Optimal Estimation of Space-Time Kinetics Reactivity Components Using a Reference Model Approach

Daryoosh Vashaee and Lobat Tayebi

Engineering Physics Department, McMaster University, Hamilton, Ontario L8S 4L7

ABSTRACT

The problem of estimating reactivity transients from an observed neutron flux transient is considered. This is relevant, for example, to analyzing a power rundown test or to estimating reactivity variations associated with some computer codes that do not specifically compute individual reactivity components. A method is presented which utilizes inverse space-time kinetics and optimal state estimators to extract the components of the reactivity transient from observed neutron flux measurements. The approach takes into account geometric characteristics and composition of the reactor core, as well as reactor operating conditions. Measurements from a limited number of in-core neutron flux detectors are the inputs used to extract reactivity components that fit a modal model of the reactor, referred to as the "reference model". An improved solution for the reactivity components is then generated using the modal approximation solution for the neutron transport equation in conjunction with optimal estimation techniques. The method has been applied to a reactivity initiated accident in which a transient is initiated by a non-uniform loss-of-coolant. This results in a spatially varying neutron overpower transient that is terminated by the asymmetric insertion of shutoff rods. In this paper an extended Kalman filter is employed to estimate the neutron flux distribution in the core and identify the reactivity components of the reference model. The reference model in the state space and the Kalman filter algorithm are shown. This filter is applied to the parameter identification of the reference model. Results of numerical simulations of the reactor transient and the optimal estimation of the reactivity components will be presented to demonstrate that the filter is capable of identifying the unknown parameters.

NOMENCLATURE

a_m	Prompt neutron modal amplitude
β_j	Delayed precursor fraction of group j
β	Prompt neutron fraction
c_m	Delayed neutron modal amplitude
D	Neutron diffusion constant
$\upsilon\Sigma_f$	Fission cross section
Λ_m	Prompt neutron generation time
λ_j	Delayed precursor decay constant
ρ_{scm}	Subcritical reactivity
ρ_{mn}	Modal reactivity
Σ_a	Absorption cross section

INTRODUCTION

The current generation of Canadian natural uranium heavy water (CANDU) power reactor consists a horizontal, cylindrical calandria filled with heavy water (D_2O) moderator/reflector, and penetrated in by 380 fuel channels (in CANDU-6) or 480 fuel channels (in CANDU-9). The fuel channel encloses fuel bundles and high temperature, high pressure heavy water coolant. These channels are contained within calandria tubes with the annulus between the two tubes filled with a gas. Absorber rods and detector guide tubes enter the core vertically. Flux detector devices are provided to measure flux and, through action of the Reactor Regulating System (RRS), induce the adjustment of the light water level in liquid zone control (LZC) units. Likewise, adjuster rods are used to control and shape the neutron flux level and also to provide excess reactivity load to compensate the Xenon build up following a power reduction. They are normally completely inserted in the core. The reactivity balance condition in such a configuration is also affected by both the long term changes due to fuel depletion and isotopic buildup, as well as by more dramatic changes due to control rod adjustment or some accidental situation. Thus, one must be able to predict and analyze the time behavior of the neutron population, commonly referred to as Nuclear Reactor Kinetics.

In the present design of CANDU reactor it is known that there exists a positive reactivity feedback mechanism referred to as the “coolant void effect” that the safety system and control system designs must be able to accommodate. The occurrence of void in the core can be due to a break in the main heat transport circuit or a result of events involving a mismatch between power generation and heat transport from the core. The void worth and void reactivity coefficient are major safety parameters.

The problem under consideration is the estimation of individual reactivity components using a limited set of transient neutron flux measurements. It is necessary to estimate the time-varying reactivity components for optimal reactor control and reactivity anomaly detection.

Reactivity coefficients are key parameters because they are directly related to reactor safety. For example, one might like to know the void reactivity coefficient during a LOCA, however a void measurement is not available and there exist other phenomena that affect the measurable variables such as the flux, temperature, pressure, and the flows. The reactivity has been traditionally determined from measurements of the neutron density using an inverse kinetics equation. Various analog and digital methods have been developed on this basis for the estimation of the reactivity.[1,2,3] However, they are not applicable to the events which involve significant changes in the flux shape. We hereby present a means to extract the space-time kinetics reactivity coefficients taking into account the special variations of the neutron flux. We will use the data obtained from the limited number of in-core flux detectors. An application of the method is presented for the case of a neutron over-power transient associated with coolant void reactivity resulting from a break in the main heat transport circuit. The transient is terminated by a reactor shutdown.

REFERENCE MODEL

In order to estimate the reactivity components in the reactor core, inverse point reactor kinetics equations (IPRKE) can be used to extract the net reactivity. However, the IPRKEs are not valid for scenarios that involve significant changes in the flux shape. Accurate space and time-dependent calculations are needed to analyze space-time kinetics in large core reactors such as CANDU. The formalism must ideally take into account geometric characteristics, core compositions, and operating conditions. For this purpose a modal approximation solution of the neutron transport equation is proposed. A solution to the neutron kinetics equations using modal expansion methods has been introduced early on in the development of nuclear technology.[4] The modal method is uniquely suited for studying problems involving reactivity perturbations because the perturbation components correspond directly to separate expansion functions. In the formulation of the modal expansion, the eigenfunctions are ordered according to the magnitude of their respective eigenvalues. The expansion is truncated at a specific minimum magnitude in the eigenvalue spectrum. The unused harmonics would be excited only by reactivity perturbations larger than the eigenvalue separation of the last used harmonic. For this reason, modal expansion with only a few functions is almost as accurate as full three-dimensional direct solution methods for many transients involving reactivity perturbations in large decoupled cores.

Figure 1 shows a schematic of the proposed model for the optimal estimation of space-time kinetics reactivity components. To demonstrate the concepts for resolving the uncertainty in parameter estimation codes and models, a generic tool that is based on a modal expansion solution of the three-dimensional neutron transport equation in the core is used. This acts as a virtual system simulating the neutron flux out from a real reactor core. As shown in the block diagram of Figure 1, the output from the virtual system is used to minimize the uncertainty in the reference model capable of dynamically adjusting varying distributed parameters inherent in the reactor core. The reference model integrates a modal description of the physical phenomena in adequate detail with a feedback control scheme designed for simultaneous parameter estimation for a nonlinear system based on extended Kalman filtering. The extended Kalman filter[5] is an "extension" of Kalman's original filter into the domain of nonlinear stochastic differential equations. The nonlinear filtering regime introduces a number of challenge to the Kalman filter that prevent application of the Kalman filter from being optimal in the same way it

is optimal in the linear regime. However, the extended Kalman filter is often a Good "ad-hoc" nonlinear filter.

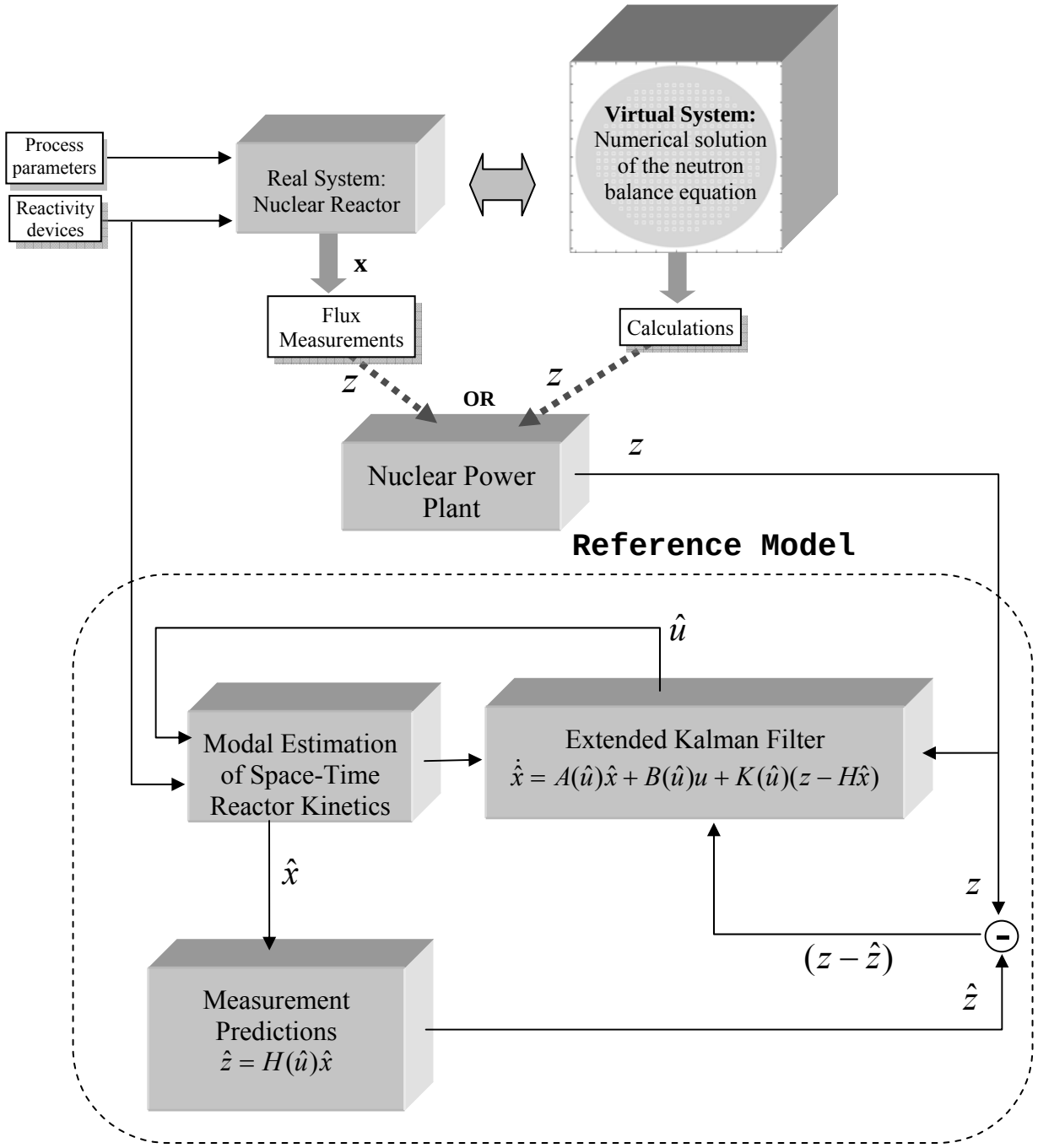


Figure 1: Block diagram of the reference model approach for the optimal estimation of space-time kinetics reactivity components.

Extended Kalman filter can be also used for parameter estimation. The problem of identifying constant parameters in a system can be handled within the Kalman filter by treating the parameters as if they were states. If the parameters are expected to be constants or slowly varying over the process time domain, a zeroth order state dynamic model is appropriate. The parameters are then augmented to the state vector. If parameters are expected to vary quickly over the process time domain, more complex models may need to be applied.

Flux measurements from the reactor core or equally generated within the virtual system are input to the reference model. The innovations term $(z-\hat{z})$ is given by the difference of the flux measurements and measurement predictions as determined from the "Extended Kalman Filter" block. The Kalman gain is determined from the current model and statistics of the inherent stochastic process and applied with the innovations term to refine the state space model. The feedback gain drives the modeled states, determined from the Modal simulation, toward the plant states. Additionally, the low order state-space model in the "Extended Kalman Filter" block also receives some control information from the "Nuclear Power Plant" block to maintain the simulation integrity. The parameter estimation algorithm refines one or more specially distributed parameters and this result is applied to the Modal simulation code. The system of Figure 1 could potentially provide a vast improvement in reactor core design and safety margins because it is capable of reducing the level of uncertainty in reactor core parameters.

The Modal description of the time-dependent flux synthesis is given by the following expression:

$$\phi(r,t)=\sum_{n=1}^{N}\psi_n^g(r)a_n(t) \qquad \text{Equ. 1}$$

where the flux modes, $\psi_n^g(r)$, contain the space and group dependence whereas the superposition coefficients, $a_n(t)$, are functions of time only. The use of only one term in the sum of Equ. 1 corresponds to the point reactor model if $\psi_n^g(r)$ is chosen as the initial multigroup flux shape. $\psi_n^g(r)$'s are the eigenfunctions of the static neutron balance equations for the non-perturbed reactor for the successive eigenvalues. Following the modal synthesis description of the neutron balance equations,[6,7] the following sets of differential equations are derived for the time-dependent prompt and delayed neutron modal amplitudes:

$$\frac{da_m}{dt}=\frac{(\rho_{scm}+\rho_{mm}-\beta)a_m}{\Lambda_m}+\frac{\sum_{n\neq m}^{M}\rho_{mn}a_n}{\Lambda_m}+\sum_j\lambda_j c_{mj} \qquad \text{Equ. 2}$$

$$\frac{dc_{mj}}{dt}=-\lambda_j c_{mj}+\frac{\beta_j}{\Lambda_m}a_m \qquad m=1,2,\cdots M$$

ρ_{mn}'s are the modal reactivities that are related to the effective incremental cross-section for the perturbation as follows:

$$\rho_{mn} \equiv \frac{\left\langle \psi_m \mid \left(\Delta \upsilon \Sigma_{f_0} - \Delta \Sigma_a + \Delta D \Omega\right) \delta(r) \mid \psi_n \right\rangle}{\left\langle \psi_m \mid \upsilon \Sigma_{f_0} \mid \psi_m \right\rangle} \qquad \text{Equ. 3}$$

where Ω is the operator defining the product of gradients, i.e. $\langle \psi_m \mid \Omega \mid \psi_n \rangle = \langle \nabla \psi_m \mid \nabla \psi_n \rangle$.

In order to utilize the extended Kalman filter, it is necessary to transform the reference model given in Equ. 2 to the state space model. Selecting the prompt and delayed neutron modal amplitudes for the state variables since their time derivatives are included in the modal model, the state x vector is defined as:

$$x = \left[[a_m][c_1][c_2]\cdots[c_M][\hat{\rho}_{mn}] \right]^T_{[(M+JM+M^2)\times 1]} \qquad \text{Equ. 4}$$

where superscript T means the transpose of the vector or matrix, and ^ means the parameter is not known. $[\hat{\rho}_{mn}]$ is the extended Kalman filter augmentation in the state vector. $[a_m]$, $[c_m]$, and $[\rho_{mn}]$ are:

$$[a_m] = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{bmatrix}_{(M\times 1)} \quad ; \quad c_m = \begin{bmatrix} c_{1m} \\ \vdots \\ c_{Jm} \end{bmatrix}_{(J\times 1)} \quad m = 1,2,..M \quad ; \quad [\rho_{nm}] = \begin{bmatrix} \rho_{11} \\ \vdots \\ \rho_{1M} \\ \rho_{21} \\ \vdots \\ \rho_{2M} \\ \vdots \\ \rho_{MM} \end{bmatrix}_{(M^2\times 1)} \qquad \text{Equ. 5}$$

Therefore the state space model can be obtained as:

$$\dot{x} = f(x,[a_m],[c_m],u,\hat{u}) = A_d([a_m],[c_m],u,\hat{u})x \qquad \text{Equ. 6}$$

where $A_d([a_m],[c_m],u,\hat{u})$ is the transition matrix:

$$A_d = \begin{bmatrix} [\rho/\Lambda]_{(M\times M)} & [\lambda_1]_{(M\times J)} & \cdots & [\lambda_M]_{(M\times J)} & [a/\Lambda_1]_{(M\times M)} & \cdots & [a/\Lambda_M]_{(M\times M)} \\ [\beta/\Lambda_1]_{(J\times M)} & [\tilde{\lambda}]_{(J\times J)} & \cdots & [0]_{(J\times J)} & [0]_{(J\times M)} & \cdots & [0]_{(J\times M)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & & \\ [\beta/\Lambda_M]_{(J\times M)} & [0]_{(J\times J)} & \cdots & [\tilde{\lambda}]_{(J\times J)} & [0]_{(J\times M)} & \cdots & [0]_{(J\times M)} \\ [0]_{(M^2\times M)} & [0]_{(M^2\times J)} & \cdots & [0]_{(M^2\times J)} & [0]_{(M^2\times M)} & \cdots & [0]_{(M^2\times M)} \end{bmatrix} \quad \text{Equ. 7}$$

J is the number of delayed precursor groups and M is the number of eigenmodes. The matrix elements are defined as follows:

$$[\rho/\Lambda] = \begin{bmatrix} \frac{\rho_{sc_1} + \rho_{11}^T - \beta}{\Lambda_1} & \frac{\rho_{12}^T}{\Lambda_1} & \cdots & \frac{\rho_{1M}^T}{\Lambda_1} \\ \frac{\rho_{21}^T}{\Lambda_2} & \frac{\rho_{sc_2} + \rho_{22}^T - \beta}{\Lambda_2} & \cdots & \frac{\rho_{2M}^T}{\Lambda_2} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\rho_{M1}^T}{\Lambda_M} & \cdots & & \frac{\rho_{sc_M} + \rho_{MM}^T - \beta}{\Lambda_M} \end{bmatrix}_{(M\times M)}$$

$$[\lambda_1] = \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_J \\ 0 & \cdots & & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & & 0 \end{bmatrix}_{(M\times J)} \cdots ; \quad [\lambda_M] = \begin{bmatrix} 0 & \cdots & & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & & 0 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_J \end{bmatrix}_{(M\times J)}$$

$$[\tilde{\lambda}] = \begin{bmatrix} -\lambda_1 & 0 & \cdots & 0 \\ 0 & -\lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & -\lambda_J \end{bmatrix}_{(J\times J)}$$

$$[\beta/\Lambda_1] = \begin{bmatrix} \beta_1 & 0 & \cdots & 0 \\ \beta_2 & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \beta_J & 0 & \cdots & 0 \end{bmatrix}_{(J\times M)} \cdots ; \quad [\beta/\Lambda_M] = \begin{bmatrix} 0 & \cdots & 0 & \beta_1 \\ \vdots & & \vdots & \beta_2 \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & 0 & \beta_J \end{bmatrix}_{(J\times M)}$$

$$[a/\Lambda_1]_{(M\times M)} = \begin{bmatrix} \frac{a_1}{\Lambda_1} & \frac{a_2}{\Lambda_1} & \cdots & \frac{a_M}{\Lambda_1} \\ 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \cdots ; \quad [a/\Lambda_M]_{(M\times M)} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ \frac{a_1}{\Lambda_M} & \frac{a_2}{\Lambda_M} & \cdots & \frac{a_M}{\Lambda_M} \end{bmatrix} \quad \text{Equ. 8}$$

u and $\hat{u}$ are the input vectors consisting the known and unknown reactivity components respectively:

$$u = \begin{bmatrix} \rho_{11} \\ \rho_{12} \\ \vdots \\ \rho_{MM} \end{bmatrix} \; ; \; \hat{u} = \begin{bmatrix} \hat{\rho}_{11} \\ \hat{\rho}_{12} \\ \vdots \\ \hat{\rho}_{MM} \end{bmatrix} \quad \text{Equ. 9}$$

The extended Kalman filter is used to generate the optimal estimates of the unknown reactivity components defined in the input vector $\hat{u}$.

KALMAN FILTER

System Model

The discrete Kalman filter is applied to the discrete model of the reference model in the state space. Because the number of the parameters to be identified is M^2, an M^2 order parameter vector is defined as $\hat{u}$ (see Equ. 9). These parameters are all that are needed to complete the reference model. In the real system, the system noise should be taken into account. The noise superimposed on the input reactivities is assumed to be the system noise. This assumption makes it easier to determine the covariance matrix of the system noise. The system model is:

$$x_{k+1} = f(x_k, u_k, \hat{u}_k, w_k) \quad \text{Equ. 10}$$

where x_k and x_{k+1} refer to $x(t_k)$ and $x(t_k + \Delta t)$ respectively. Δt is the sampling period. Using Equ. 6, we can estimate:

$$f(x_k, u_k, \hat{u}_k) = \{1 + \Delta t A_d([a_m],[c_m],u,\hat{u},0)\}x_k + Gw_k \quad \text{Equ. 11}$$

w_k is the system noise and assumed to be a zero mean white Gaussian noise which is independent of x_k, with covariance Q. G is the Jacobian matrix of partial derivatives of f with respect to w. Let the available discrete time measurements be modeled as:

$$z_k = Hx_k + \upsilon_k \quad \text{Equ. 12}$$

where

$$H = \begin{bmatrix} \psi_1(r_1) & \psi_2(r_1) & \cdots & \psi_M(r_1) & 0 & \cdots & 0 \\ \psi_1(r_2) & \psi_2(r_2) & & \psi_M(r_2) & \vdots & \ddots & \vdots \\ \vdots & & \ddots & \vdots & & & \\ \psi_1(r_d) & \psi_2(r_d) & \cdots & \psi_M(r_d) & 0 & \cdots & 0 \end{bmatrix}_{[d\times(M+JM+M^2)]} \qquad \text{Equ. 13}$$

and the measurement noise υ_k is a zero mean white Gaussian noise that is independent of x_k and w_k, with covariance R.

Equ. 12 means that the neutron flux is measured at d locations in the core, which following the modal expansion of Equ. 1, can be written in the space-time domain as:

$$z(r_d, t) = \sum_{m=1}^{M} \psi_m(r_d) a_m(t) + \upsilon(t) \qquad \text{Equ. 14}$$

The system model is shown in Figure 2. Now that the complete system model is obtained, an extended Kalman filter can be applied.

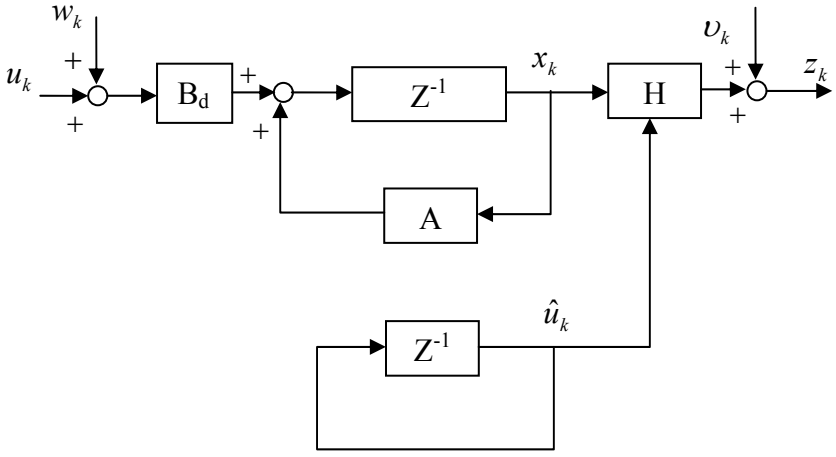


Figure 2: System Model for the Kalman Filter

Filtering algorithm

The-principle of the Kalman filter is as follows. At time stage k, the predictions of the state and the parameter vector x is calculated from the input u_{k-1} and the estimation of the state vector $\hat{x}_{k-1}$ and parameter vector $\hat{u}_{k-1}$ assuming that there is no noise. The covariance matrix of the prediction error P_k^-, that of the estimation error P_k and Kalman gain matrix K_k are updated. Finally, the estimations of the state and the parameter vector is obtained by revising the predictions by the difference between the measurements of the neutron flux and the predictions via the Kalman gain matrix, which is determined so that the covariance matrix of the estimation error is the smallest value. The Kalman filter algorithm is summarized as follows:

1. Starting value $\hat{x}_0, P_0$

2. $\hat{x}_k^- = f(\hat{x}_{k-1}, u_{k-1}, \hat{u}_{k-1}, 0)$

3. $P_k^- = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T$

4. $K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1}$

5. $\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$

6. $P_k = (I - K_k H_k) P_k^-$

7. Repeat steps 2 to 6

A_k : Jacobian matrix of partial derivatives of f with respect to x_k

$\hat{x}_k$: Estimation of the state vector

$\hat{x}_k^-$: Prediction of the state vector

$\hat{u}_k$: Estimation of the parameter vector

I : Unit matrix

0 : Zero matrix

P_k^- : Covariance matrix of the prediction error

P_k : Covariance matrix of the estimation error

K : Kalman gain matrix

The starting values of the estimations of the state and parameter vectors and the covariance matrix of the estimation error, and the covariance matrices of the system and measurement noises should be set before filtering.

BENCHMARK TRANSIENT PROBLEM

Virtual System:

The virtual system consists of a modal description of the space-time kinetics of the reactor core. In order to apply the reference model method to a trip event the following scenario was considered. This problem is a revisit of the benchmark activity presented in [8]. The reactivity transient represents a uniform loss-of-coolant followed by a subsequent asymmetric insertion of shutdown reactivity devices.

The transient starts with a non-uniform loss-of-coolant that results in the increase of coolant void reactivity and is followed by an asymmetric insertion of shutoff rods which results in an increase in the thermal absorption cross section in one-half of the core. Figure 3 shows the layout of the reactor, which consists of inner (regions 3,4,13, and 16) and outer fueled regions surrounded by a D_2O reflector (regions 1, 6, 7, 9, 10, 12, 14, 18, 19, 20, 22, 23, 25, and 26). The axes lengths are normalized from 780cm to 50cm. Other parameters are accordingly normalized in the calculations.

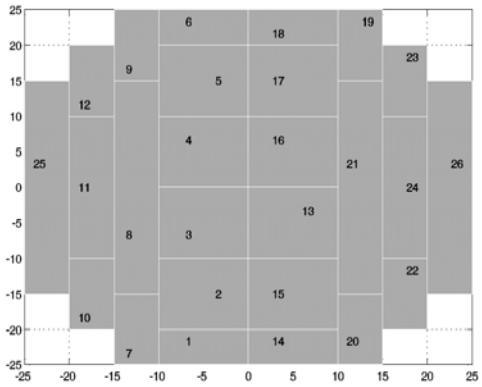
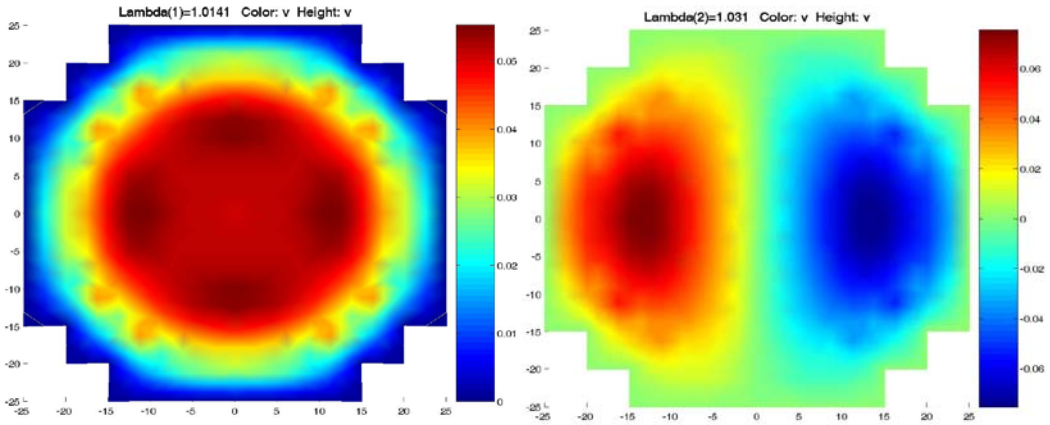


Figure 3: Reactor core configuration

The first four thermal modes are calculated using a finite element solution of the eigenvalue Equ. 1 within the MATLAB environment for a two-dimensional CANDU reference core. Figure 4 shows the calculation result for the thermal neutron distributions for the first four modes.



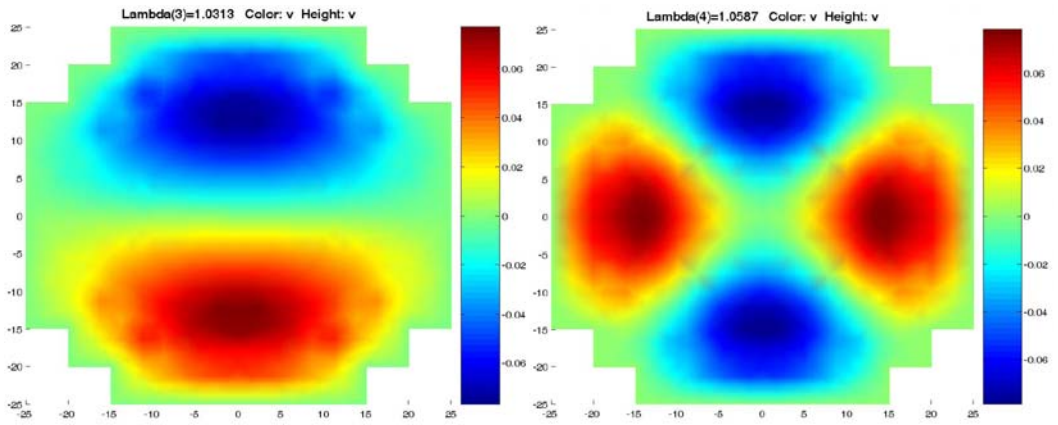


Figure 4: Thermal neutron spatial density distribution for the eigenmodes one to four. Lamda is the corresponding eigenvalue.

The two-group cell parameters and other data for the problem are given in Table 1 and

Table 2. Six groups of delayed neutrons were used in the simulation. The initiating perturbation is represented by an exponential decay decrease in the thermal absorption cross section, Σ_2, in regions 2, 3, 4, 5, 8, and 11 of the form:

$$\Sigma_2 = -5.8667\times10^{-5}(1-e^{-t/0.4}) \quad cm^{-1} \qquad \text{Equ. 15}$$

At t=0.6 shutoff rods are inserted that results in an increase $\Delta\Sigma_2=1.23\times10^{-4}$ cm^{-1} progressively in regions 1 to 6 and 13 to 18 from top to bottom. This simulates the insertion of shutoff rods at a constant velocity of 520 cm/s in the y direction. This movement is parallel to the x axis and moves from regions 6+18 to 1+14. During this transient significant flux distortion and delayed neutron holdback occurs.

Table 1: Lattice Parameters

Region	Group g	D_g (cm)	Σ_g (cm^{-1})	$\upsilon\Sigma_{f\,g}$ (cm^{-1})	$\Sigma_{1\to2}$ (cm^{-1})
1, 6, 7, 9, 10, 12, 14, 18, 19, 20, 22, 23, 25, 26	1	1.310	1.018×10^{-2}	0.0	1.018×10^{-2}
	2	0.8695	2.117×10^{-4}	0.0	
2, 5, 8, 11, 15, 17, 21, 24	1	1.264	8.154×10^{-3}	0.0	7.368×10^{-3}
	2	0.6328	4.014×10^{-3}	4.523×10^{-3}	
3, 4, 13, 6	1	1.264	8.154×10^{-3}	0.0	7.368×10^{-3}
	2	0.9328	4.100×10^{-3}	4.462×10^{-3}	

Table 2: Delayed Neutron Data

β_i	λ_i (s^{-1})
4.170×10^{-4}	0.01244
1.457×10^{-3}	0.03063
1.339×10^{-3}	0.11390
3.339×10^{-3}	0.30790
8.970×10^{-4}	1.1980
3.200×10^{-4}	3.2120

The results are calculated in terms of total and regional fission yields versus time:

$$\frac{\int_{v_i}[\nu\Sigma_{f2}\phi_2(\bar{r},t)dv]}{\int_{v_i}[\nu\Sigma_{f2}\phi_2(\bar{r},0)dv]}$$

where v_i is the area of region i. The calculated time dependent thermal flux in the y direction at x=375 cm is plotted in Figure 5.

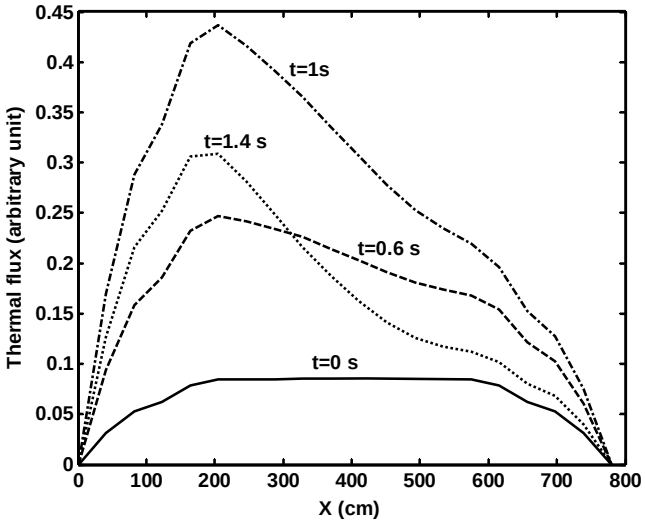


Figure 5: Time-dependent thermal flux shape.

Figure 6 shows the calculated relative power in regions 12+13, 16+17, and the total core.

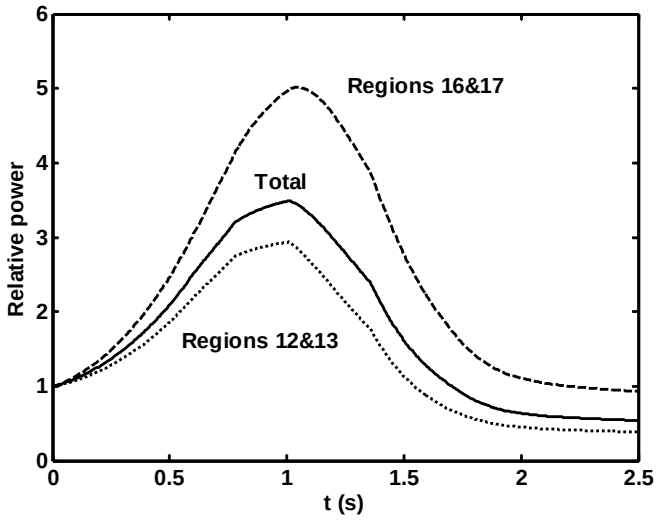


Figure 6: Time-dependent relative power.

Reference Model:

The calculated space-time dependent thermal flux from the virtual system is the input to the modeled detectors located in the core. We use the data from 13 detectors distributed in the core as shown in Figure 7. Since the higher order modes are not excited in the transient, the number of detectors is not as important as the location of the detectors and their ability to measure the excited mode shapes. Therefore, systematic errors from systematic loss of detector signals are not affecting the estimation results. The output from the modeled detector of the operating reactor consists of both true neutronic signal and random noise. The random noise comes from random neutronic fluctuations in the core and the electronic hardware. The effect of the noise component at any particular detector at any particular time is a random phenomenon. It can either increase or decrease the output of a detector with respect to the true neutronic signal. The random error component of the detector signal is completely unpredictable and cannot be filtered out using an electronic circuit. The effect of random detector error is simulated by adding a set of independent random errors to the thermal flux amplitude at the detectors locations. The distribution of the random error in each set is a normalized Gaussian distribution with a specified deviation. To evaluate the robustness of the Kalman filter estimator, an excessively large measurement noise with amplitude of about 50% of the peak value of that of the thermal flux at the center point is assumed for all the detectors. In reality noise levels are below 10%. The modeled detector reading for the one located at the center of the core is shown in Figure 8. Also shown are estimates generated with smaller, more realistic noise level of 5%. Detector readings are finally input to the reference model where the reactivity components are to be extracted.

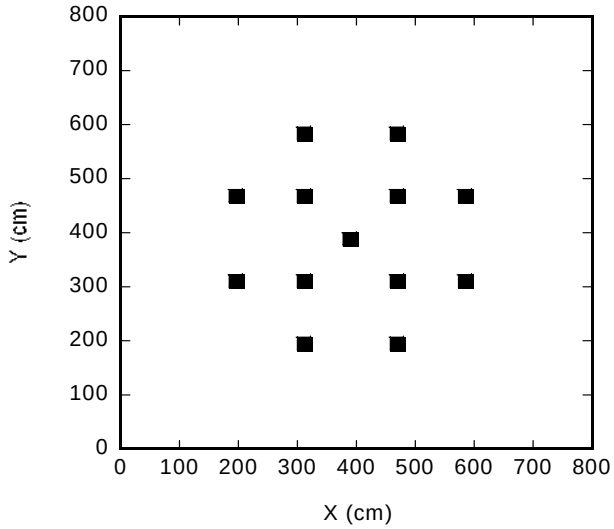


Figure 7: Detector locations.

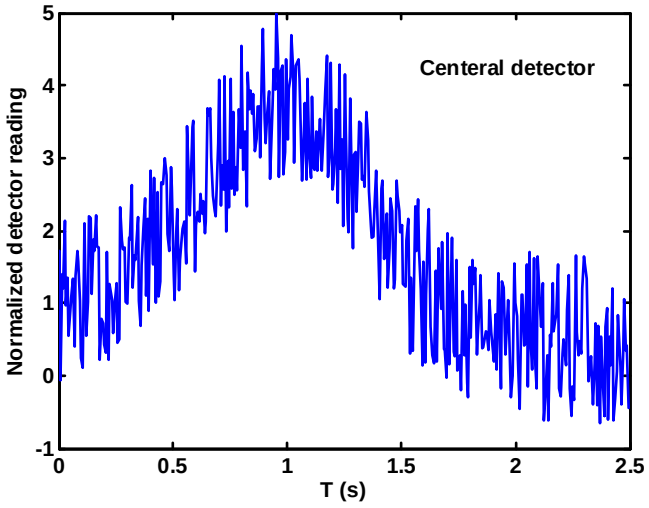


Figure 8: The measured central detector reading.

Applying the extended Kalman filter estimator as described in the previous section to the measured detectors readings in the described benchmark activity, we arrive at the estimated value of the coolant void reactivity during the LOCA. The shut-off rod reactivities are assumed to be as known input to the optimal estimator filter and the coolant void reactivities are left as the unknown parameters to be estimated. Figure 9 and Figure 10 show the coolant void reactivity estimations. Dynamic reactivity shown in Figure 9 is defined as:

$$\rho_{Dynamic}^{void} = \rho_{11}^{void} + \sum_{m=1}^{M} \rho_{1m}^{void} \frac{a_m}{a_1} \qquad \text{Equ. 16}$$

and ρ_{11}^{void} shows the self-coupling of the fundamental mode due to the coolant void perturbation effect. Eight thermal modes have been used in the calculations that were generated as static eigenfunctions (i.e., λ modes) using the finite element method described in the previous section.

The fit is obviously good considering the large amplitude of the measurement noise shown in Figure 8. The initial estimations of the reactivity coefficients have a time delay that is due to the zeroth order approximation of the reactivity variations made in the extended Kalman filter. The filter needs to make a history before updating the reactivity coefficient to a higher value. The delay, and thus the estimated values, is improved after about one second when the reactivity variation is relaxed. As a future research, one may improve the estimations with higher order approximations or assuming a shape function for the reactivity variation if applicable. For the later purpose, it is possible to estimate the reactivity transient with a zeroth order approximation and use the estimated value as the initial shape function of the reactivity variation in another extended Kalman filter. Therefore, with an iterative scheme one can optimize the estimated values of the reactivity coefficients.

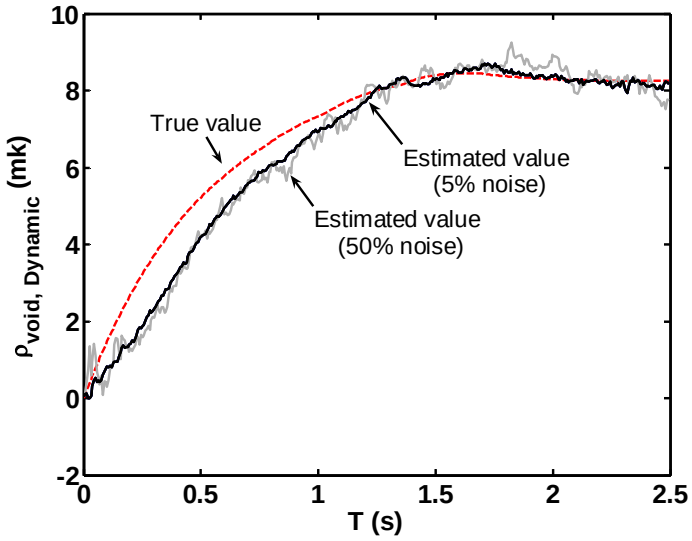


Figure 9: Coolant void net dynamic reactivity.

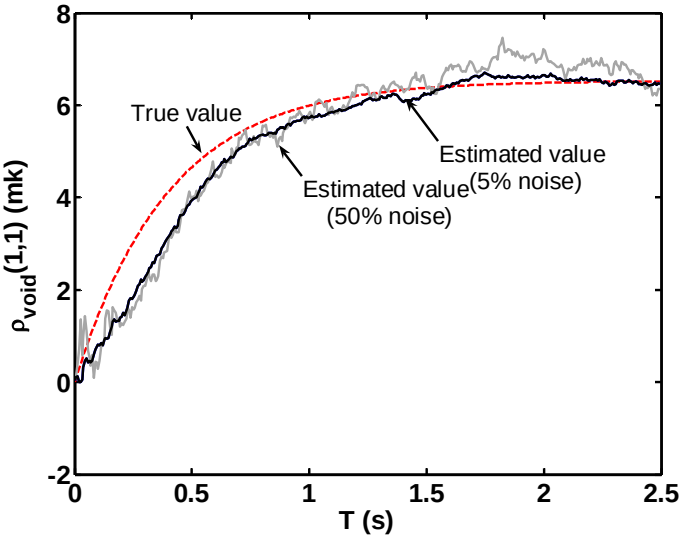


Figure 10: Coolant void self-coupling reactivity of the first mode.

CONCLUSION

An optimal estimation method for CANDU core reactivity estimation has been developed in this study. It was shown that extended Kalman filer can be used to generate robust estimates of the unknown reactivity components such as that of coolant void in a LOCA. The method was applied to a trip event due to transient power increase linked to positive reactivity excursions caused, for example, by a loss of regulation or by a break in the main heat transport circuit. Coolant void reactivity coefficients in the reference model were estimated using extended Kalman filter. Using the date from a virtual system based on modal description of the neutron flux, it was verified that the estimated values are very close to the true values used in the virtual system generating the measurement data.

This paper gives the basic steps to the parameter identification of the reactor core space-time kinetics using the Kalman filter and the followings are left for future research. In the benchmark experiment of this paper, it was assumed that the reactivity coefficients were constant or slowly varying over the process time domain. Therefore, a zeroth order sate dynamic model was appropriate. This approximation can be improved by higher order approximation or iterative estimation of the reactivity coefficients variations. Extensions of this work can be tested to extract other reactivity coefficients such as fuel temperature coefficients in a rapid power pulse to confirm the applicability of the extended Kalman filter.

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