Improving the Accuracy of Neutron Flux Calculations in a CANDU Reactor by Combining the Diffusion Equation with In-Core Detector Readings

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Abstract

The neutron flux in a CANDU nuclear reactor needs to be known both for refuelling decisions and for on-line flux mapping. Traditionally, different methods have been employed for the two purposes. While core-follow calculations have been performed off-line by solving the steady-state diffusion equation, flux mapping calculations have been performed on-line by fitting a linear combination of flux shapes (modes) to the detector readings from 102 vanadium detectors. Of the two, the first method is more accurate, while the second is faster. The present work introduces a new method, which combines the diffusion equation with detector readings to determine the neutron flux. Results for a one-energy-group one-dimensional implementation of the method are presented and shown to be more accurate than those of the diffusion-only method.

1. INTRODUCTION

The neutron flux in CANDU reactors is found using different methods depending on the desired accuracy and execution time. For design and fuel management calculations the flux is calculated off-line by solving the diffusion equation. For on-line reactor regulation, the flux is *mapped* by fitting a linear combination of flux shapes (modes) to detector readings from 102 Vanadium detectors. The flux modes are obtained beforehand as first- and higher-order solutions to the diffusion equation for a core with *time-average* neutronic cross sections. As the name implies, the time-average neutronic cross sections are averages over time of the true time-dependent cross sections whose values change over the life of the reactor, as it is being refuelled. As a consequence, flux mapping cannot reproduce accurately the fine spatial dependence of the real instantaneous neutron flux in the reactor.

The detailed solution of the diffusion equation can, in principle, provide the needed spatial detail. However, in reality, the cell macroscopic cross sections employed by the diffusion code are obtained from a *core follow* calculation. A core follow calculation determines the macroscopic cross sections by performing burnup calculations for each

bundle in a flux equal to the one calculated by a prior diffusion calculation. Parameters such as fuel temperature, coolant temperature and density, position of reactivity devices are determined based solely on calculation results. There is currently no way to account for actual core conditions that differ from the ones predicted by the core-follow calculation. If the true local parameters such as fuel temperature, coolant temperature and density were known, then the instantaneous flux in the reactor could, in principle, be determined accurately. However, in reality it is impossible to obtain readings of these parameters for each fuel bundle. What can be done instead is to use the vanadium-detector readings, which represent the actual reactor flux (consistent with the true local parameters), to improve the results of the diffusion-only calculations.

Hong and Kim[1] proposed to achieve this by augmenting the generalized eigenvalue problem representing the discrete form of the diffusion equation for a multiplicative medium:

$$M\Phi - \frac{1}{k}F\Phi = 0 \tag{1}$$

by the equations representing the detector readings:

$$D\Phi = \phi_D \tag{2}$$

where ϕ_D is the column vector of detector readings. Equation (2) represents the fact that any detector reading is a linear combination of the flux values in its neighbourhood.

The resulting linear system is:

$$\begin{bmatrix} M - \frac{1}{k}F \\ D \end{bmatrix}\Phi = \begin{bmatrix} 0 \\ \phi_D \end{bmatrix} \tag{3}$$

In moving from Eq. (1) to Eq. (3), the original eigenvalue problem was transformed into a fixed-source problem.

The linear system in Eq. (3) is obviously overdetermined. To remove the overdetermination of the system, a simple minimization of the residual in a least-squares sense can be used by calculating:

$$Min\left\{\left(\begin{bmatrix} M - \frac{1}{k}F \\ D \end{bmatrix}\Phi - \begin{bmatrix} 0 \\ \phi_D \end{bmatrix}\right)^T \left(\begin{bmatrix} M - \frac{1}{k}F \\ D \end{bmatrix}\Phi - \begin{bmatrix} 0 \\ \phi_D \end{bmatrix}\right)\right\} \tag{4}$$

It can be easily shown that the minimum is attained for:

$$\left[M^T - \frac{1}{k} F^T \quad D^T \right] \begin{bmatrix} M - \frac{1}{k} F \\ D \end{bmatrix} \Phi = \left[M^T - \frac{1}{k} F^T \quad D^T \right] \begin{bmatrix} 0 \\ \phi_D \end{bmatrix} \tag{5}$$

which can be rewritten as:

$$\left[\left(M^T - \frac{1}{k} F^T \right) \left(M - \frac{1}{k} F \right) + D^T D \right] \Phi = D^T \phi_D \tag{6}$$

The above are called the *normal* equations. In this particular form, they represent a fixed source problem. Additionally, the condition number of the matrix is very large, because of the near singularity of $\left(M - \frac{1}{k} F \right)$. The latter assertion is easily verified by noting that $\left(M - \frac{1}{k} F \right) \Phi = 0$ admits a non-identically-zero solution.

In what follows, an alternative method of combining diffusion and detector readings is presented, one which preserves the eigenvalue nature of the original problem and which results in a linear system with a better conditioning number.

2. METHOD

To combine the diffusion equation with the in-core detector readings while preserving the eigenvalue-eigenvector nature of the problem, a homogeneous form for Eq. (2) is developed.

To do that, Eq. (2) is rewritten in such a way as to show its coefficients explicitly:

$$\sum_{j=1}^{N} d_{ij} \Phi_j = \phi_{Di} \tag{7}$$

In reality, coefficients d_{ij} are zero, with the exception of a few nodes in the vicinity of the detector. To make the above equation homogeneous, both its sides are divided by a linear combination of detector readings, normally those of the nearest neighbours.

$$\frac{\sum_{j=1}^{N} d_{kj} \Phi_j}{\sum_{i=1}^{N_D} c_i \sum_j d_{ij} \Phi_j} = \frac{\phi_{Dk}}{\sum_{i=1}^{N_D} c_i \phi_{Di}} = \alpha_k \tag{8}$$

A homogeneous form equivalent to Eq. (2) is now obtained:

$$\frac{\sum_{j=1}^{N} d_{kj}\Phi_j}{\sum_{i=1}^{N_D} c_i \sum_{j=1}^{N} d_{ij}\Phi_j} = \alpha_k \tag{9}$$

Equation (9) can now be further processed, yielding, in sequence:

$$\sum_{j=1}^{N} d_{kj}\Phi_j = \alpha_k \sum_{i=1}^{N_D} c_i \sum_{j=1}^{N} d_{ij}\Phi_j \tag{10}$$

$$\alpha_k \sum_{i=1}^{N_D} c_i \sum_{j=1}^{N} d_{ij}\Phi_j - \sum_{j=1}^{N} d_{kj}\Phi_j = 0 \tag{11}$$

$$\sum_{j=1}^{N}\sum_{i=1}^{N_D} \alpha_k c_i d_{ij}\Phi_j - \sum_{j=1}^{N} d_{kj}\Phi_j = 0 \tag{12}$$

$$\sum_{j=1}^{N}\left(\sum_{i=1}^{N_D} \alpha_k c_i d_{ij} - d_{kj}\right)\Phi_j = 0 \tag{13}$$

By making the notation:

$$\sum_{i=1}^{N_D} \alpha_k c_i d_{ij} - d_{kj} = \tilde{d}_{kj} \tag{14}$$

equation (14) can be written in the more compact form:

$$\sum_{j=1}^{N} \tilde{d}_{kj}\Phi_j = 0 \tag{15}$$

or, in matrix notation:

$$\tilde{D}\Phi = 0 \tag{16}$$

Combining equations (1) and (16), one can write the overdetermined system:

$$\begin{bmatrix} M \\ \tilde{D} \end{bmatrix}\Phi = \frac{1}{k}\begin{bmatrix} F \\ 0 \end{bmatrix}\Phi \tag{17}$$

The corresponding normal system obtained by applying the least-squares method is:

$$\begin{bmatrix} M^T & \widetilde{D}^T \end{bmatrix} \begin{bmatrix} M \\ \widetilde{D} \end{bmatrix} \Phi = \frac{1}{k} \begin{bmatrix} M^T & \widetilde{D}^T \end{bmatrix} \begin{bmatrix} F \\ 0 \end{bmatrix} \Phi \qquad (18)$$

Equation (18) can be simplified to:

$$\left(M^T M + \widetilde{D}^T \widetilde{D} \right) \Phi = \frac{1}{k} \left(M^T F \right) \Phi \qquad (19)$$

It can easily be seen that eq. (19) is very similar to the diffusion equation (2). The only important difference is the presence of the additional term $\widetilde{D}^T \widetilde{D}$ which incorporates the detector readings. Such an equation is expected to enjoy the same good numerical properties as the diffusion equation, namely a good condition number of the left hand side matrix, the one that needs to be inverted at each inverse power iteration step.

3. CALCULATIONS AND RESULTS

For a preliminary test of its functionality and accuracy, the method was implemented in a one-dimensional one-energy group finite-difference MATLAB program. Calculations were subsequently performed for a simple geometrical model, comparing the errors of the combined method with the errors of the diffusion-only method.

The employed geometrical model consists of twelve regions, each 50-cm long, representing a CANDU reactor with an infinite number of 12-bundle channels. Detectors are assumed to be placed at bundle positions 3 and 10.

For a first test, two separate material-property distributions were considered. The first one has identical bundle properties for all regions, roughly representative of a natural-uranium 37-element fresh-fuel CANDU bundle with a coolant density of 0.85 g/cm^3. The second one has macroscopic cross sections varying from bundle to bundle, corresponding to the coolant density increasing linearly from 0.30 g/cm^3 to 0.85 g/cm^3 between bundle 1 and bundle 12 respectively. This represents roughly a situation where saturated coolant enters the channel at bundle 12 and its void fraction increases linearly towards bundle 1. While this is not a very realistic model, it provides a good test of the method's ability to capture the effect of changes in local parameters on the neutron flux.

The first material-property distribution corresponds to what is usually available for an off-line calculation: bundle properties based on calculated local thermal-hydraulic parameters. The second material-property distribution corresponds to the "true" local thermal-hydraulic parameters that exist in the core, but are unknown to the analyst. The latter information is only implicitly available through the flux detector readings.

Three flux shapes were calculated.

1) Flux shape based on the "true", non-uniform, parameters and calculated using pure diffusion. This served as the reference and was also used to calculate the detector readings.
2) Flux shape based on the uniform properties and calculated using the diffusion-only method.
3) Flux shape based on both the uniform properties and detector readings and calculated using the combined method.

Based on the above shapes, the percent error at each bundle position was calculated as:

$$\varepsilon = \left(\frac{\phi_i}{\phi_i^{ref}} - 1 \right) \times 100 \tag{20}$$

The Root Mean Square (RMS) percent error was calculated using:

$$RMS\% = \frac{\sqrt{\frac{\sum_{i=1}^{12} \left(\phi_i - \phi_i^{ref} \right)^2}{12}}}{\frac{\sum_{i=1}^{12} \phi_i^{ref}}{12}} \times 100 \tag{21}$$

Additionally, the condition number of the system matrix A defined as:

$$cond = \|A\| \cdot \|A^{-1}\| \tag{22}$$

(where $\|A\|$ is the induced Euclidean norm) was also computed for the combined method and for the method described in reference 1.

The condition number of the system matrix was found to be 3.9 for the combined method presented here and 8.3×10^7 for the method presented in reference 1. The results of the flux calculations are presented in Figure 1. The errors are shown in Table 1, and Figure 2.

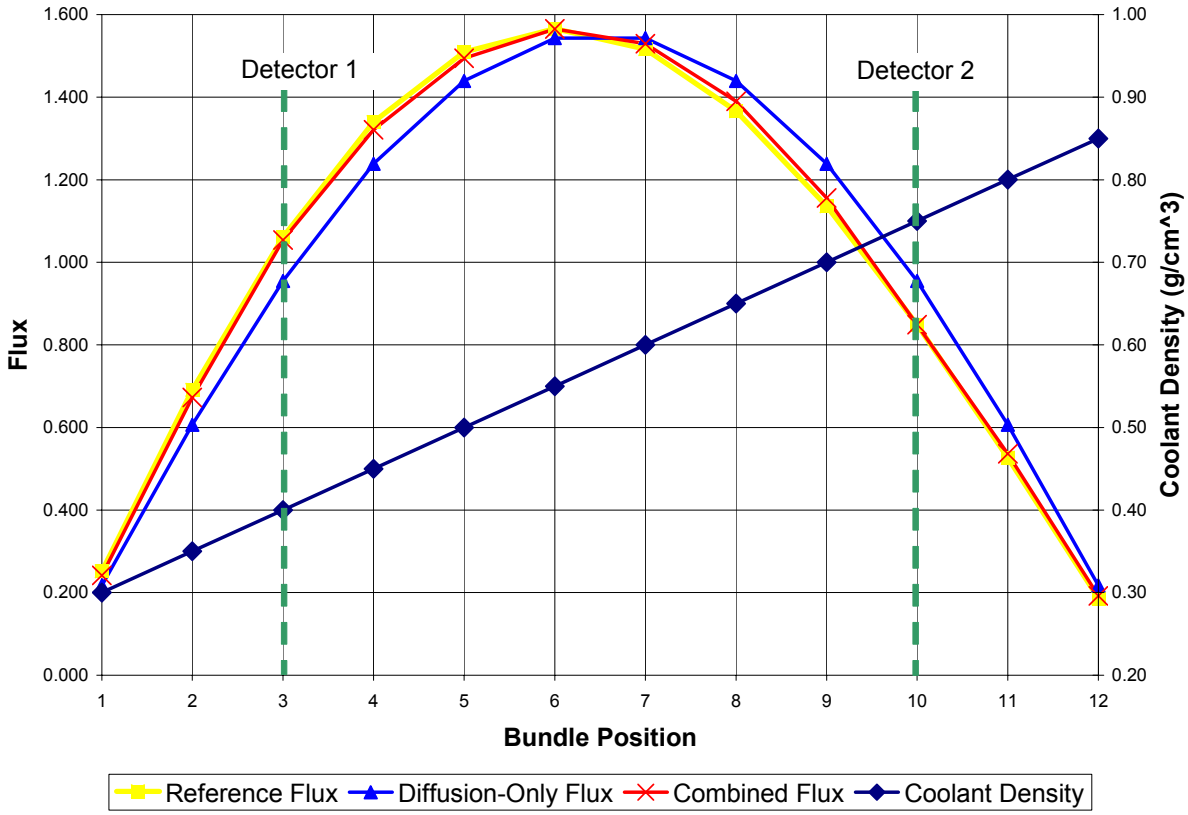

Figure 1: Flux Results for Diffusion-Only and Combined Method

Table 1: Errors for Diffusion-Only and Combined Methods

Method	RMS % error	Max % error
Diffusion-Only	7.7	16.7
Combined	1.4	-4.0

It can be seen in Figure 1 that the combined flux is closer to the reference flux than the one calculated using diffusion only. Table 1 shows the root mean square percent error and the maximum percent error. It can be observed that both are reduced to roughly one quarter when the combined method is used compared to when the diffusion-only method is used. Figure 2 shows the percent error at each bundle location. It can be observed that the error drops significantly at the two detector locations, which is a consequence of the fact that those are the points at which we require the calculated flux to be as close as possible to the (exact) detector readings.

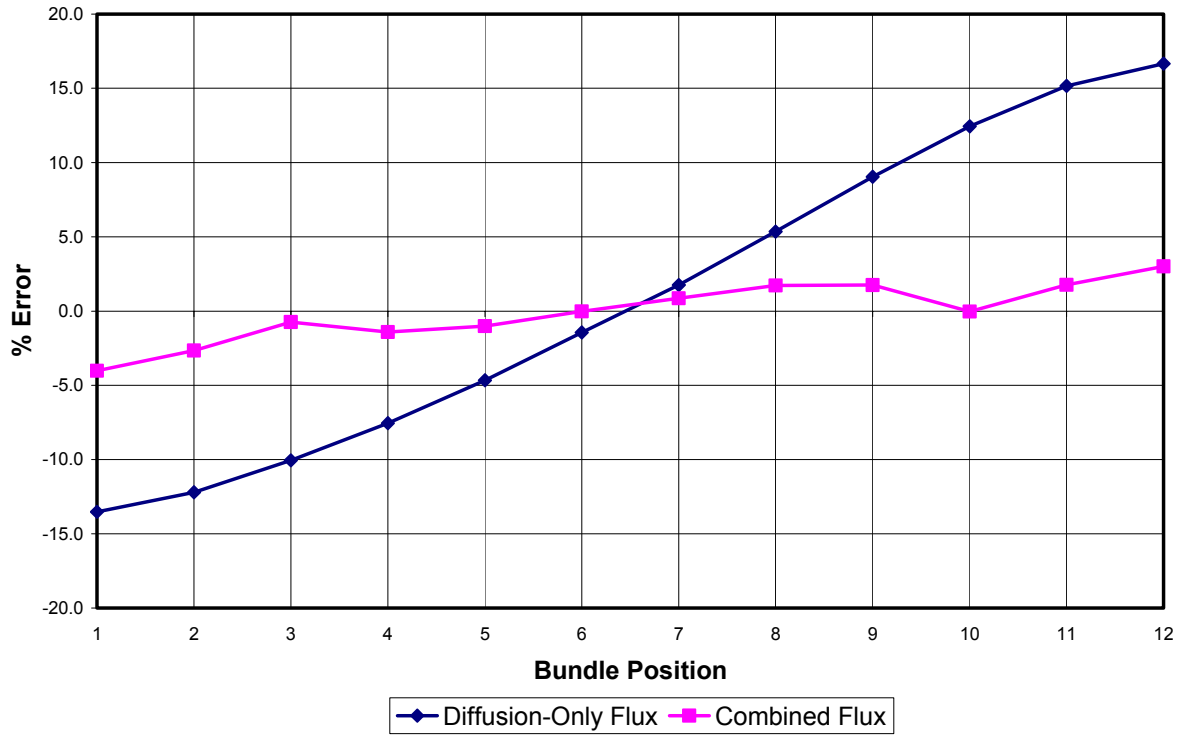

Figure 2: Errors for Diffusion-Only and Combined Methods

To subject the method to a more severe, however unrealistic, test, a second non-uniform parameter distribution was used. In this one, the effect of voiding on cross sections was artificially "amplified" by a factor of 6. That is, for each cross section corresponding to coolant density ρ, the "amplified-effect" macroscopic cross section was defined as:

$$\Sigma_i^{ampl} = \Sigma(\rho_0) + 6 \times \left[\Sigma(\rho_i) - \Sigma(\rho_0)\right] \tag{22}$$

where ρ_0 is the coolant density for the uniform-property case and ρ_i is the coolant density for bundle-position i in the non-uniform-property case.

For this second test, the condition number of the system matrix was found to be 3.9 for the combined method presented here and 8.8×10^7 for the method of reference 1. The flux results for this severe test are shown in Figure 3 and the errors in Table 2.

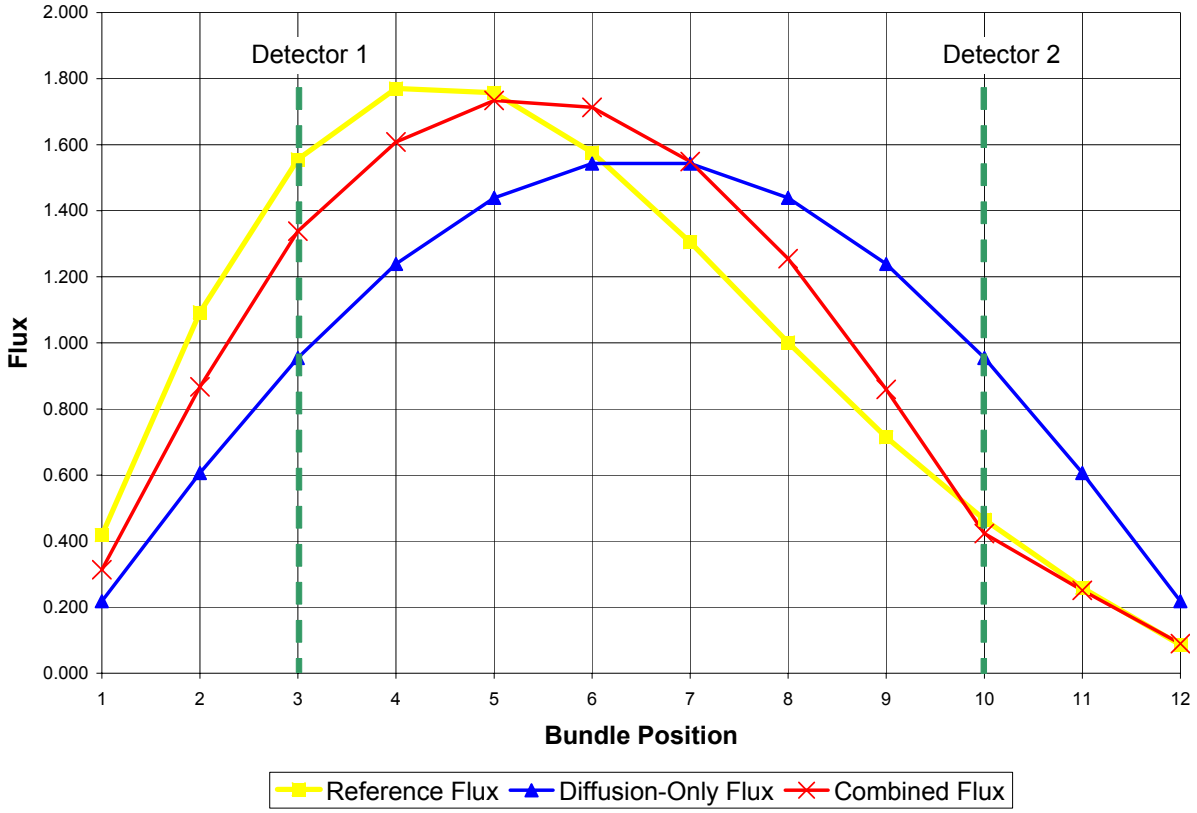

Figure 3: Flux Results for Diffusion-Only and Combined Methods

Table 2: Errors for Diffusion-Only and Combined Methods

Method	RMS % error	Max % error
Diffusion-Only	40.0	154.4
Combined	15.8	25.4

It can be seen both from Figure 3 and Table 2 that the combined method continues to provide significantly better results than the diffusion-only method even for the extreme conditions of the second test. Both the maximum percent error and RMS percent error are reduced to less than half.

4. CONCLUSION

A new method of calculating the neutron flux, one that combines the diffusion equation with detector readings was developed. The method was shown to be more accurate than pure diffusion calculations and to possess very good numerical properties.

REFERENCES

1. I. S. Hong and C. H. Kim, “Advanced Online Flux Mapping of CANDU PHWR by Least-Squares Method” NSE, 150, 299–309, (2005)