NEW LAWS OF PRACTICE FOR LEARNING AND ERROR CORRECTION

Romney B. Duffey
Atomic Energy of Canada Limited
E-mail: duffeyr@aecl.ca

Abstract
Relevant to design, operation and safety is the determination of risk and error rates. We provide the detailed comparison of our new learning and statistical theories for system outcome data with the traditional analysis of the learning curves obtained from tests with individual human subjects. The results provide a consistent predictive basis for the learning trends emerging all the way from timescales of many years in large technological system outcomes to actions that occur in about a tenth of a second for individual human decisions. Hence, we demonstrate both the common influence of the human element and the importance of statistical reasoning and analysis.

Individual Learning and Practice

We wish to predict the error rate and learning response relevant to the design, operation and safety of nuclear reactors. We provide a consistent predictive basis for the learning trends emerging all the way from timescales of many years in large technological system outcomes to actions that occur in about a tenth of a second for individual human decisions. Hence, we demonstrate both the common influence of the human element and the importance of statistical reasoning and analysis.

We examine here comparisons with the data and empirical analysis that underpins the use of empirical learning curve correlations ("laws of practice") to describe individual skill and learning. These correlations of thousands of experimental data determine separately and independently (a) the error rate for repetitive tasks, and (b) the instantaneous response time reductions for recognizing patterns and recall for successive trials. Interestingly, the dependence of the learning trend on the number of trials, t, is different for each type. The fundamental technical questions are then not only why the learning curves have the dependencies they have, but also why they are different.

We assume that these differing responses represent just two distinct types of learning behaviour. For error reductions that depend on prior learning of repetitive skills, we postulate that the error correction *rate* (systematic learning and unlearning) is due to the learning hypothesis, and hence the error or success rate varies with the *accumulated* experience or practice. But for the different response time reductions that depend on an instantaneous judgement or learned response, we postulate that the error correction *probability* (faster solution and the most likely) is due to discerning order and patterns, and hence the error or success rate varies with the *depth* of experience or practice.

With the present work, the empirical but highly successful correlations that have been developed in psychology and the cognitive sciences are therefore placed on a firmer theoretical and practical basis. The instantaneous decisions from the resulting skill, knowledge and rule acquisition are reflected in the distribution of the outcomes we observe externally from the systems they inhabit, decreasing with practice and experience. This is consistent with Ohlsson's Theory of Error Correction [1], where rule correction and unlearning provide the mechanisms for error correction and skill acquisition at the individual level.

Therefore, we now compare our new and predictive general theory to these published correlations[1].

Comparison to Error Reduction Data

Consider the first case when the number of successes increases, or failures decrease, with increased repetitive learning. Our analysis for predicting the outcome rates from homo-technological systems is based on the Learning Hypothesis where the rate of reduction of the error rate with experience is proportional to the error rate. The result is an exponential form for the failure rate and for the Universal Learning Curve (ULC). The best representation of the world data is in Know the Risk [2]:

$$E^* = \exp{-3\,N^*} \qquad (1)$$

Here, $E^* = (1-\lambda/\lambda_m)/(1-\lambda_0/\lambda_m)$, the ratio of the failure rate, λ at any experience, ε, to the failure rate, λ_0, at the initial experience, and to the minimum failure rate, λ_m, achieved at the maximum experience, ε_T, and N^* is the non-dimensional experience, $\varepsilon/\varepsilon_T$. For correlating all the disparate data, the maximum experience can also be taken as either that at which the minimum outcome rate was attained, or as the most experience achieved with that system. The learning rate constant or slope factor, -3, is obtained from a fit to some 200 years of outcomes covering some 800 data points from multiple technological systems.

The observed result of skill testing on human subjects shows a non-linear relation exists between practice and the amount of error reduction, or conversely the increasing numbers of successes, N_S. This ubiquitous relation has been termed the “law of practice”. In fact, Stevens and Savin [3] examined many such learning experiments which examined the improvement in success with practice for stylized tasks like learning to run a maze, write upside down, toss balls at a target, typeset words, memorize word strings, etc., etc.

Stevens and Savin [3] fitted all these data sets with a series of *totally* empirical power laws that correlated the number of successful or correct responses, N_S, as a function of practice, t:

$$N_S = a\,t^m \qquad (2)$$

where the fitting constants are, a, and, m, with the number of trials, t, as the practice measure. Generally, success improves with practice (“practice makes nearly perfect”).

[1] The author thanks Professor Stellan Ohlsson for providing the referenced papers and data sources for this analysis.

For analysis, we chose several task types that were tested for these human learning activities. The experimental tasks, practice range and the published fitted values for, a, and, m, are shown in Table 1.

Table 1 Practice correlation values for data fits given by Stevens and Savin ($N_S = a\ t^m$)

Task	**Constant, a**	**Slope, m**	**Start, t_0**	**Stop, t_T**	**Units**
Syllables	5.7	1.56	1	20	# Trials
Ball tosses	0.0356	1.25	200	20,000	# Tosses
Typesetting	46.4	1.10	12.7	1,227	# Hours
Invert writing	25	1.18	1	70	# Minutes
Coding	16	1.18	1	20	# Minutes

Note the deliberately chosen large spread in task type, durations, numbers of tests, and apparent values of the correlating parameters. In order to compare these "success" data to the Learning Hypothesis we really need a failure rate, λ, being an effective *rate* of unsuccessful outcomes. Although the failure rate is not given explicitly in the Stevens and Savin paper, nor is the number of failures, N_F, we can determine the needed non-dimensional error rate, E*, by working backwards from the correlations for the number of successes, N_S. Given the fact that the rate of failures with increasing trials, t, is related straightforwardly and conversely to the rate of the number of successes, we have:

$$dN_F/dt = -\ dN_S/dt = -am\ t^{m-1} \qquad (3)$$

By definition, this differential rate of failure with increasing trials, dN_F/dt, is proportional to, λ, the usual failure rate, so also,

$$dN_F/dt\ \alpha\ \lambda\ (a,m,t). \qquad (4)$$

To obtain the differentials that give the failure rates, we assume the (a,m,t) correlations given by Stevins and Savin for, N_S, do indeed fit the data well to a very high degree of accuracy , as shown in the Figure 1 given in the original paper [3]. Since the actual data were plotted but not tabulated, we can take the fitted (a,m,t) power laws for, N_S, as reasonable substitutes for the original data points, which they are by definition. Noting that the minimum failure rate, λ_m, occurs at the maximum practice, the equivalent expression for the non-dimensional error rate ratio, E*, for any amount of practice, t, from the power law correlation becomes,

$$E^* = \frac{(1-\lambda/\lambda_m)}{(1-\lambda_0/\lambda_m)} \Rightarrow \frac{(1-\{t/t_T\}^{m-1})}{(1-\{t_0/t_T\}^{m-1})} \qquad (5)$$

Here, t_0, is the initial, and, t_{max}, the maximum amount of practice, $t^* = t/t_T$, is the non-dimensional practice for any number of trials, and the constant product, am, has cancelled out everywhere.

The needed values of t_0, t_{max}, and, m, are all reported for the empirical correlations (see the Table 1), so we can back calculate, E* for any given non-dimensional practice, t*. The result is shown in the Figure 1, where the points are actually typical values calculated from the Table 1 correlations for the entire ranges of practice given.

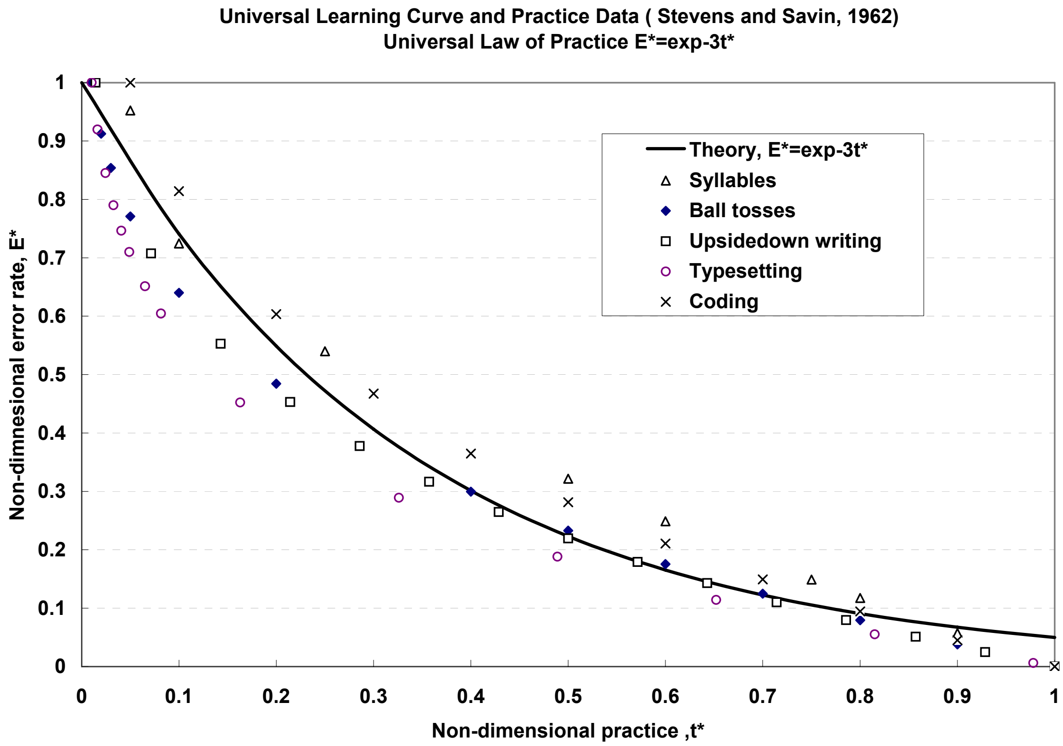

Figure 1: The Universal Law of Practice, E*=exp-3t*.

We observe an *extraordinary* fact. The individual practice curves align almost *exactly* with the totally independently derived Universal Learning Curve that fits all the world's homo-technological system outcome data based on the Learning Hypothesis for over 200 years of data [2]. The experience parameter in the ULC is simply transformed to the non-dimensional practice, t*, so the non-dimensional error *rate* ratio is,

$$E^* = \exp - 3\,t^* \qquad (6)$$

Formally, this result has established the equivalence between "accumulated experience" for a system and "practice" for an individual as the relevant and corresponding measures for learning rate trends. Because of its extraordinarily general basis, we may call this new curve the Universal Law of Practice (ULP). The major implication is then that the failure rate and bathtub-shaped error probability for the *individual* have exactly the same forms and dependencies on experience (practice) as derived for the *system* outcomes [2]. The individual is indeed an integral part of the system, so much so that *the system behaviour mirrors precisely the same learning trends.*

In addition this comparison shows a key point: the learning behavior at the system level is *exactly the same form* as that which occurs at the individual level. The Learning Hypothesis holds true and is independently validated.

Comparison to Response Time Data and the Consistent Law of Practice

Consider now the second case where the response time is an instantaneous measure of how quickly new things or tasks are learned or recognized. Our prediction of the variation of learning response time with experience is based on the statistical theory that shows the probability distribution of errors is exponential with depth of experience. Based on the statistical treatment of outcomes, the distribution with experience that emerges is also the most likely. From that distribution, we derived the Information Entropy as a measure of the degree of order and uncertainty (learning) and the emergence of learning patterns. We coupled this result with the known Hick-Hyman Law for the observed effects of Information Entropy on learning response time, RT [4] and [5].

Hence, we derive the information entropy from the statistical error state theory [2] and [6], and couple that with the Hick-Hyman expression [2] for the response time as a function of the information entropy or random stimulus. Thus, we now know that the response time varies explicitly with the experience depth, ε, in any interval as,

$$RT_j\ (\varepsilon) = a_j + b_j\ (p_0 + p_0^*\ e^{-\beta\varepsilon})^2\ (1/4 - ½ \ln \{p_0 + p_0^*\ e^{-\beta\varepsilon}\}) \qquad (7)$$

where the, a, and b, constants derive from the Hick-Hyman law. Physically *and hence mentally*, the terms in brackets represent the most probable *distribution* of outcome errors with depth of experience. The parameters that are observation interval dependent are the probabilities of error state occupancy, p_0 and p_0^*, and the learning constant, β, for the most probable (i.e., observed) distribution. By grouping the constant terms (a_j, b_j, p_0 etc.) together we may write this expression as:

$$RT_j\ (\varepsilon) = A_j + B_j\ e^{-\beta\varepsilon} + C_j\ e^{-2\beta\varepsilon} \qquad (8)$$

where A_j, B_j and C_j are constants and the logarithmic term is taken to be relatively small.

The observed result from tests using human subjects is also a non-linear relation between practice and the response time reduction. This relation has also been termed the "law of practice", and is ubiquitous. The major study by Heathcote et al. [7] examined 40 studies with over 7,900 learning series from 475 subjects of such learning experiments. These tests were for many different learning situations, measuring improvement in response time with practice for stylized tasks like memory search, counting, mental arithmetic, visual search, motor learning, etc., etc. They correlated the response times, RT, using a *totally* empirical exponential and/or power law function assumed to be of the form, using the same notation as above:

$$RT(t) = A + (B\ e^{-mt})/t^c \qquad (9)$$

where A , B, and c, are constants. The forms chosen were justified by heuristic reasoning about the type and nature of the presumed learning processes. The options are for an exponential choice, $c = 0$; for a power law choice, $m = 0$; or for a mixed variation. The values of the "constants" varied with the experiment and were fitted to each test series. Moreover, they showed that - at least for the RT data reviewed - an exponential (with $c = 0$) was generally a somewhat better fit than a power law (with $m = 0$), contrary to the previous wisdom and conventional choices. This exponential form is very similar to the MERE failure rate expression, but the basis and application is entirely different. Our result is based on the statistical analysis of outcomes, where the exponential probability distribution with experience naturally emerges from the uncertainty.

The comparable statistical entropy theory result for the RT can be derived by substituting numbers of trials or practice, t, as the τ units of the depth of experience/practice variable, ε. So we have the ABC Law of Practice:

$$RT(t) = A_j + B_j e^{-\beta t} + C_j e^{-2\beta t} \qquad (10)$$

By inspection, these two RT "laws of practice" (the empirical exponential fit equation (9) and the information entropy theory equation (10)) have extraordinarily similar forms. Sufficiently so that essentially identical fits to data can be obtained when we self evidently take $\beta \equiv m$, assuming that "depth of experience", ε, equivalent to "practice", t. To prove that fact directly preferably requires access to, and analysis of all the original data, which we do not have. But the excellent agreement can be shown conclusively in principle as follows.

There was one subset test for counting (Heathcote et al's Figure 4 Count 3 dataset) [7], with some 12,000 trials, where the numerical values for A and B were listed for an exponential correlation ($c = 0$). Again we have made the entirely reasonable assumption that the published correlation fits to, and is an acceptable substitute for the actual data points for the present purpose of establishing the comparison in the learning trends. As a direct numerical test, the present entropy theory was therefore fitted to Heathcote et al's RT data correlation curve adopting the same slope, m, and by normalizing the A, B and C values to the published values.

In Figure 2, for this specific counting case, against the non-dimensional practice, t^*, we plot the non-dimensional $RT/RT(0)$ response time ratios calculated both by the exponential correlation ($c = 0$ in equation (9)) and from the entropy theory (equation (10)).

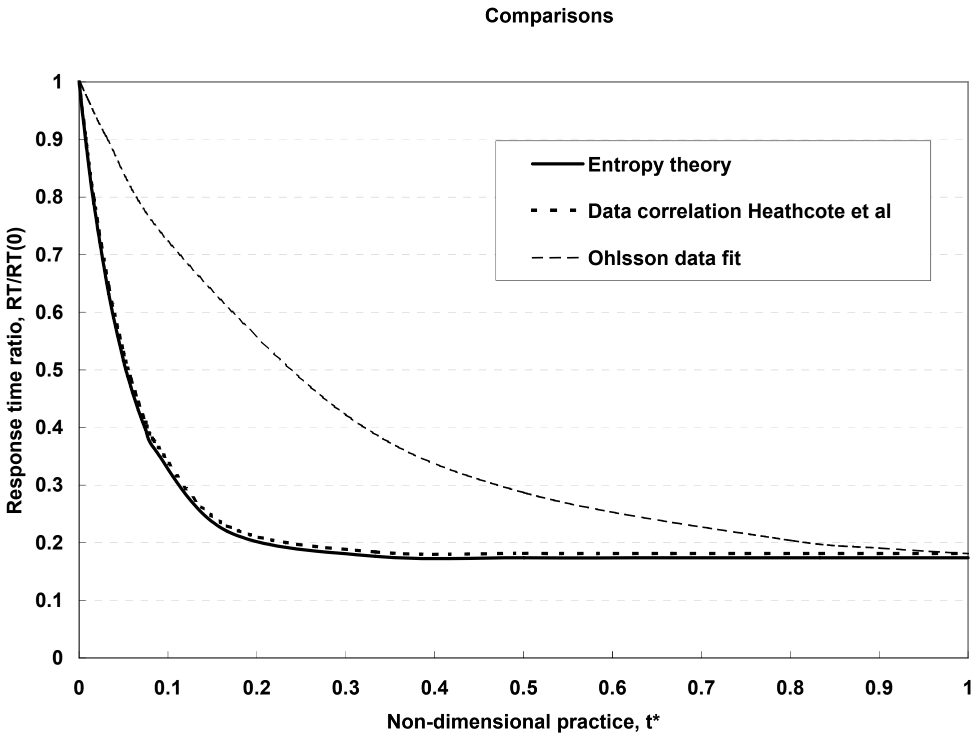

Figure 2: The Consistent Law of Practice: Comparison of entropy theory with RT data correlation.

The curves from the two expressions are clearly totally indistinguishable. Just to show other generally similar trends, for comparison we show the correlation of Ohlsson's RT data that was completely independently developed.

Recently, to account for differing data and models, Brown has proposed yet another more complex but still empirical form for the response time, RT [8]. This form is actually a ratio of exponential terms which, using the same notation as above, expanding the exponential term in the denominator, multiplying out the terms and regrouping the resulting constants, can be written as the infinite series,

$$RT(t) = A' + B' e^{-mt} + C' e^{-2mt} + \qquad (11)$$

This series in equation (11) is also *identical* in form to our approximate expression equation (10) for RT that was derived from the information entropy, using the Hick-Hyman RT law. The implication is that *the instantaneous decision-making and error correction observed for individuals is reflected in the outcome learning trends observed for the entire system* [9].

Therefore, in response to an anonymous reviewer's question, the human heterogeneity effects or "psychological factors" are naturally taken into account by the inherent randomness. The external emergence of the collective learning response of the entire system mirrors the unobserved myriad of individual decisions and interactions occurring within it. The observed learning patterns represent the emergence of structure, or of "order from disorder", with and according to the most likely distribution of errors. The theoretical models and resulting correlations therefore naturally include the uncertainty due to human decisions and actions. They are based precisely on the use and description of that very same uncertainty, as measured and quantified by the information entropy [9]. What we observe is the most likely learning behavior and laws, simply because that is what we observe.

Conclusions

In a sweeping generalization, we have shown a consistent relationship between human performance in individual learning with the outcomes, errors and accidents in entire technological systems. Moreover, the distinctly different dependencies on practice (experience) are explained for both repetitive learning and instantaneous response time experiments.

Utilizing the Learning Hypothesis and the statistical information entropy in the presence of learning, we provide a technical rationale for the form of the previously developed empirical correlations and "laws of practice" used in psychology for fitting error reduction and response time data.

The consistent and new Universal Law of Practice for error reduction, skill acquisition, and system outcome rates is proposed as:

$$E^* = \exp - 3\, t^* \qquad (12)$$

where, t*, is the non-dimensional practice, and is equivalent to the "accumulated experience", N*, for all homo-technological systems. This form and trend of reducing errors with practice is identical to the Universal Learning Curve for homo-technological systems outcomes reducing with experience, and established the importance of accumulated experience on learning and error correction.

For the response time, RT variation, we propose the new consistent Law of Practice, as given by the probability distribution from the statistical error state theory coupled with the Hick-Hyman law. For fitting to any RT data, it is clear that we may use the approximate simplified expression as a new correlating consistent ABC Law of Practice, without any loss of accuracy:

$$RT_j = A_j + B_j\, e^{-\beta t} + C_j\, e^{-2\beta t} \qquad (13)$$

where A_j, B_j and C_j are constants determined from comparison to the data, as usual. The practice, t, is equivalent to the "depth of experience" for homo-technological systems.

These new results are also consistent with the vast body of data reported for human subjects, and the empirical forms of practice correlations adopted to date. An additional and powerful reason for using this new approach is the entire theoretical concept and basis, that suggests and requires that order (learning curves) emerge from the disorder and random (neural) learning processes. This concept is also consistent with and derived from the established Hick-Hyman model, and there is an equivalence shown between "depth of experience" and "practice" in instantaneous cognitive decision-making by individuals.

Thus, we have successfully linked the mental learning processes with the observed physical error distributions of outcomes using the information entropy measure. These conclusions, analysis and results all clearly support the present learning hypothesis and statistical learning theory approach in any and all homo-technological systems, as does the validation shown against all the published data trends.

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