A SIMPLE AND FAST CALCULATION METHOD FOR CANDU DECAY HEAT POWER

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Abstract

A method for a simple and fast calculation of CANDU bundle specific decay powers has been developed by employing the Alternating Conditional Expectation (ACE) algorithm. The data samples are generated by performing ORIGEN-S runs. The decay heat power equation developed by such a method is much simpler than that presented in the ANSI/ANS-5.1-2005 standard. And through its application in decay heat power calculations of a CANDU6 reactor, it has been demonstrated that the results agree well with those provided by the ORIGEN-S code which is considered to be one of the most accurate methods for reactor decay heat powers.

Key Words: CANDU, Decay Heat Power, Simple Method, ACE

1. Introduction

Generally, nuclear reactor decay heat power calculations can be done through either solving neutron physics equations with numerical algorithms, or using empirical formulas. The ORIGEN-S [1] code is considered to be one of the applications providing accurate decay heat power results, and it has been verified and validated in Canadian nuclear industry.

So far, the most accurate empirical formula should be the decay heat power equations presented in the ANSI/ANS-5.1-2005 standard [2], which is used widely internationally as decay heat power standards for light water reactors containing ^{235}U, ^{238}U and plutonium. The standard sets forth values for the decay heat power from fission products and ^{239}U and ^{239}Np following shutdown of the reactors. Other applicable decay heat power equations or formulas, such as those described in references [3] and [4], are very simple, but not so accurate compared with the ORIGEN-S code and the standard.

The decay heat power calculations through the ORIGEN-S code are usually very time-consuming if bundle specific decay heat powers are required for the entire reactor core. Moreover, the code is not easy to be incorporated into safety analysis programs where decay heat powers are needed. Although the ANSI/ANS-5.1-2005 standard is accurate and relatively simple, it is intended to be applied in light water reactors. In other words, it is not applicable for bundle specific decay heat power calculations in CANDU-type reactors if bundle operating conditions are beyond those of the standard.

A new method based on the Alternating Conditional Expectation (ACE) algorithm [5] has been used to develop CANDU bundle specific decay heat power formulas or equations.

Section 2 describes the format of the decay heat power equation and a summary of the ACE algorithm. Section 3 describes the ORIGEN-S data samples and ACE data mapping results, and in Section 4 the application of the developed decay heat power equation in a CANDU6 reactor. A discussion of the results and conclusions are summarized in Section 5. The ACE algorithm has been incorporated into CHAOS-W code [6].

2. Simple Decay Heat Power Equation and ACE Algorithm

The empirical decay heat power equation to be used is a simple polynomial equation with the following generic form:

$$P_d = \sum_{k=0}^{M} b_k \left\{ \sum_{j=1}^{n} \left[\sum_{i=0}^{N_j - 1} a_{ji} \left(x_j - C_j \right)^{\mathrm{i}} \right] - D \right\}^k \qquad (1)$$

Where, a --- Polynomial coefficients, total number is n x N_j,
b --- Polynomial coefficients, total number is M,
C --- Known constants, total number is N_j,
D --- Known constant,
P_d --- Decay heat power,
x --- Independent variables, total number is n.

The independent variables adopted in the current equation are the ^{238}U mass percentage of the fuel bundle, operating power, operating period and cooling time (time after shutdown), respectively. A variable ^{238}U mass percentage takes into account the effect of different fuel compositions on decay heat powers. The operating power and period express the irradiation history (operating history) of the bundles. These four variables are considered to be key factors for decay heat powers of the bundles in the simulated CANDU reactor (see Sections 3 and 4). After ACE data mapping is performed based on the data samples presented in Section 3, a specific decay heat power equation is derived in Equation (5).

The polynomial coefficients and constants in equation (1) can be derived by using the ACE algorithm, which was developed by Breiman and Friedman in 1985 [5]. It is for estimating the transformations of a response and a set of predictor variables in multiple regression analysis that produce the maximum linear effect between the (transformed) independent variables and the (transformed) response variable. These transformations can give the data analyst insight into the relationships between these variables so that relationship between them can be best described and non-linear relationships can be uncovered.

In the ACE algorithm, the dependent variable Y and the independent variables X_1, X_2, …, X_p are replaced by function $\theta(Y)$ and $\phi(X_1), \phi(X_2), ..., \phi(X_p)$, which are optimal transformations of system variables and minimize the mean square error (MSE) for a set of N data samples $\{ (X_{ji}, Y_i), j = 1,2,..p \ \ and \ \ i = 1,2,..., N \}$.

$$e^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\theta(Y_i) - \sum_{j=1}^{P}\phi(X_{ji})\right]^2 \quad (2)$$

The objective of the ACE algorithm is to find these transformations through an iterative procedure including inner and outer iteration.

A first-order locally weighted regression algorithm is adopted to achieve data smoothing at every stage of the iterative procedure in order to improve accuracy. In the ACE algorithm, the local linear regression within a specific data window, employed as a data smoothing operation, plays the role of a filter and removes noise or uncertainties in the data samples. The specific data window is expressed by the window factor f_w .

The iterative convergence criterion is defined in terms of differential errors $\varepsilon_c = \left|e^2(\theta,\phi)^t - e^2(\theta,\phi)^{t-1}\right|$ where t is the iteration index. After implementation of the iterative procedure, the independent and dependent variables are represented in the transformations $\theta(Y)$ and $\phi(X_j)_{j=1,2,\ldots,p}$. A polynomial regression analysis is then performed in each transform domain to determine the fitting coefficients for $\theta(Y)$ and $\phi(X_j)$, and finally the overall analytical fit is obtained:

$$Y = \theta^{-1}\left[\sum_{j=1}^{P}\phi(X_j)\right] \quad (3)$$

3. Data Samples and Mapping Results

The present data samples for decay heat powers used in ACE algorithm are derived from the calculation results of ORIGEN-S code based on a CANDU6 reactor core. In total 3000 data samples have been generated in current data mapping by randomly selecting the values of four variables: ^{238}U mass percentage in fuel composition, bundle operating power, bundle operating time and cooling time. A large number of calculations and analyses have been performed to determine which parameters have great effects on the decay heat powers. The value ranges of these four variables are listed in Table 1.

Table 1 Variables Used in Data Samples

Variable	Meaning	Range	Variable Type
X_1	^{238}U mass percentage (U-Mass %)	99.2 ~ 99.8	Independent
X_2	Bundle operating power (kw)	20 ~ 1000	Independent
X_3	Bundle operating period (days)	0.01 ~ 700	Independent
X_4	Cooling time (sec)	0.01 ~ $1x10^{10}$	Independent
Y	Decay Heat Power	To be calculated	Dependent

For each independent variable, 3000 values have been generated randomly. Based on statistical inference theory, the characteristics calculated from the sample are close to the characteristic of the population, provided that the sample has been selected randomly. Thus these 3000 sets of independent variables were used in 3000 ORIGEN-S runs to create 3000 decay heat power values. Then these 3000 data samples have been generated by performing data conversions in equation (4) where ln means natural logarithm.

$$
\begin{aligned}
X_1 &= 0.0 + \frac{1}{(99.8 - 99.2)} \times (X_1 - 99.2) \\
X_2 &= X_2 / 1000.0 \\
X_3 &= \ln(X_3 \times 3600 \times 24) \\
X_4 &= \ln(X_4) \\
Y &= \frac{Y}{X_2} \times 100\%
\end{aligned}
\tag{4}
$$

These data samples are shown in Figures 1, including the relationships between independent variables and decay heat power (dependent variable).

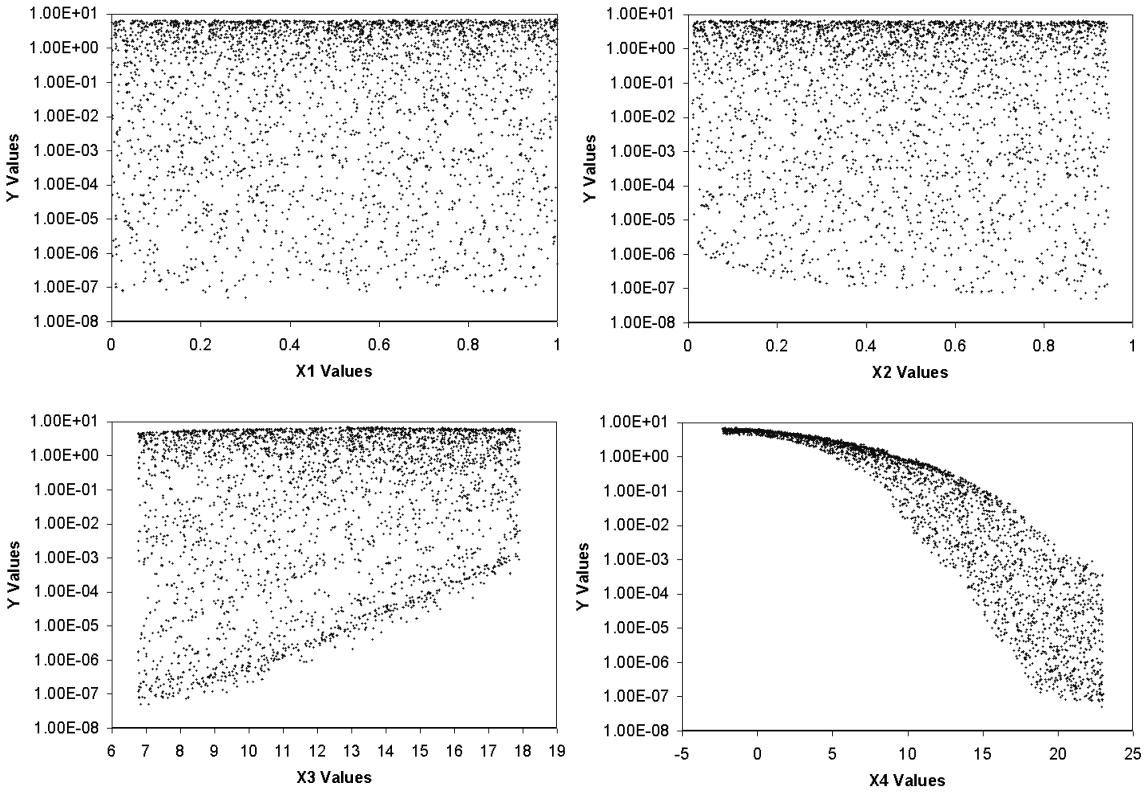

Figure 1 3000 Data Samples Derived from Calculation Results of ORIGEN-S

Figure 1 shows that the relationships between the dependent variable and independent variables are highly non-linear. It would be difficult to find the functional form by using general data mapping methods rather than ACE algorithm described in Section 3. After having the data samples, the selection of data for mapping completely depends on what is expected for decay heat powers. For example, if decay heat power calculation focuses on the first 720 seconds (X_4=6.58) after shutdown, the derived equation coefficients of equation (1) by ACE algorithm are presented in Table 2.

Shown in Figure 2 are ϕ and θ values for independent and dependent variables, where 1167 of the 3000 data samples were adopted in ACE data mapping due to X_4 being less

than 6.58. These curves indicate the variations of decay heat power versus the fuel composition, operating power, operating period and cooling time.

From Table 2, it can be seen that the derived decay heat power polynomial is simple and the highest power is only 14. In Section 4, application of the ACE decay heat power equation will show that the computation based this equation is accurate and fast.

Therefore, starting with the generic equation (1), the derived decay heat power equation is expressed as follows:

$$P_d = \sum_{k=0}^{13} b_k \left\{ \sum_{j=1}^{4} \left[\sum_{i=0}^{N_j - 1} a_{ji} \left(x_j - C_j \right)^{\mathrm{i}} \right] - D \right\}^k \tag{5}$$

Where, $x_1 = \dfrac{(X_1 - 99.2)}{(99.8 - 99.2)}$, $x_2 = X_2 \times 0.0001$, $x_3 = \ln(X_3 \times 86400)$ and $x_4 = \ln(X_4)$

The four independent variables adopted in the current equation are:

X_1 = ^{238}U mass percentage (U-Mass %),
X_2 = Bundle operating power (kw),
X_3 = Bundle operating period (days), and
X_4 = Cooling time after shutdown (sec)

The equation coefficients a_{ji}, C_j, b_k, and D are presented in Table 2. The applicable ranges for variables X_1, X_2, X_3 and X_4 are described in Table 1. The calculated decay heat power P_d is a percentage of the bundle operating power.

4. Application of Decay Heat Power Equation

The derived decay heat power equation was applied for a CANDU6 reactor containing 4560 fuel bundles that experienced different real operating power histories. The reactor total decay heat powers are shown in Figure 3, where the calculation results obtained from the ORIGEN-S code and the ANS-5.1-2005 standard are also given. Obviously, the results of the developed equation agree well with those of the ORIGEN-S code. The error analysis results based on 4560 fuel bundles, 380 channels and the entire core are presented in Figure 4. Compared with the decay heat powers calculated by the ORIGEN-S code, the maximum relative error of total core decay heat power for ACE-based equation is -5.1% (3.6% for ANS-5.1-2005), the maximum average relative error of channel decay heat powers for ACE-based equation is 6.4% (5.6% for ANS-5.1-2005), the maximum average relative error of bundle decay heat powers for ACE-based equation is 6.9% (17.6% for ANS-5.1-2005).

Figure 4 also demonstrates that ACE-based decay heat power equation can give bundle and channel powers closer to those of the ORIGEN-S code than the ANS-5.1-2005 standard, because the effects of fuel compositions on decay heat powers can be easily taken into account by the ACE-base equation. The ORIGEN-S calculation results show that the decay heat power error caused by different fuel types can be up to 4.5% for the same irradiation history and cooling time.

Table 2 a_{ji}, C_j, b_k and D of equation (1) for cooling time less than 720 seconds

i,k	a_{1i}	a_{2i}	a_{3i}	a_{4i}	b_k
0	-1.05491e-02	-8.77673e-03	2.17019e-01	2.25228e-02	**4.28748e+00**
1	1.17640e-01	-1.10987e-01	8.27456e-02	-4.14214e-01	**1.54046e+00**
2	9.16278e-02	6.47638e-02	-2.03499e-02	-1.26431e-02	**-6.45168e-02**
3	-3.17174e-01	3.18603e-01	-1.19607e-03	6.43770e-03	**-1.09439e-01**
4	-2.31351e-01	-1.50496e+00	-1.93268e-04	-3.48949e-04	**5.60716e-02**
5	5.55761e+00	-1.01198e+01	2.12075e-04	2.67513e-04	**1.93984e-01**
6	1.32024e+01	5.79833e+01	-3.14884e-06	-2.73456e-06	**-3.24893e-03**
7	-3.51900e+01	1.08301e+02	-2.00584e-05	-8.17733e-05	**-1.55671e-01**
8	-1.08335e+02	-7.60385e+02	2.67086e-06	3.89018e-06	**-3.43962e-02**
9	1.14931e+02	-5.49685e+02	1.07540e-06	7.27846e-06	**5.88089e-02**
10	3.61325e+02	5.36512e+03	-1.89960e-07	-2.75205e-07	**2.52415e-02**
11	-1.38829e+02	1.16880e+03	-2.85680e-08	-2.97529e-07	**-5.98112e-03**
12	-4.35121e+02	-1.95654e+04	5.46303e-09	6.06208e-09	**-4.92484e-03**
13		-6.90444e+02	2.97081e-10	4.68976e-09	**-7.26458e-04**
14		2.85048e+04	-5.79407e-11		
	C_1	C_2	C_3	C_4	**D**
	0. 5024298	**0. 4797158**	**12.2619320**	**2.2822629**	**0.0061580**

The calculations to obtain the above results were performed on a computer with CPU speed 1.0 GHz. For the entire core (4560 bundles) and 98 cooling time intervals (totally 62208000 seconds i.e. 720 days), it takes 18 hours, 3.6 hours and 0.22 hours respectively to complete the calculations by using the ORIGEN-S code, the ANS-5.1-2005 standard and ACE-based equation. The computation speed rate is about 82:16:1. It can be concluded that ACE-based decay heat power calculation method is much faster. When getting the coefficients for the equation (5), no more than ten minutes computational time is needed for 1167 data samples. And such calculations are done once for specific fuel types characterized by the given data samples.

5. Summary and Conclusion

The ACE method can be of practical value in determining CANDU bundle specific decay heat powers, especially when accurate and fast calculations are needed. The derived simple decay heat power equation can be easily applied alone or incorporated into a system analysis code in which decay heat power is coupled with thermal hydraulic simulations. For different calculation requirements or conditions, it is only necessary to provide a set of coefficients and constants as described above by using this methodology.

The data and results presented in Section 4 demonstrate that the ACE method is reliable and results in a simple decay heat power equation. In fact, such a methodology can be applied to any parameter range due to the powerful data mapping functions of ACE algorithm, which has been being used widely in areas of data mapping since 1985. According to the work done in this paper, there should be no technical obstacles for its application in developing CANDU decay heat power formulas.

To obtain the coefficients and constants in simple calculation equation (1) by using ACE algorithm and make the data mapping error minimum, having a set of reasonable fitting data samples is important. The data samples presented in Section 3 are selected randomly. Although random sampling provides some reassurance that the data sample expresses the characteristics of the population, the data sample is not optimal. Other methods such as genetic algorithm (GA) could be adopted to optimize the data samples.

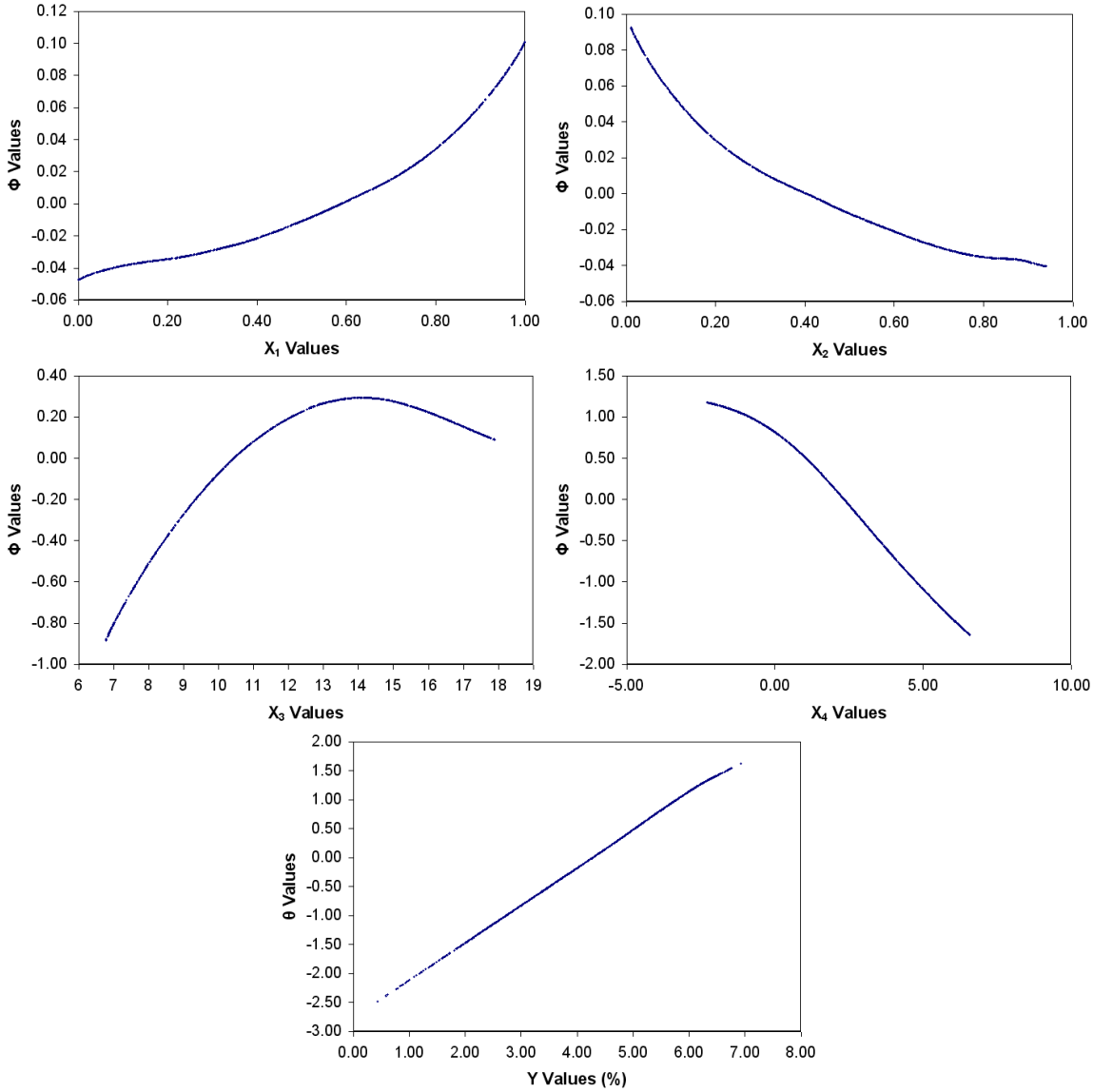

Figure 2 ϕ and θ values for independent and dependent variables

If a CANDU-6 core has different types of fuel bundles (e.g. 37-element fuel and CANFLEX fuel) and only one decay heat power equation derived from ACE algorithm has to be used for all these fuel types, then the data samples used for ACE mapping should contain enough information to reflect the effects of fuel types on decay powers. For example, if the fuel bundle geometric parameter is justified to have significant effects on decay powers, it should be considered to be another independent variable in the

equation (1). An alternative is to develop two decay power equations using the ACE algorithm for 37 element and CANFLEX fuels respectively whenever the data sample can be available through other calculation codes like ORIGEN-S. Due to lack of decay power data samples relevant to CANFLEX fuel bundles, the decay power equation developed using ACE algorithm in the current CHAOS-W [6] is not suitable for CANFLEX fuel bundles. However the methodology presented in this paper is applicable.

The ACE algorithm is generic and can be applied to other reactors such as CANDU-9, ACR, PWR, etc, if the data mapping is performed for these types of reactors.

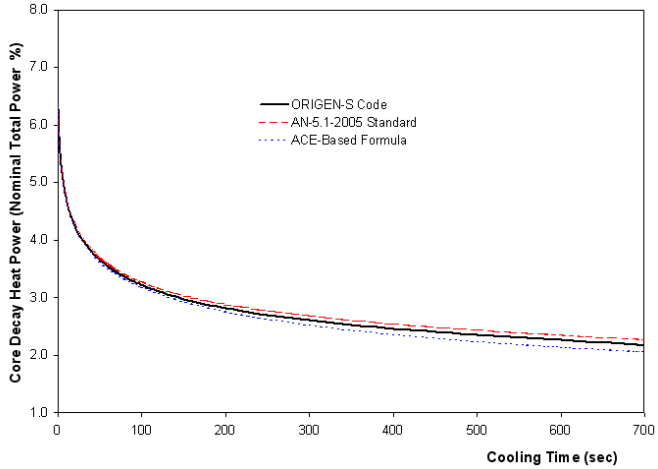

Figure 3 Decay Heat Powers for Cooling Time 720 Seconds

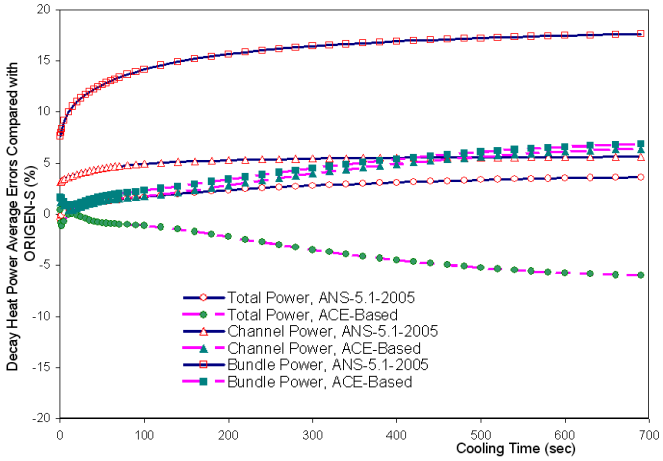

Figure 4 Result Errors between ORIGEN-S and ACE Data Mapping

6. Reference

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