A Strategy for Determination of Test Intervals of k-out-of-n Multi-channel Systems

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Abstract

State space models for determination of the optimal test frequencies for k-out-of-n multi channel systems are developed in this paper. The analytic solutions for the optimal surveillance test frequencies are derived using the Markov process technique. The solutions show that an optimal test frequency which maximizes the target probability can be determined by decomposing the system states to 3 states based on the system configuration and success criteria. Examples of quantification of the state probabilities and the optimal test frequencies of a three-channel system and a four-channel system with different success criteria are presented. The strategy for finding the optimal test frequency developed in this paper can generally be applicable to any k-out-of-n multi-channel standby systems that involve complex testing schemes.

1. Introduction

Multi-channel voting logics are generally applied to the systems of nuclear power plants to increase reliability and testability. Such multi-channel design also reduces spurious operations of the systems which cause undesirable states of the plants. The special safety systems in Canadian Deuterium Uranium (CANDU) power plants are incorporated with the multi-channel voting logic design feature. The reactor protection systems (RPS) and the engineered safety feature actuation systems (ESFSAS) of pressurized water reactor (PWR) plants are also equipped with the multi-channel voting logics.

The special safety systems and the RPS/ESFAS are standby systems which are not supposed to activate during the normal plant operation. Therefore, failures cannot be revealed before they are called upon to function. Therefore, the surveillance tests during normal operation for insuring the proper functioning of the systems have to be carried out. Various fault tree models are introduced and used in the industry to quantify the unavailability and to confirm satisfaction of the unavailability target of the systems [1][2][3][4]. Fault tree techniques are most useful in the analysis of complex systems. The technique assumes instantaneous test and recovery, which ignores the effects of the test duration. This assumption leads to inaccuracies in quantifying the unavailability. The inaccuracy increases when the test frequency increases. Availability models and reliability models can be found in the literature which propose new techniques to overcome the drawback of the classical fault tree methods [5][6][7][8][9]. However, these models cannot directly be applicable to evaluate the test effect on reliability and availability of the system. To determine self-checking intervals for power transmission line protection systems, Markov process models are also introduced suggesting different test strategies [10][11][12]. A dynamic fault tree model of shutdown system number 1 (SDS1) was proposed to derive the test duration effect on the unavailability [13]. A Markov process model to overcome the inaccuracy of the fault tree model of SDS1 was also proposed in the previous research [14]. The Markov process model can quantify the test effect on the unavailability and the spurious trip probability which are essential information for determination of the surveillance test interval. The proposed dynamic fault tree model and the Markov model are only applicable to the SDS1 channel logics having 2-out-of-3 voting logics.

In this paper, a new strategy for determination of the surveillance test interval of k-out-of-n systems is proposed. The strategy using analytical model can generally be applicable to the reparable systems designed with identical multi-channel sub-systems. The strategy can be applicable to optimize the test frequencies of multi-channel systems in the operating nuclear power plants as well as to design the initial test frequencies of multi-channel systems in the newly designing CANDU plants. Examples for determination of the surveillance test intervals using the proposed method are also presented.

2. Optimal test frequency for reparable multi-channel systems

Using the Markov process and state space representation of the probability of a reparable system, a new strategy for finding the optimal test frequency is developed. The notations used in this paper are presented below and the state space model for developing the optimal test frequency follows.

Notations

S_i : Discrete variable for indicating state i

n : The total number of state

S : A Set of all the state spaces, $S = \{S_1, S_2, S_3, \cdots S_n\}$

S_D : A Set of the desired state space

S_T : A Set of the test state space
S_F : A Set of the failure state space
$P_R(A)$: The steady state probability of an event A occurring
P_D : The steady state probability of residing in the desirable states,
P_T : The steady state probability of residing in the test states
P_F : The steady state probability of residing in the failure states
μ_{TF} : State transition rate by the test
μ_{TD} : State transition rate by the recovery
λ_f : State transition rate by component failures

2.1 State classification (desired, failed, and test state)

Let the set $S = \{S_1, S_2, ..., S_N\}$ of all possible states of a *k*-out-of-*n* system be grouped into three sub-sets: S_D, S_F, and S_T, where S_D represents the desired state space that can be controlled by adjusting the test state space (S_T), and S_F represents the set of all the failure states of the system. Let P_D denote the steady state probability of residing in the desirable states (S_D), P_T denote the steady state probability of residing in the test state (S_T), and P_F denote the steady state probability of residing in the failure state (S_F). Then, the system is assumed to be in one of the subset state S_D, S_F, or S_T in any point of time so that $P_D + P_F + P_T = 1$. If the state transitions are assumed to follow memory-less, stationary characteristics, the state residing probability can be quantified using so called the Markov process analysis.

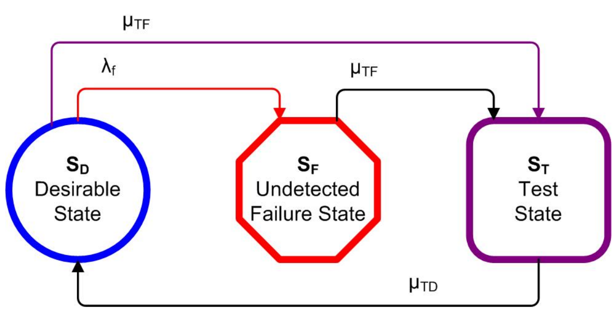

Figure 2-a: 3-States Diagram of a Repairable System

Let's consider the state space diagram representing states of a *k*-out-of-*n* system shown in Figure 2-a. The state transition rate by component failures, the state transition rate by the test, and the state transition rate by the recovery are shown as well. Here, the desired state space, S_D, depends on the success criteria of the system. For example, S_D can represent the system states that are in the moment at which no channel failure event has occurred or one channel failure event has occurred in a system of 2-out-of-3 success criteria. Let's assume that the state transition rate by the test, and the state transition rate by the recovery can be controlled, however, that of component failures is given and cannot be independently controlled even though one can replace the components with more reliable parts to reduce the component failure rate. Then, the optimal test frequency which achieves the maximum P_D can be determined by controlling the P_T of residing in the test state S_T.

2.2 Determination of the optimal test frequency

Let $\overline{V}$ denote the stochastic transition probability matrix of the three-state model in Figure 2-a. The steady state probability of the system can be described [15]

$$\left[P_D \quad P_F \quad P_T\right]\overline{V} = \left[P_D \quad P_F \quad P_T\right], \tag{2-1}$$

where

$$\overline{V} = \begin{bmatrix} 1-(\lambda_f+\mu_{TF}) & \lambda_f & \mu_{TF} \\ 0 & 1-\mu_{TF} & \mu_{TF} \\ \mu_{TD} & 0 & 1-\mu_{TD} \end{bmatrix}. \tag{2-2}$$

Expanding Eq. (2-1) with Eq. (2-2) and using the summation of the probability

$$P_D + P_F + P_T = 1 \tag{2-3}$$

The steady-state probability can be obtained as

$$P_D = \frac{\mu_{TF}\mu_{TD}}{(\mu_{TF} + \lambda_f)(\mu_{TD} + \mu_{TF})} \tag{2-4a}$$

$$P_F = \frac{\lambda_f \mu_{TD}}{(\mu_{TF} + \lambda_f)(\mu_{TD} + \mu_{TF})} \tag{2-4b}$$

$$P_T = \frac{\mu_{TF}(\mu_{TF} + \lambda_f)}{(\mu_{TF} + \lambda_f)(\mu_{TD} + \mu_{TF})}. \tag{2-4c}$$

As the objective of the surveillance test is to get maximum probability of residing in the desired state of the system, the optimal test rate can be decided to the value of μ_{TF} which brings the maximum probability of P_D. Differentiating P_D with respect to μ_{TF} using Eq. (2-4a) gives

$$\frac{dP_D}{d\mu_{TF}} = \frac{\mu_{TD}(\lambda_f \mu_{TD} - \mu_{TF}^2)}{(\mu_{TF} + \lambda_f)^2(\mu_{TD} + \mu_{TF})^2} \tag{2-5}$$

Equating Eq. (2-5) to zero gives the test frequency which results in *Max* (P_D) such that

$$\mu_{opt} = \sqrt{\lambda_f \mu_{TD}} \tag{2-6}$$

2.3 Profile of the steady state probability

The profiles of the steady state probability of residing in each state of a repairable system are generated using Eq. (2-4), Eq. (2-6) and shown in Figure 2-b, Figure 2-c, and Figure 2-d. The probabilities are quantified with various component failure rates and test frequencies. In Figure 2-b, it can be seen that the optimal test frequency should be increased if higher component failure rate is found. However, in the same component failure rate, performing more surveillance tests over the optimum test frequency will only reduce the probability of residing in the desired state.

The unavailability can be referred from Figure 2-c. The unavailability can be reduced by performing more tests and repairing the failed channel more often as it can be seen in the figure. The unavailability is monotonously decreasing as increasing the test frequency. Figure 2-d shows the combined probabilities of residing in the desired state and residing in the test state. The online test states are considered as the states that are still functioning. Therefore, to get the availability information, one has to refer Figure 2-d. It can be noted that the availability will be increased by performing more tests and recovering the failed components in the figure. The availability is monotonously increasing as the test frequency increases. The availability or unavailability profile does not bring the information of the optimal test frequency as they are monotonous functions with respect to the test frequency. This is why one needs to propose analyzing the desired state probability profile as a function of the test frequency.

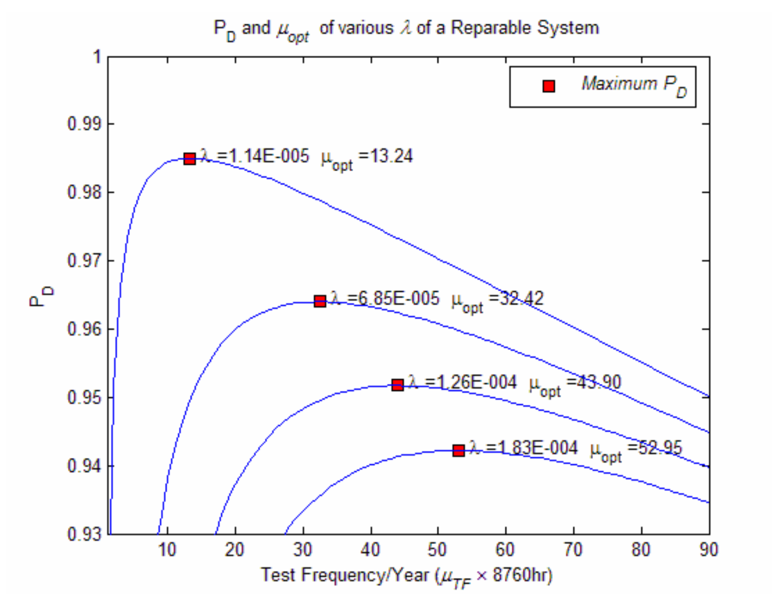

Figure 2-b: Probability of residing in the desired state of a repairable system

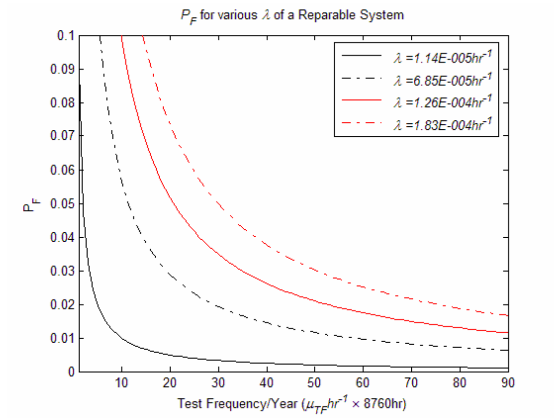

Figure 2-c: Probability of failed state

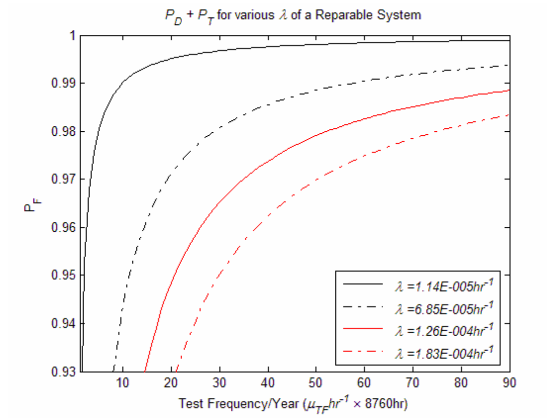

Figure 2-d: Probability of (P_D+P_T)

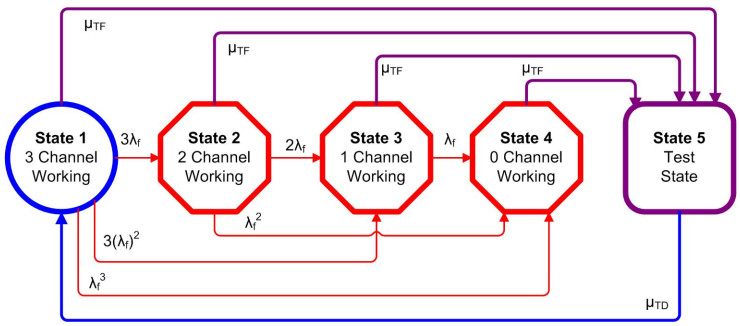

Figure 3-a: 5-States model of a three identical channel system

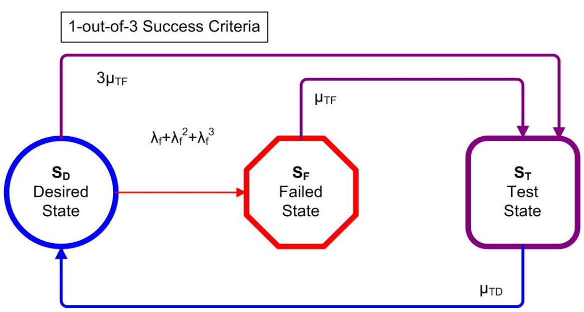

Figure 3-b: 3-States model
(1-out-of-3 success criteria)

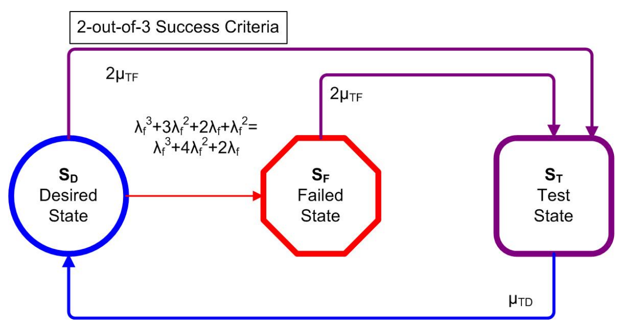

Figure 3-c: 3-States model
(2-out-of-3 success criteria)

3. Optimal test frequency of three identical channel systems

3.1 (1-out-of-3) success criteria

The transition diagram of a three identical channel system is shown in Figure 3-a with 5 state spaces, together with the state transition rates of the component failures, the tests, and the recovery. If the success criteria of the system is 1-out-of-3, the state transition diagram can be reduced to a diagram having 3 state spaces as shown in Figure 3-b. In the 3 states representation, the state transition caused by a component failure can be rewritten as

$$\lambda = \lambda_f + \lambda_f^2 + \lambda_f^3 \tag{3-1}$$

Subsequently, the stochastic transition probability matrix $\overline{V}$ can be composed

$$\overline{V} = \begin{bmatrix} 1-(\lambda + 3\mu_{TF}) & \lambda & 3\mu_{TF} \\ 0 & 1-\mu_{TF} & \mu_{TF} \\ \mu_{TD} & 0 & 1-\mu_{TD} \end{bmatrix} \tag{3-2}$$

Solving Eq. (2-1) with Eq. (2-3) and Eq. (3-2), the steady-state probability can be obtained as

$$P_D = \frac{\mu_{TF}\mu_{TD}}{(\lambda\mu_{TF} + 3\mu_{TF}^2 + \lambda\mu_{TD} + \mu_{TD}\mu_{TF})} \tag{3-3a}$$

$$P_F = \frac{\lambda\mu_{TD}}{(\lambda\mu_{TF} + 3\mu_{TF}^2 + \lambda\mu_{TD} + \mu_{TD}\mu_{TF})} \tag{3-3b}$$

$$P_T = \frac{\mu_{TF}(\lambda + 3\mu_{TF})}{(\lambda\mu_{TF} + 3\mu_{TF}^2 + \lambda\mu_{TD} + \mu_{TD}\mu_{TF})} \tag{3-3c}$$

Differentiating the P_D with respect to μ_{TF} using Eq. (3-3a) gives

$$\frac{dP_D}{d\mu_{TF}} = \frac{\mu_{TD}(-3\mu_{TF}^2 + \lambda\mu_{TD})}{(\lambda\mu_{TF} + 3\mu_{TF}^2 + \lambda\mu_{TD} + \mu_{TD}\mu_{TF})^2} \tag{3-4}$$

Then, the optimal test frequency of the system can be derived to

$$\mu_{opt} = \sqrt{\frac{\lambda\mu_{TD}}{3}}, \lambda = \lambda_f + \lambda_f^2 + \lambda_f^3 \tag{3-5}$$

3.2 (2-out-of-3) success criteria

The 5-states transition diagram of a three identical channel system shown in Figure 3-a can be reduced to a 3-states diagram as shown in Figure 3-d, if the success criteria of the system is 2-out-of-3. The state transition caused by the component failure can be rewritten

$$\lambda = \lambda_f^3 + 4\lambda_f^2 + 2\lambda_f \tag{3-6}$$

Then, the stochastic transition probability matrix $\overline{V}$ can be written

$$\overline{V} = \begin{bmatrix} 1-(\lambda + 2\mu_{TF}) & \lambda & 2\mu_{TF} \\ 0 & 1-2\mu_{TF} & 2\mu_{TF} \\ \mu_{TD} & 0 & 1-\mu_{TD} \end{bmatrix} \tag{3-7}$$

Solving Eq. (2-1) with Eq. (2-3) and Eq. (3-7), the steady-state probability can be obtained as

$$P_D = \frac{2\mu_{TF}\mu_{TD}}{(2\mu_{TF}+\lambda)(\mu_{TD}+2\mu_{TF})} \tag{3-8a}$$

$$P_F = \frac{\lambda\ \mu_{TD}}{(2\mu_{TF}+\lambda)(\mu_{TD}+2\mu_{TF})} \tag{3-8b}$$

$$P_T = \frac{2\mu_{TF}}{(\mu_{TD}+2\mu_{TF})} = \frac{2\mu_{TF}(2\mu_{TF}+\lambda\ )}{(2\mu_{TF}+\lambda)(\mu_{TD}+2\mu_{TF})} \tag{3-8c}$$

Differentiating P_D with respect to μ_{TF} using Eq. (3-8a) gives

$$\frac{dP_D}{d\mu_{TF}} = \frac{2\mu_{TD}(\lambda\mu_{TD}-4\mu_{TF}^2)}{(2\mu_{TF}+\lambda)^2(\mu_{TD}+2\mu_{TF})^2} \tag{3-9}$$

Then, the optimal test frequency giving the *Max*(P_D) can be derived from

$$\mu_{opt} = \sqrt{\frac{\lambda\mu_{TD}}{4}}, \quad \lambda = \lambda_f^3 + 4\lambda_f^2 + 2\lambda_f \tag{3-9}$$

3.3 Examples of optimal test frequencies in a three-identical-channel system

The profiles of the steady state probability of residing in each state of a 3-channel system are shown in Figure 3-d, Figure 3-e, and Figure 3-f. The probabilities are quantified with various component failure rates and test frequencies. It also shows the maximum probabilities and the optimal test frequencies at each component failure rates and in the two different success criteria developed in the previous sections. It can be seen in Figure 3-d that to achieve higher state residing probability of the desired state, more tests are required for the 2-out-of-3 success criteria than the 1-out-of-3 success criteria in the same 3-channel system having the same component failure rate. However, under the same success criteria and the same hardware condition, performing more surveillance tests over the optimum test frequency will only reduce the probability of residing in the desired state.

Figure 3-e shows the combined probabilities of residing in the desired state and residing in the test state. The probabilities are obtained under the two different success criteria of the three channel system. It can be seen that the 1-out-of-3 success criterion leads to higher availability than the 2-out-of-success criterion. The unavailability of the 3-channel system can be referred from Figure 3-e. The unavailability can be reduced by performing the test and recovering the failed channel as indicated in Figure 3-f. The figure also shows that the 1-out-of-3 success criterion brings lower unavailability than the 2-out-of-success criterion as expected. But, the optimal test frequency information cannot be derived from the unavailability or availability profile as mentioned in Section 2.3.

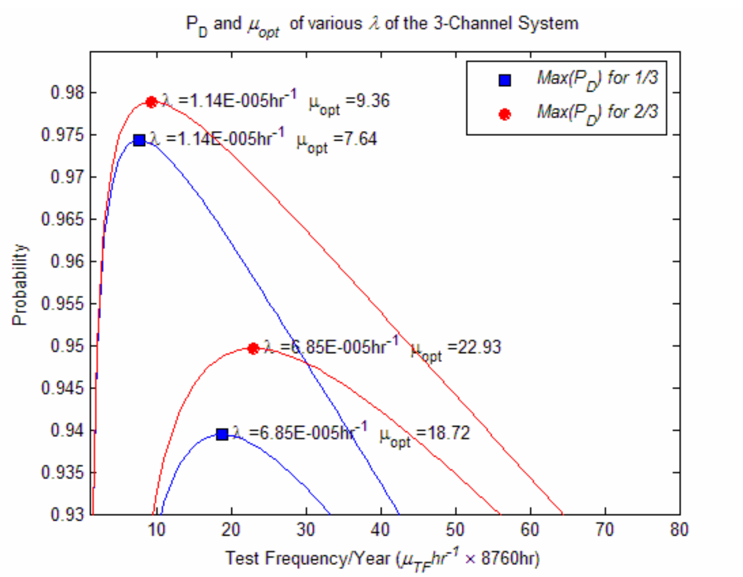

Figure 3-d: Probability of residing in the desired state of the three channel system

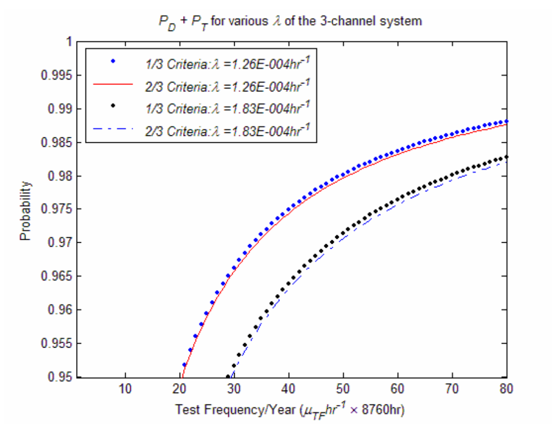

Figure 3-e: Probability of (P_D+P_T)

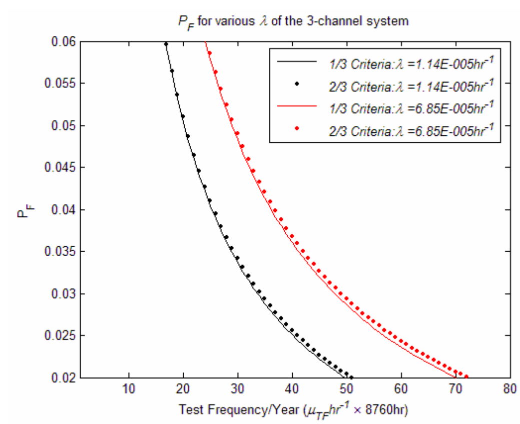

Figure 3-f: Probability of failed state

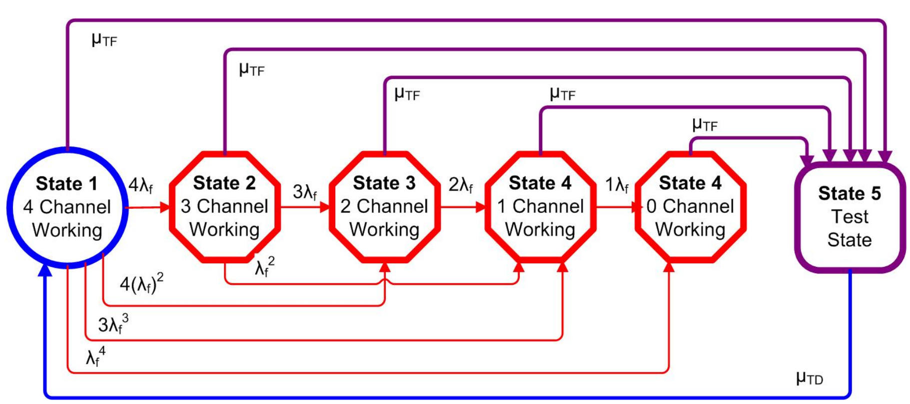

Figure 4-a: 6-states model of a four identical channel system

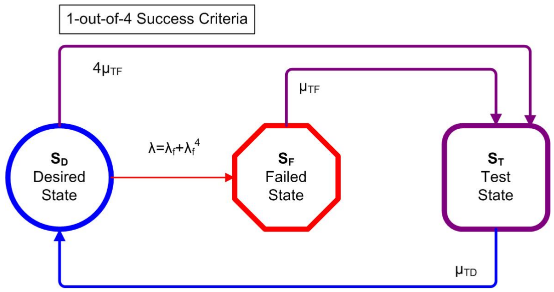

Figure 4-b: 3-states model
(1-out-of-4 success criteria)

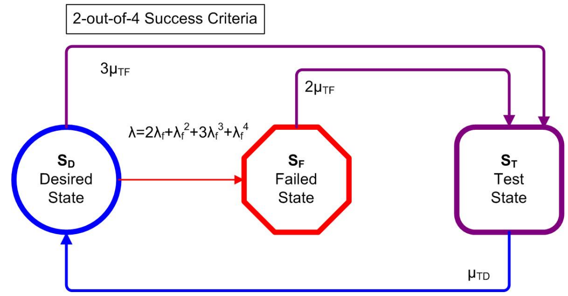

Figure 4-c: 3-states model
(2-out-of-4 success criteria)

4. Optimal test frequency for four-identical-channel systems

The state transition diagram of a four-identical-channel system is shown in Figure 4-a with 6 state spaces. The state transition rates of the component failures, the tests, and the recovery are also shown in the figure. The diagram can be simplified based on the required success criteria. In this section, examples of deriving the optimal test frequency for the 1-out-of-4 and 2-out-of-4 success criteria are presented.

4.1 (1-out-of-4) success criteria

If the success criteria requirement of the system is 1-out-of-4, then the state transition diagram can be reduced to the diagram having 3 state spaces as shown in Figure 4-b. After the state reduction to the 3 states representation, the state transition caused by the component failure can be rewritten

$$\lambda = \lambda_f^4 + \lambda_f \tag{4-1}$$

Thus, the stochastic transition probability matrix $\overline{V}$ is

$$\overline{V} = \begin{bmatrix} 1-(\lambda + 4\mu_{TF}) & \lambda & 4\mu_{TF} \\ 0 & 1-\mu_{TF} & \mu_{TF} \\ \mu_{TD} & 0 & 1-\mu_{TD} \end{bmatrix} \tag{4-2}$$

Solving Eq. (2-1) with Eq. (2-3) and Eq. (4-2), the steady-state probability can be obtained as

$$P_D = \frac{\mu_{TF}\mu_{TD}}{(\lambda\mu_{TF} + 4\mu_{TF}^2 + \lambda\mu_{TD} + \mu_{TD}\mu_{TF})} \tag{4-3a}$$

$$P_F = \frac{\lambda\mu_{TD}}{(\lambda\mu_{TF} + 4\mu_{TF}^2 + \lambda\mu_{TD} + \mu_{TD}\mu_{TF})} \tag{4-3b}$$

$$P_T = \frac{\mu_{TF}(\lambda + 4\mu_{TF})}{(\lambda\mu_{TF} + 4\mu_{TF}^2 + \lambda\mu_{TD} + \mu_{TD}\mu_{TF})} \tag{4-3c}$$

Differentiating the P_D with respect to μ_{TF} using Eq.(4-3a) gives

$$\frac{dP_D}{d\mu_{TF}} = \frac{\mu_{TD}(-4\mu_{TF}^2 + \lambda\mu_{TD})}{(\lambda\mu_{TF} + 4\mu_{TF}^2 + \lambda\mu_{TD} + \mu_{TD}\mu_{TF})^2} \tag{4-4}$$

Then, the optimal test frequency of the system is

$$\mu_{opt} = \sqrt{\frac{\lambda\mu_{TD}}{4}}, \lambda = \lambda_f^4 + \lambda_f \tag{4-4}$$

4.2 (2-out-of-4) success criteria

After the state reduction, the state transition caused by the component failure of the 3-states model can be rewritten

$$\lambda = 2\lambda_f + \lambda_f^2 + 3\lambda_f^3 + \lambda_f^4 \tag{4-5}$$

The stochastic transition probability matrix $\overline{V}$ can be written

$$\overline{V} = \begin{bmatrix} 1-(\lambda + 3\mu_{TF}) & \lambda & 3\mu_{TF} \\ 0 & 1-2\mu_{TF} & 2\mu_{TF} \\ \mu_{TD} & 0 & 1-\mu_{TD} \end{bmatrix} \tag{4-6}$$

Solving Eq. (2-1) with Eq. (2-3) and Eq. (4-6), the steady-state probability can be obtained as

$$P_D = \frac{2\mu_{TF}\mu_{TD}}{(2\lambda\mu_{TF} + 6\mu_{TF}^2 + \lambda\mu_{TD} + 2\mu_{TD}\mu_{TF})} \tag{4-7a}$$

$$P_F = \frac{\lambda\mu_{TD}}{(2\lambda\mu_{TF} + 6\mu_{TF}^2 + \lambda\mu_{TD} + 2\mu_{TD}\mu_{TF})} \tag{4-7b}$$

$$P_T = \frac{2\mu_{TF}(\lambda + 3\mu_{TF})}{(2\lambda\mu_{TF} + 6\mu_{TF}^2 + \lambda\mu_{TD} + 2\mu_{TD}\mu_{TF})} \tag{4-7c}$$

Differentiating the P_D with respect to μ_{TF} using Eq.(4-7a) gives

$$\frac{dP_D}{d\mu_{TF}} = \frac{2\mu_{TD}(-6\mu_{TF}^2 + \lambda\mu_{TD})}{(2\lambda\mu_{TF} + 6\mu_{TF}^2 + \lambda\mu_{TD} + 2\mu_{TD}\mu_{TF})^2} \tag{4-8}$$

Then, the optimal test frequency of the system is

$$\mu_{opt} = \sqrt{\frac{\lambda\mu_{TD}}{6}}, \quad \lambda = 2\lambda_f + \lambda_f^2 + 3\lambda_f^3 + \lambda_f^4 \tag{4-9}$$

4.3 Examples of optimal test frequencies in a four-identical-channel system

The profiles of the steady state probability of residing in each state of the 4-channel system are shown in Figure 4-d, Figure 4-e, and Figure 4-f. It can be seen that the maximum probabilities and the optimal test frequencies at each chosen component failure rate and two different success criteria developed in the previous sections. It can be seen from Figure 4-d that more tests are required for the 2-out-of-4 success criteria than the 1-out-of-4 success criteria in the same 4-channel system having the same component failure rate to achieve higher state residing probability of the desired state. However, in the same success criteria and in the same hardware condition, performing more surveillance tests over the optimum test frequency will only lower the probability of residing in the desired state. The unavailability of the 4-channel system can be referred from Figure 4-e. The figure shows that the 1-out-of-4 success criterion has lower unavailability than the 2-out-of-4 success criterion. Figure 4-f shows the probabilities of residing in the desired state or residing in the test state that has developed with the two different success criteria of the three channel system. It can be read that the 1-out-of-4 success criterion brings higher availability than the 2-out-of-4 success criterion in the same system.

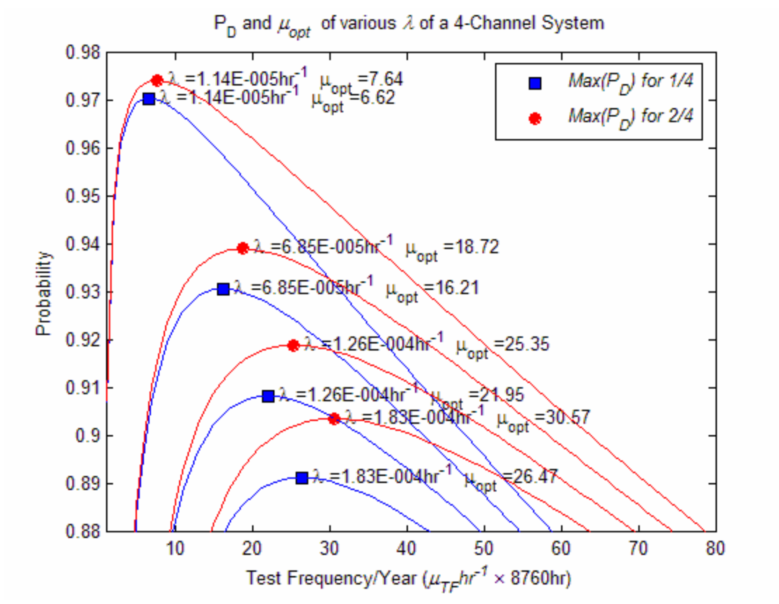

Figure 4-d: Probability of residing in the desired state of the four channel system

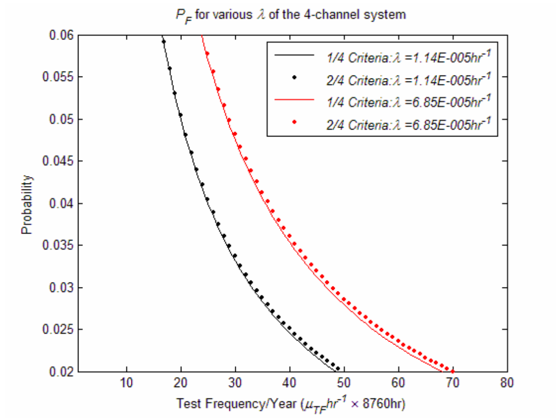

Figure 4-e: Probability of failed state

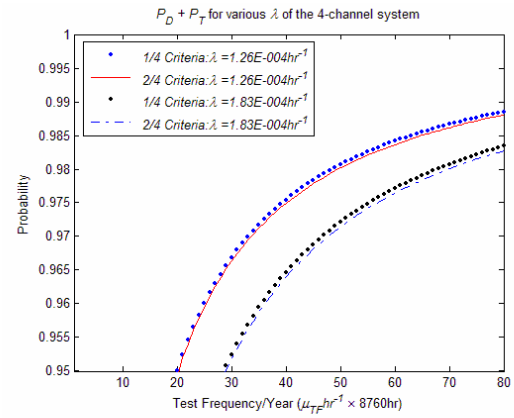

Figure 4-f: Probability of (P_D+P_T)

5. Conclusions

A strategy for determination of the optimal test frequency for an identical multi-channel system is proposed in this paper. The strategy suggests decomposing all the states of a multi-channel system to three sub-sets of states: the desired, the failed, and the test state, to get the optimal test frequency information. The strategy provides answers to how many surveillance tests should be performed to gain the maximal benefit in terms of the probability of staying in the desirable state. Based on the examples of analytic solutions for the optimal surveillance test interval of a k-out-of-n multi-channel reparable system, it can be concluded that the optimal surveillance test frequency is a function of the component failure rates and the test duration. It also depends on the success criteria requirements in the same hardware configuration.

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