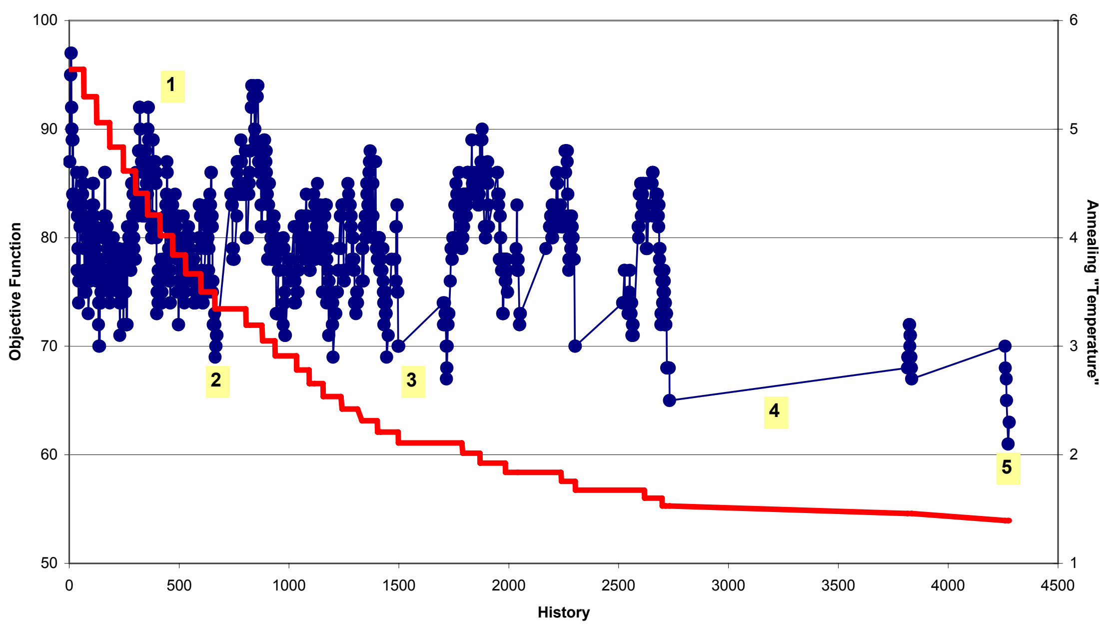

Figure 1 Results from DETPLASA

First of all, the figure is showing two plots. The first one (in blue) shows the objective function (on the primary y-axis) as a function of accepted history (x-axis). Note that each history represents a certain sequence of cases. The second plot (in red) reflects the annealing temperature (on the secondary y-axis) also as a function of accepted history.

In the beginning of the process (illustrated by point 1 in the figure), the temperature is still high, therefore a big variation in the objective function can be observed. At this stage, a relatively large increase in the objective function of two consecutive histories is still acceptable. Point 2 is one example of this increase where an increase of 15 detectors (from 69 detectors to 84 detectors) between consecutive accepted histories is acceptable. Point 2 also illustrates the ability of the algorithm to escape from local minima (at 69 detectors). As the temperature is decreased, the variation in the objective function decreases too. Points 3 and 4 show that as the temperature continues to decrease, the allowable increase between consecutive accepted histories goes down to 5 and 3, respectively. The algorithm finally finds a quasi-optimum solution of 61 detectors, shown as Point 5 in the figure.

4 Conclusion

A new approach for optimisation of the detector layout for the ROP system of a CANDU reactor is introduced in this paper. For a small system where both the number of available potential detectors and the number of cases to be evaluated are limited, both the traditional approach and the new approach will be able to find a global minimum, which is the best solution for the problem.

As the number of cases and the number of potential detectors increase significantly, the use of the traditional methodology becomes computationally prohibitive. It has been shown in this paper that, for a large problem, the simulated annealing approach could produce an acceptable result. This solution might not be the global minimum number of detectors for this problem. However, this result should be considered a success since the algorithm is able to find a solution from a pool of around 2500 detectors at the beginning of the process.

5. Further Development

Results presented in this paper are related to a deterministic design process for designing a detector layout for the ROP system. The quality of the system must be evaluated using a probabilistic assessment where a trip setpoint for the system is calculated. Currently, this assessment is performed after the optimisation process has arrived at a solution. Incorporating the probabilistic assessment into the SA algorithm (by modifying the objective function) is one of the improvements that can be done.

In the near future, there are several other investigations that will be performed to improve the new methodology:

1. The execution time will increase significantly[6] once the probabilistic assessment is incorporated into the algorithm. Therefore, the performance of other stochastic optimisation algorithms such as genetic algorithm (GA), tabu search (TS), and ant

[6] Currently, for the example shown in section 3.2, the execution time is less than 5 minutes on a 2 GHz PC. Depending on the size of the problem (*i.e.*, the number of detectors in the system and the number of cases to be evaluated), a typical probabilistic assessment could run between 2 to 4 minutes. This means that evaluations of around 4000 histories will be completed in around one week.

colony system (ACS) should be evaluated to ensure that the "fastest" algorithm is utilized in the design process.

2. Another option that can be explored to reduce the overall execution time is to utilize parallel processing. To explore this option, the parallel simulated annealing algorithm will be implemented and its performance will be examined.

6. References

[1] Kirkpatrik, S., Gelatt, C., and Vecchi, M., "Optimization by Simulated Annealing," *Science*, **220**, 671-680, 1983.