MATLAB/SIMULINK Model of CANDU Reactor for Control Studies

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Abstract

In this paper a MATLAB/SIMULINK model is developed for a CANDU type reactor. The data for the reactor are taken from an Indian PHWR, which is very similar to CANDU in its design. Among the different feedback mechanisms in the core of the reactor, only xenon has been considered which plays an important role in spatial oscillations. The model is verified under closed loop scenarios with simple PI controller. The results of the simulation show that this model can be used for controller design and simulation of the reactor systems. Adding models of the other components of a CANDU reactor would ultimately result in a complete model of CANDU plant in MATLAB/SIMULINK.

Keywords: Reactor Simulation, MATLAB/SIMULINK, Nodal Model.

1 Introduction

Nuclear reactors are complicated systems and very difficult to model mathematically. To describe the behavior of a nuclear reactor accurately, one has to solve neutron transport equations, or the multi-group diffusion equation. Numerous computer codes have been developed to solve these equations for different configurations of core geometry, materials, fuel, moderator, coolant, etc. Proficiency in these codes usually takes a long time and preparation for each run requires a large data file.

Although these kind of codes are extremely valuable in reactor designs and physics calculations, they are neither efficient nor easy to use for controller design and dynamic studies. The reason is that most of these codes are based on solving partial differential equations, while most of control theory is based on ordinary differential equations. Control system design based on frequency domain methods, or state space (time domain) approaches require the system to be represented in terms of ordinary differential equations.

MATLAB/SIMULINK is a powerful numerics and simulations software package that is widely accepted and used in academia and in industries. It is a great tool to study the performance of the control systems and analyze their behavior. Implementing a reactor model in MATLAB/SIMULINK

can help control engineers to investigate the behavior of the reactor and its interaction with control systems, as well as helping them with design of new controllers for the reactor and verifying their performance. Although the results obtained with this type of models may not be exact, they are good enough to serve as the first step in modeling/simulation process. Once the preliminary results are obtained, the more accurate codes can be utilized to zoom into more detailed aspects of the problem and trim the parameters of the controller.

This model is aimed at spatial transients and control. For this purpose the thermal-hydraulic feedback effects play a small role and can be neglected [1]. In the model that follows, among different feedback mechanism that exist in a nuclear reactor, only xenon feedback will be considered.

In Section 2 the mathematical model of the reactor is described and some important concepts are introduced. Then, in Section 3 the steady state equations will be solved which give the initial condition for solving the differential equations. Later, in Section 4 the structure of the Simulink model is examined and then some simulation examples will be presented.

2 Mathematical Model

The mathematical model of the reactor used for this study is the model of a 500 MWe Pressurized Heavy Water Reactor (PHWR), which is a replica of CANDU 6. The model is developed by Dr. A. P. Tiwari and his colleagues [1–3]. This model is a *nodal model* obtained from the two-group diffusion equation along with some simplifying assumptions. Each node is considered to be relatively large here; physically it is assumed that each zone of the reactor is one node of the model.

The central point of this model is a quantity called coupling coefficient between zone i and zone j. The coupling coefficient between zones i and j is defined as [1]

$$\alpha_{ij} = \begin{cases} \dfrac{DvlA_{ij}}{d_{ij}V_i} & i \neq j \\ 0 & i = j \end{cases}, \tag{1}$$

where D is the diffusion coefficient; v is the speed of thermal neutron; l is the prompt neutron lifetime; A_{ij} is the interface area between zones i and j; V_i is the volume of zone i; and d_{ij} is the center-to-center distance of zones i and j. The coupling coefficient between two non-neighboring zones is assumed to be zero.

The final form of dynamic equations expressing the behavior of a PHWR core is

$$\frac{\mathrm{d}P_i}{\mathrm{d}t} = \left(\rho_i - \beta - \frac{\bar{\sigma}_{\mathrm{X}i}X_i}{\Sigma_{\mathrm{a}i}} - \sum_{j=1}^{Z}\alpha_{ij}\right)\frac{P_i}{l} + \sum_{j=1}^{Z}\alpha_{ji}\frac{P_j}{l} + \sum_{h=1}^{m_d}\lambda_h C_{ih} \tag{2}$$

$$\frac{\mathrm{d}C_{ih}}{\mathrm{d}t} = \frac{\beta_h}{l}P_i - \lambda_h C_{ih} \tag{3}$$

$$\frac{\mathrm{d}I_i}{\mathrm{d}t} = \gamma_{\mathrm{I}}\Sigma_{\mathrm{f}i}P_i - \lambda_{\mathrm{I}}I_i \tag{4}$$

$$\frac{\mathrm{d}X_i}{\mathrm{d}t} = \gamma_{\mathrm{X}}\Sigma_{\mathrm{f}i}P_i + \lambda_{\mathrm{I}}I_i - \left(\lambda_{\mathrm{X}} + \bar{\sigma}_{\mathrm{X}i}P_i\right)X_i \tag{5}$$

where $i = 1, 2, \cdots, Z$ and $h = 1, 2, \cdots, m_d$ and Z is the number of the zones (for CANDU or PHWR, $Z = 14$) and m_d is the number of the delayed neutron precursors (usually $m_d = 6$).

Table 1: *Volumes of the zones of* 500 MWe PHWR *reactor*

zone	1,2,6,7,8,9,13,14	3,5,10,12	4,11	Total
Volume (m^3)	14.5760	14.0902	8.7456	190.46

Table 2: *Parameters of the* 500 MWe PHWR *for all zones*

$l = 790\,\mu s$	$E_f = 32\,pJ$	$\Sigma_a = 0.32341\,m^{-1}$	$\Sigma_f = 0.1262\,m^{-1}$
$\lambda_I = 28.78 \times 10^{-6}\,s^{-1}$	$\lambda_X = 21 \times 10^{-6}\,s^{-1}$	$\sigma_X = 1.2 \times 10^{-22}\,m^2$	$\nu = 2.418$

3 Steady State Analysis

When the reactor is critical and at equilibrium, the power level in zones doesn't change and other variables (delayed neutron precursors, iodine and xenon) are at equilibrium with power in each zone. Because of the equilibrium condition, all the derivative terms on the LHS of equations (2)–(5) vanish.

Applying this condition on delayed neutron precursors, iodine, and xenon equations, their equilibrium values are calculated as

$$C^*_{ih} = \frac{\beta_h P^*_i}{\lambda_h l}\,; \tag{6}$$

$$I^*_i = \frac{\Sigma_{fi}\gamma_I P^*_i}{\lambda_I}\,; \tag{7}$$

$$X^*_i = \frac{\Sigma_{fi}(\gamma_I + \gamma_X)P^*_i}{\lambda_X + \bar{\sigma}_{Xi}P^*_i}\,, \tag{8}$$

where P^*_i is the equilibrium power in zone i. Considering the above equations, the equilibrium equation for power in zone i becomes

$$\left(\rho^*_i - \frac{\bar{\sigma}_{Xi}X^*_i}{\Sigma_{ai}} - \sum_{j=1}^{Z}\alpha_{ij}\right)P^*_i + \sum_{j=1}^{Z}\alpha_{ji}P^*_j = 0\,. \tag{9}$$

For 500 MWe PHWR, the volumes of the zones are given in Table 1. Note that these volumes do not agree with what is reported in [1, 2, 4]. Especially, the volumes of zones 3, 5, 10, and 12 are about 3 m^3 different from what one would calculate using geometrical dimensions of the core. Table 2 gives all the other parameters of the reactor for all zones. The value of ν is for fresh fuel when there is still no plutonium buildup in the reactor.

To calculate the steady state power *power method* can be used. This method is generally used to calculate the steady state solution of multi-group diffusion equation [5]. The basic idea of this method is to separate the terms of neutron production and neutron loss, and setup an iterative scheme to solve the resulting equation.

The reactivity ρ_i is defined as

$$\rho_i = \frac{k_{\infty i} - 1}{k_{\infty i}}\,, \tag{10}$$

where $k_{\infty i}$ is the infinite medium multiplication factor given as

$$k_{\infty i} = \frac{\nu\Sigma_{fi}}{\Sigma_{ai}}\,. \tag{11}$$

To make the reactor critical, or to measure its distance from criticality, ν is divided by k_{eff}. The reactivity in terms of infinite medium multiplication factor considering k_{eff}, will be

$$\rho_i = 1 - \frac{k_{\text{eff}}\Sigma_{\text{a}i}}{\nu\Sigma_{\text{f}i}}\,. \tag{12}$$

Substituting this definition for ρ_i in Eqn. (9), gives (to keep the notation tidy, the * superscript is omitted in the following equations)

$$\left(1 - \frac{k_{\text{eff}}\Sigma_{\text{a}i}}{\nu\Sigma_{\text{f}i}} - \frac{\bar{\sigma}_{\text{X}i}}{\Sigma_{\text{a}i}}\frac{\Sigma_{\text{f}i}(\gamma_{\text{I}}+\gamma_{\text{X}})P_i}{\lambda_X + \bar{\sigma}_{\text{X}i}P_i} - \sum_{j=1}^{Z}\alpha_{ij}\right)P_i + \sum_{j=1}^{Z}\alpha_{ji}P_j = 0\,. \tag{13}$$

By keeping the terms with positive sign in one side, and moving rest of the terms to the other side, Eqn. (13) can be written as

$$P_i + \sum_{j=1}^{Z}\alpha_{ji}P_j = \left(\frac{k_{\text{eff}}\Sigma_{\text{a}i}}{\nu\Sigma_{\text{f}i}} + \frac{\bar{\sigma}_{\text{X}i}}{\Sigma_{\text{a}i}}\frac{\Sigma_{\text{f}i}(\gamma_{\text{I}}+\gamma_{\text{X}})P_i}{\lambda_X + \bar{\sigma}_{\text{X}i}P_i} + \sum_{j=1}^{Z}\alpha_{ij}\right)P_i\,. \tag{14}$$

The unknowns in this equation are k_{eff} and the zonal powers P_i. To solve for these unknowns an iterative scheme is set up and then the solution is started from an initial guess and it is run until the scheme converges to the solution within a predefined tolerance. The iteration scheme is

$$P_i^{(n)} + \sum_{j=1}^{Z}\alpha_{ji}P_j^{(n)} = \left(\frac{k_{\text{eff}}^{(n)}\Sigma_{\text{a}i}}{\nu\Sigma_{\text{f}i}} + \frac{\bar{\sigma}_{\text{X}i}}{\Sigma_{\text{a}i}}\frac{\Sigma_{\text{f}i}(\gamma_{\text{I}}+\gamma_{\text{X}})P_i^{(n+1)}}{\lambda_X + \bar{\sigma}_{\text{X}i}P_i^{(n+1)}} + \sum_{j=1}^{Z}\alpha_{ij}\right)P_i^{(n+1)} \tag{15}$$

$$k_{\text{eff}}^{(n+1)} = k_{\text{eff}}^{(n)}\frac{\sum_{j=1}^{Z}P_j^{(n+1)}}{\sum_{j=1}^{Z}P_j^{(n)}}\,. \tag{16}$$

The second equation is used to update k_{eff} at each step. In the power equation, all the quantities at n-th iteration are known. RHS of power equation is nonlinear in terms of $P_i^{(n+1)}$. To solve it, the following variables at each iteration are defined:

$$a_i^{(n)} \triangleq P_i^{(n)} + \sum_{j=1}^{Z}\alpha_{ji}P_j^{(n)} \tag{17}$$

$$b_i^{(n)} \triangleq \frac{k_{\text{eff}}^{(n)}\Sigma_{\text{a}i}}{\nu\Sigma_{\text{f}i}} + \sum_{j=1}^{Z}\alpha_{ij} \tag{18}$$

$$c_i \triangleq \frac{\bar{\sigma}_{\text{X}i}\Sigma_{\text{f}i}(\gamma_{\text{I}}+\gamma_{\text{X}})}{\Sigma_{\text{a}i}} \tag{19}$$

Substituting $a_i^{(n)}$, $b_i^{(n)}$, and c_i in Eqn. (15) simplifies the form of this equation to

$$a_i^{(n)} = \left(b_i^{(n)} + \frac{c_i P_i^{(n+1)}}{\lambda_{\text{X}} + \bar{\sigma}_{\text{X}i}P_i^{(n+1)}}\right)P_i^{(n+1)}\,, \tag{20}$$

and the latter is simplified furthermore to get a quadratic equation in $P_i^{(n+1)}$:

$$\left(c_i + \bar{\sigma}_{\mathrm{X}i} b_i^{(n)}\right) P_i^{(n+1)^2} + \left(\lambda_{\mathrm{X}} b_i^{(n)} - \bar{\sigma}_{\mathrm{X}i} a_i^{(n)}\right) P_i^{(n+1)} - \lambda_{\mathrm{X}} a_i^{(n)} = 0\,. \tag{21}$$

It is easy to see that this equation will have two real roots with different signs, because

$$P_{i,1}^{(n+1)} P_{i,2}^{(n+1)} = -\frac{\lambda_{\mathrm{X}} a_i^{(n)}}{c_i + \bar{\sigma}_{\mathrm{X}i} b_i^{(n)}} < 0\,, \tag{22}$$

where $P_{i,1}^{(n+1)}$ and $P_{i,2}^{(n+1)}$ are the two roots of the equation. Only positive root is meaningful and the negative root is discarded at each iteration.

Two tolerances have been defined, and the iteration will go on until both are fulfilled. The first tolerance is on k_{eff} and is defined as

$$\mathrm{TOL}_k = \left| \frac{k_{\mathrm{eff}}^{(n+1)} - k_{\mathrm{eff}}^{(n)}}{k_{\mathrm{eff}}^{(n)}} \right| = \left| \frac{k_{\mathrm{eff}}^{(n+1)}}{k_{\mathrm{eff}}^{(n)}} - 1 \right| < \epsilon_k\,, \tag{23}$$

and the second tolerance is defined on power vector as

$$\mathrm{TOL}_P = \max_i \left| \frac{P_i^{(n+1)} - P_i^{(n)}}{P_i^{(n)}} \right| = \left\| \frac{\boldsymbol{P}^{(n+1)} - \boldsymbol{P}^{(n)}}{\boldsymbol{P}^{(n+1)}} \right\|_\infty < \epsilon_P\,. \tag{24}$$

In Eqn. (24), the division of vectors is meant to be element-wise division.

For the initial guess it is assumed that all the zones produce the same power, i.e.,

$$P_i^{(0)} = \frac{P_{\mathrm{b}}}{Z} \qquad i = 1, 2, \ldots, Z. \tag{25}$$

where P_b is the bulk or global power of the reactor. This is obviously not the only choice, and the iterations can be started from any power level. To insure that the iterations converge to the correct solution, the solution from the first run is considered as initial guess and the problem is solved again. This procedure is repeated several times to make sure that we have obtained the correct solution within the prescribed tolerances. The solution is reported in Table 3. The vector $\boldsymbol{P}$ is the eigenvector of the reactor. All the solutions to the steady state power problem should lie along this eigenvector. So, once the problem is solved for, say 100%FP, it can be simply scaled to get the steady state power at any lower levels.

4 Transient Analysis

For the transient analysis of the system numerical integration techniques have to be applied, because symbolic solution to the set of differential equations is out of question due to the complexity and size of the problem. A Simulink model is developed which can be used to numerically integrate the differential equations using MATLAB's different solvers.

Table 3: *Zonal powers and reactivities of* 500 MWe PHWR *(all powers in* MW*),* $\epsilon_P = 10^{-8}$, $\epsilon_k = 10^{-5}$

k_{eff}	0.9203179	P_1^*	137.7572	P_8^*	137.7572
k_∞	1.0252372	P_2^*	137.7572	P_9^*	137.7572
ρ(mk)	24.616	P_3^*	133.1620	P_{10}^*	133.1620
		P_4^*	82.6473	P_{11}^*	82.6473
		P_5^*	133.1620	P_{12}^*	133.1620
		P_6^*	137.7572	P_{13}^*	137.7572
		P_7^*	137.7572	P_{14}^*	137.7572

4.1 Description of the Simulink Model

To build the Simulink model Eqn. (2) can be rewritten as

$$\frac{\mathrm{d}P_i}{\mathrm{d}t} = \left(\rho_i - \beta - \frac{\bar{\sigma}_{\mathrm{X}i} X_i}{\Sigma_{\mathrm{a}i}}\right)\frac{P_i}{l} + \sum_{h=1}^{m_d} \lambda_h C_{ih} - \sum_{j=1}^{Z} \alpha_{ij}\frac{P_i}{l} + \sum_{j=1}^{Z} \alpha_{ji}\frac{P_j}{l}\,. \tag{26}$$

If we ignore the last two terms on the RHS of the equation, the rest of the equation looks exactly like a point kinetics model with xenon feedback. The last two terms are just modeling the coupling between the zones.

For each zone, a point kinetics model is developed and they are connected to each other through the coupling matrix. The building block of each zone is shown in Fig. 1. The interconnection between different zones and adding the coupling term is shown in Fig. 2.

The model is masked and the initial conditions of all the integrators are calculated using the method described in Section 3 in the initialization code of the masked block. The outputs of the model, are 14-dimensional vectors $\boldsymbol{P}$ and $\boldsymbol{X}$. Both these vectors are normalized by their equilibrium values. This provision makes the model closer to the control and instrumentation algorithms used in CANDU reactors. The closed loop system which includes the reactor, a 14-dimensional PI controller and demand power generator is shown in Fig. 3.

4.2 Closed Loop Simulation

In this section the results of two transient simulation on the reactor will be presented. These simulation are merely to show the effectiveness of the model, and don't exactly represent the behavior of the CANDU core in response to these transients. Liquid zone controllers play a very important role during transients and they haven't been modeled here. The controller is supposed to be a simple PI controller, whereas in reality it is much more complex than a bare PI controller.

First simulation models a disturbance in one of the zones, namely zone 4. The demand power of zone 4 starts from steady state value (100%) and then it is reduced to 75% in one hour and immediately it is increased to 100% in one hour. It has been assumed that all other zones have a constant demand power.

The power P_4, reactivity ρ_4, and xenon concentration X_4 are plotted in Fig. 4. Power of zone 4 is following the demand power very closely. The reactivity is varying in order to enable the reactor to follow the demand power. The variation of xenon has a much longer time constant and take a long time until the xenon concentration settles at the new equilibrium value. The overshoot of

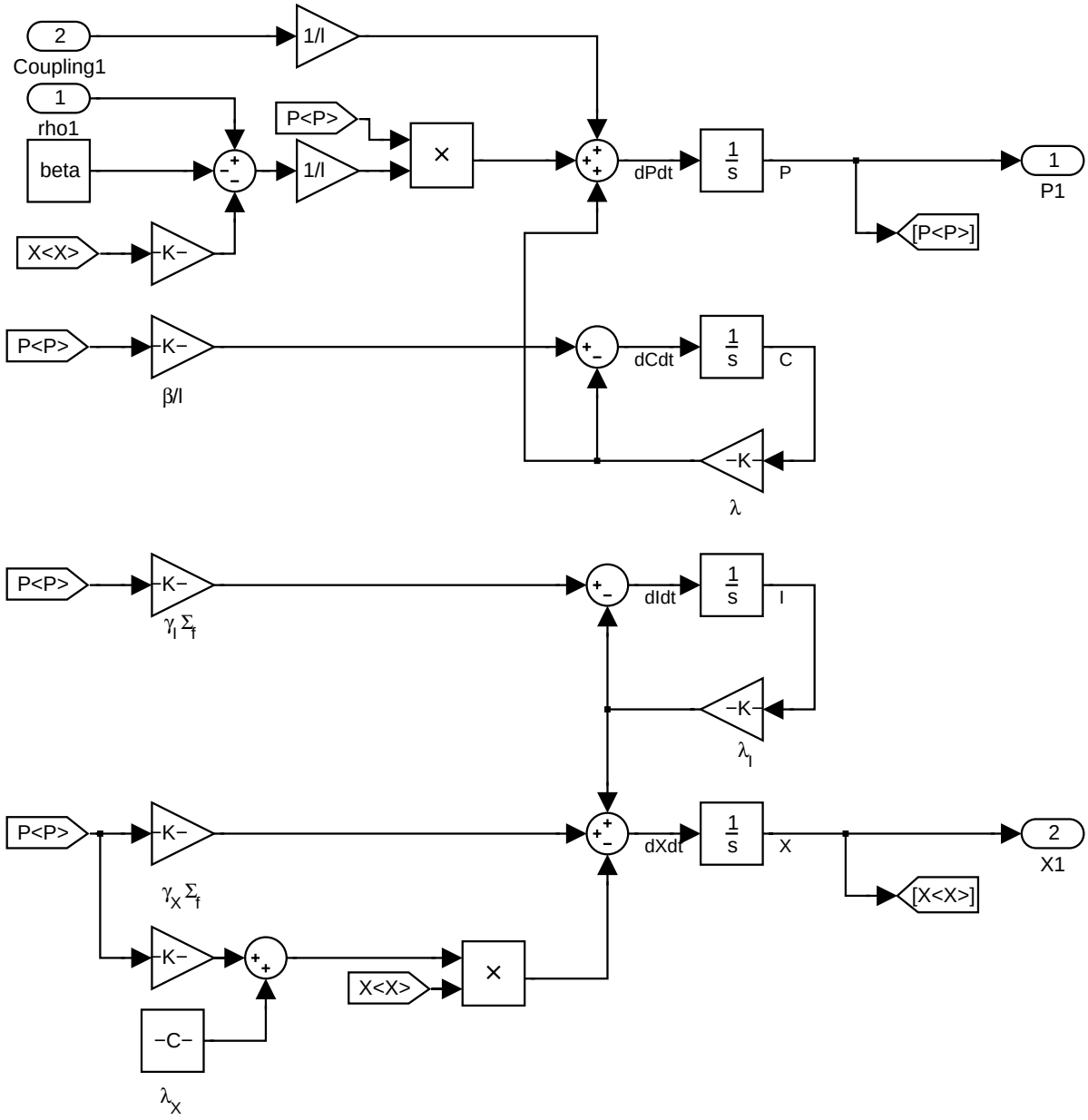

Figure 1: *Implementation of dynamics equations of each zone*

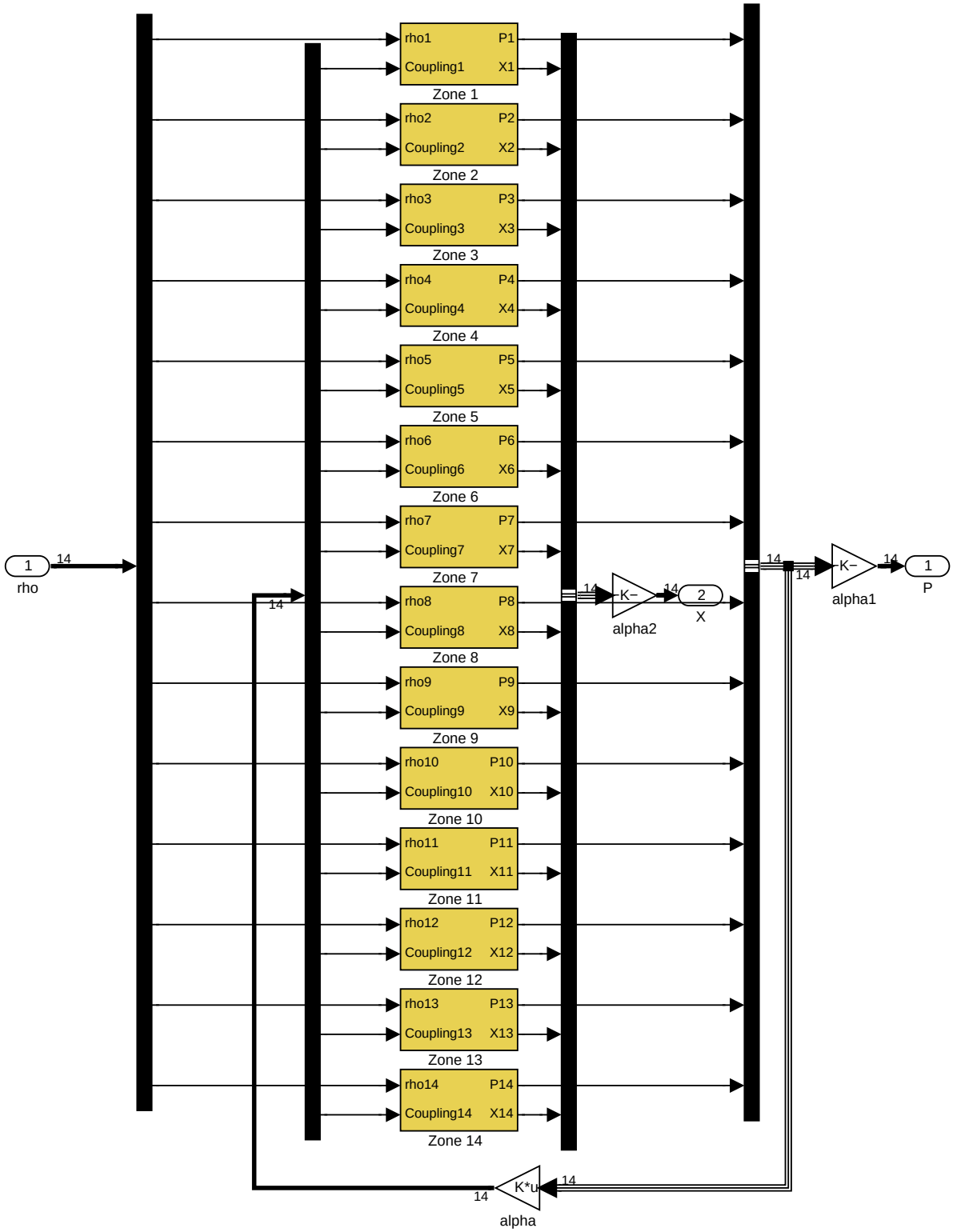

Figure 2: *Interconnection of zones*

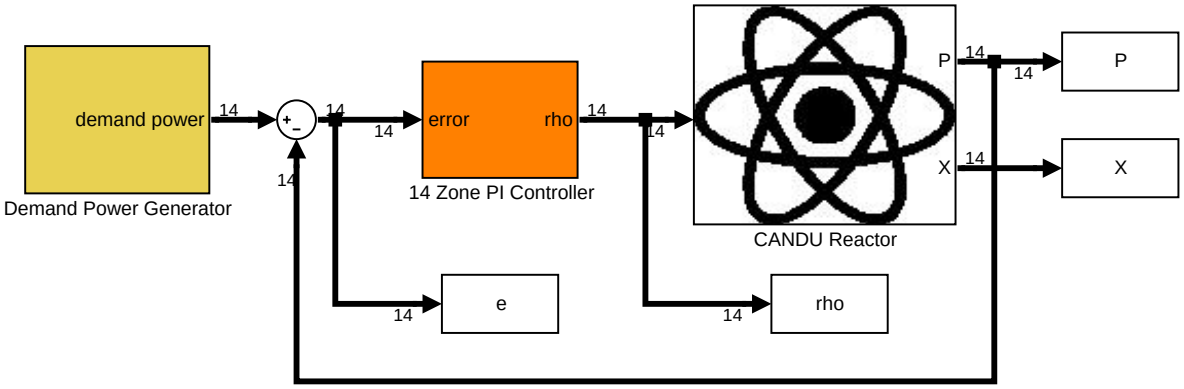

Figure 3: *Closed loop configuration of the reactor*

xenon depends on the depth of power transient. If disturbance of demand power reduces the power level from 100% to 50% instead 75%, there would be a larger overshoot and the xenon would require longer time to settle down on the new equilibrium point.

Another interesting feature would be to look at one of the neighboring zones and see the effect of this disturbance on it. Fig. 5 which shows the variables of zone 3. Considering the fact that this zone has a relatively large interface area with zone 4, Fig. 5 reveals an interesting behavior of the reactivity. Because, all the other zones have constant demand power, the control system tries to keep their power constant, but the power level in zone 4 has decreased and there are not as many neutrons as before, so less neutrons will migrate from zone 4 to the neighboring zones. To keep up the power of the neighboring zones at constant level, the control system increases the reactivity to compensate the shortage of neutrons. The power and xenon in these zones also undergo a very tiny transient, but from practical point of view they remain constant.

The second simulation is mimicking the daily load cycle under 12-3-6-3 regime. The power is kept at 100% for 12 hours then it is reduced to 75% in 3 hours and kept constant at this level for 6 hours. Final stage is increasing the power to 100% in 3 hours which completes the 24 hours cycle. The 12-3-6-3 load cycling pattern or other similar variants are common in studying load following capability of reactors.

The model in Fig. 3 has been used again to run the simulation with the difference that the "demand power" block is modified so that all zones follow the 12-3-6-3 pattern between 100% and 75%. Obviously, if all zones follow this pattern, the global or bulk power will also follow the pattern. Fig. 6 shows the power, reactivity, and the xenon concentration for zone 4 as an example. Note that the reactivity plotted is the net reactivity to overcome the variation of the load and the constant part which is compensating xenon is not plotted. In practice, the small variation of reactivity around an operating point can be furnished by liquid zone controllers. All the other zones behave in the same manner.

5 Conclusion

This paper discusses the development and implementation of a MATLAB/SIMULINK based model for a CANDU reactor. The mathematical model is derived from nodal approximation to two-group dif-

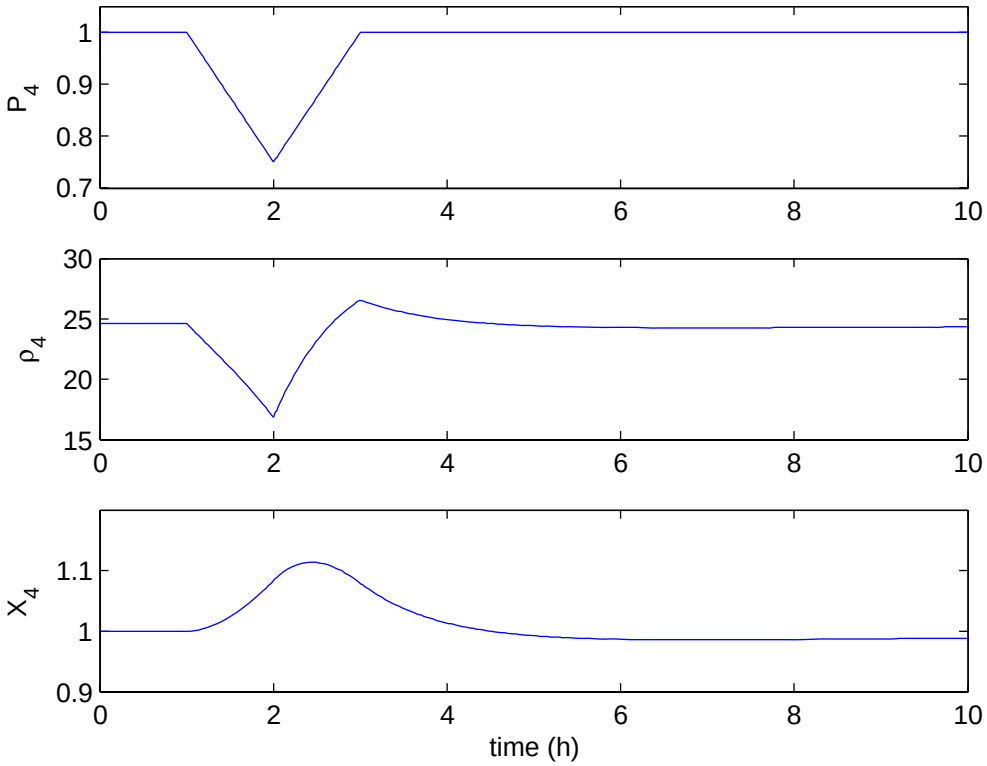

Figure 4: *Variables of zone 4 during a disturbance in its demand power*

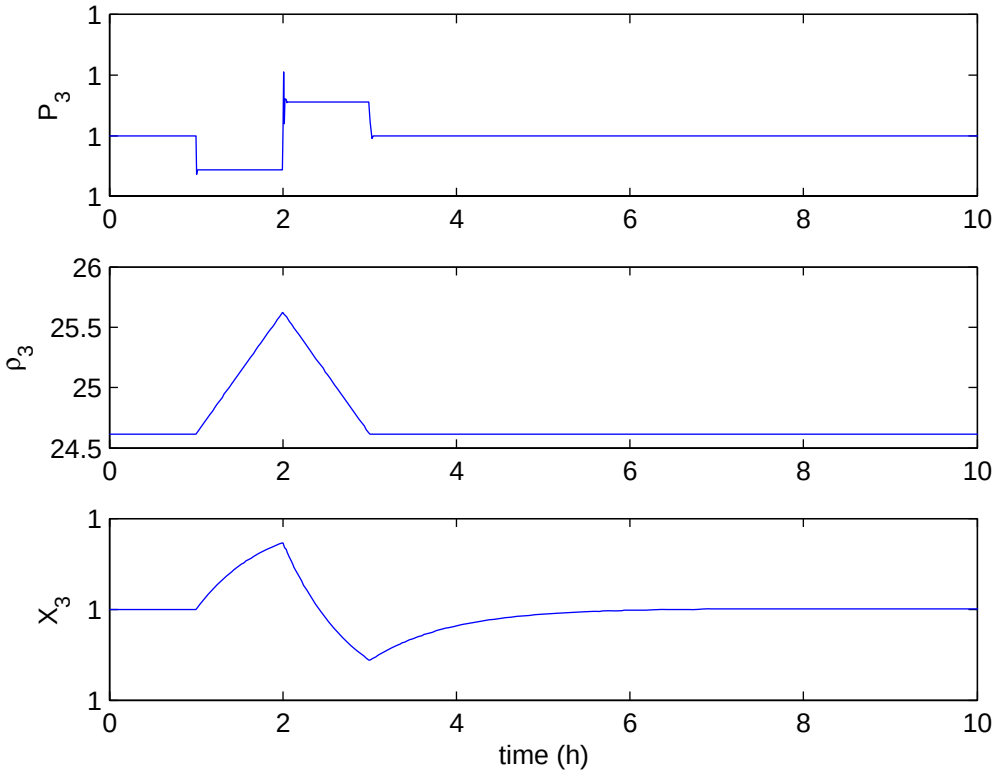

Figure 5: *The effect of disturbance in zone 4 demand power on zone 3*

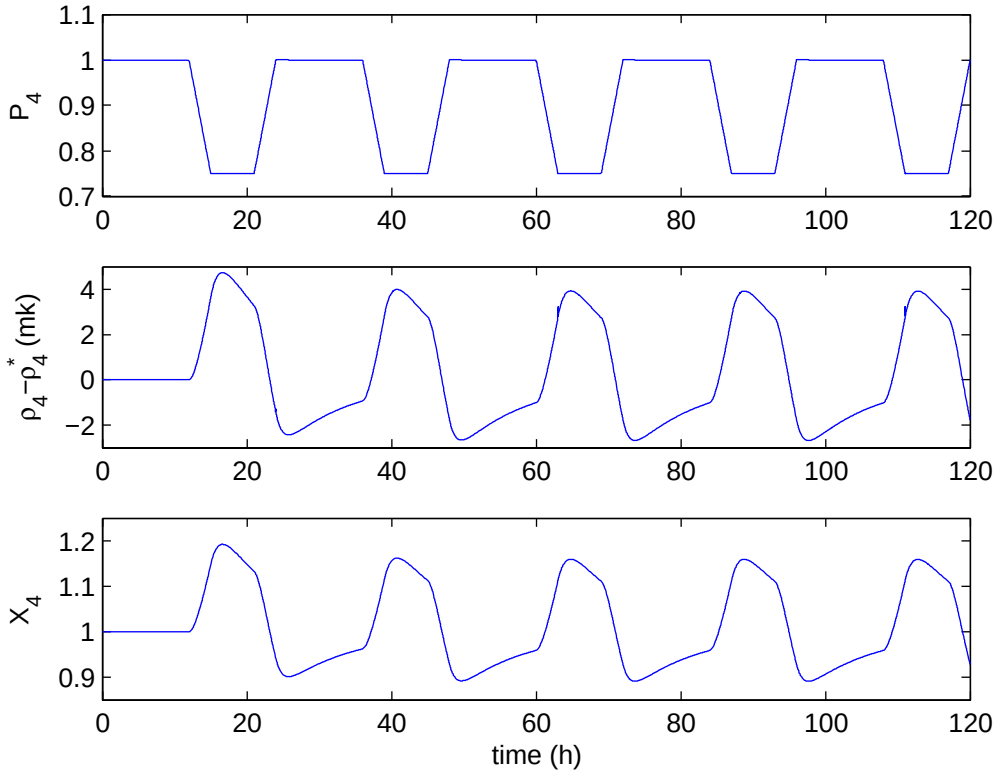

Figure 6: *Variation of power, reactivity, and xenon concentration in a 12-3-6-3 load cycling*

fusion equation. Among the several feedback mechanisms present in a nuclear reactor, only xenon feedback has been modeled. The results of the simulations show that the model can be used to explore the behavior of CANDU reactors during transients caused by load variations or xenon oscillations. The most important feature of the model is that it is implemented in MATLAB/SIMULINK which are widely accepted tools in universities and industries and it can be linked to the models of the other components of a reactor. Currently, the model of liquid zone compartments and their control algorithms is being developed in our group which will complement the CANDU reactor model.

For the two simulations that were done, the simulation speed is very fast. There is a short delay in the beginning of the simulation and this is the period when the initialization function of masked subsystem (reactor) calculates the steady state values. There are ways to shorten this delay, but it is not a very serious issue to be worried about.

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