TRACEABILITY AND MATHEMATICAL MODELING OF CROSS CORRELATION FLOW MEASUREMENT

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Abstract

Traceability is important in all measurements, and especially flow measurement, given industry requirements for uncertainty. How traceability is achieved depends on the flow measurement method used. For flow measurement based on the transport velocity of turbulent structures, an accurate description of behavior of turbulent structures in the flow is important. This is the case with cross correlation flow measurements.

Advanced Measurement and Analysis Group Inc. (AMAG) uses the cross correlation flow meter CROSSFLOW around the world. At AMAG, a mathematical model has been developed, describing cross correlation flow measurement, for various flow conditions. Results predicted by the model, were compared to laboratory test results from Utah State University Water Laboratory and the AMAG Flow Laboratory. There was good agreement between the modeled results and laboratory results.

1. Introduction

All measurements have an uncertainty. To obtain this uncertainty, the measured value with unknown uncertainty must be comparable to a different measured value of the same phenomena with known uncertainty, which we may colloquially refer to as *actual value*. The deviation of the measured value from the actual value, is used to deduce the uncertainty of the measured value. For example, to determine the uncertainty of a flow meter, the flow meter is set up to conduct measurements in a laboratory, where a weight tank is used to measure the same flow. The weight tank's uncertainty is known. The deviation of the flow meter's measurement from the weight tank's measurements, are used to calculate uncertainty.

If such a process is conducted, then when the flow meter is used in the field, one can claim traceability to the laboratory where calibration was conducted, with particular uncertainty, as long as the measurement conditions in the laboratory are representative of the flow conditions in the field. If there is less confidence that the laboratory flow conditions are representative of field flow conditions, the same uncertainty cannot be claimed. Claiming a particular uncertainty in a field measurement, requires traceability to a laboratory setting as described above, and claiming such traceability requires that the measurement condition in the laboratory is sufficiently representative of the measurement conditions in the field.

In order to determine whether a laboratory test is representative of a field condition, a good understanding of the measurement method must exist, in order to understand which phenomena affect the measurement, and whether those phenomena affect the measurement in the same way in the laboratory and in the field. This depends on what is being measured, and what method is used.

For example, establishing uncertainty and traceability for a transit time flow meter, is different from establishing uncertainty and traceability for a weight tank flow meter, because different phenomena affect the different flow measurement methods. For example, weight tank flow measurement does not depend on speed of sound.

Cross correlation flow measurement is based on measuring the transport velocity of turbulent structures. Therefore, achieving traceability for a cross correlation flow meter, requires an understanding of the space-time development of turbulent structures.

Advanced Measurement and Analysis Group Inc. (AMAG) is a Canadian company, that provides products and services for power plants around the world. Mostly, AMAG provides products and services for non-intrusive cross correlation flow measurement, using the flow meter CROSSFLOW, a technology developed and manufactured by AMAG. At AMAG, a mathematical model was developed, of cross correlation flow measurement. Since traceability depends not only on flow conditions, but on the flow meter as well, it is necessary for the mathematical model to simulate not only flow, but to simulate flow measurement as well. This model, based on simulating the change in turbulent structures through space and time, can be used to simulate cross correlation flow measurement in different flow conditions. Laboratory testing was conducted to test the model, and results were in good agreement with modeled results.

This paper discusses the importance of traceability in flow measurement, and how the mathematical modeling of cross correlation flow measurement assists achieving traceability. There will be a description of the operating principle of CROSSFLOW, and its applications. Recent laboratory test results and their comparison to simulated flow measurement results will be shown.

2. The importance of Traceability

All flow measurements in industry, regardless of flow measurement technology, must be traceable to a laboratory setting, in order to ensure that the flow measurement is accurate. To achieve such traceability, it is not sufficient to use a flow meter that performs well in a laboratory. The same flow meter may achieve 0.5% uncertainty in a specific laboratory setting, but only be capable of achieving 2% uncertainty in an industrial condition where it is installed, due to differences between laboratory and field conditions. For a flow measurement uncertainty in the field to be equivalent to a laboratory setting, the flow meter must perform well in a laboratory setting that is representative of the flow conditions in the field where the flow meter is intended to be used.

Since different flow measurement technologies are based on different operating principles, the method for determining whether laboratory conditions are representative of field conditions, depends on the type of flow meter used. For example, for some flow meters it may be sufficient for a laboratory to have the same piping geometry as in the field, while for other flow meters, the same Reynolds number may be necessary as well.

Since cross correlation flow measurement is based on measuring the transport velocity of turbulent structures, the space-time development of turbulent structures in the laboratory must be representative of the space-time development of turbulent structures in the field. To achieve this, AMAG conducts rigorous research of the space-time development of turbulent structures within flow conditions where measurement is conducted. Research conducted in this regard, includes mathematical modeling of flow measurement in plant flow conditions and laboratory flow conditions, to optimize laboratory testing in order to ensure laboratory conditions are representative of field conditions.

Through such research, one can ensure that flow measurements are traceable to a laboratory setting, and the uncertainty observed in the field is equivalent to the uncertainty observed in laboratory testing. This paper presents recent laboratory test results, and their comparison to simulations of cross correlation flow measurements. The simulations were conducted using a mathematical model of cross correlation flow measurement developed at AMAG.

3. Cross correlation flow measurement

Non-intrusive methods of measuring flow rate in conduits or pipes using ultrasonic signal have been gaining popularity since the 1970s. One of these methods, the cross correlation method, has been known for a long time [1], but it became popular only more recently, due to recent advances in computation power. [2].

In the simplest design of the ultrasonic cross correlation flow meter, two ultrasonic waves, separated by a known axial distance, are transmitted diametrically through the pipe, as shown in Figure 3.1.

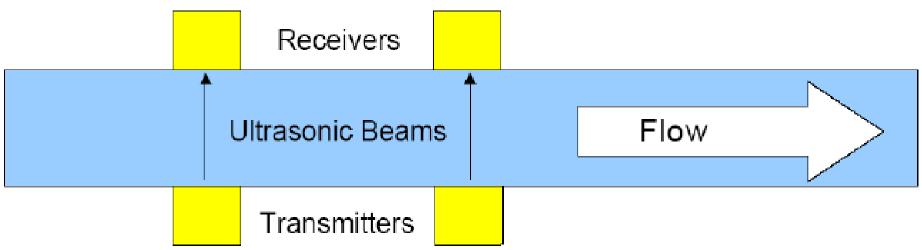


Figure 3.1: Simplest cross correlation flow measurement set up.

Each wave is modulated by a turbulent field, which is naturally present in the flow.
The phase of the ultrasonic waves is modulated by the turbulence velocity component along the direction of propagation of the ultrasonic waves. In single phase flow with constant temperature, this modulation is described by the following equation [2,3]:

$$\phi(t) = \frac{f}{C^2} \int_0^D v_t(\varsigma, t) d\varsigma \qquad (3.1)$$

In (3.1) f is the frequency of the ultrasonic waves, which has a magnitude much higher than turbulent spectrum frequencies. C is the speed of sound, v_t is the turbulence velocity component along direction of the ultrasonic wave's propagation, ς is the spatial variable of integration along direction of the ultrasonic wave's propagation, and t is time. If the signal, $\phi(t)$, is obtained in two flow cross-sections $z = z_1$ and $z = z_2$, the cross-correlation function of signals $\phi(z_1,t)$ and $\phi(z_2,t)$, over time T, can be calculated as follows:

$$R(\tau) = \int_0^T \phi(z_1,t)\cdot\phi(z_2,t+\tau)dt \qquad (3.2)$$

If the distance between two ultrasonic waves, $l = z_2 - z_1$, is sufficiently small, the signals $\phi(z_1,t)$ and $\phi(z_2,t)$ maintain similarity to each other, but are shifted in time by a certain time delay, τ^*. In such a case, the function $R(\tau)$ has a well defined maximum at $\tau = \tau^*$, and a measured flow velocity v_m can be introduced as:

$$v_m = \frac{l}{\tau^*} \qquad (3.3)$$

The meaning of v_m can be better understood from the following considerations: The function $R(\tau)$ can be presented as a double integral of two-point correlation functions, by substituting (3.1) into (3.2), and changing the order of integration, as is illustrated in Figure 3.2 and shown below [2]:

$$R_0(\varsigma,\xi,\tau) = \int_0^T \phi(z_1,\varsigma,t)\cdot\phi(z_2,\xi,t+\tau)dt$$
$$R(\tau) = \int_0^D\int_0^D R_0(\varsigma,\xi,\tau)d\varsigma d\xi \qquad (3.4)$$

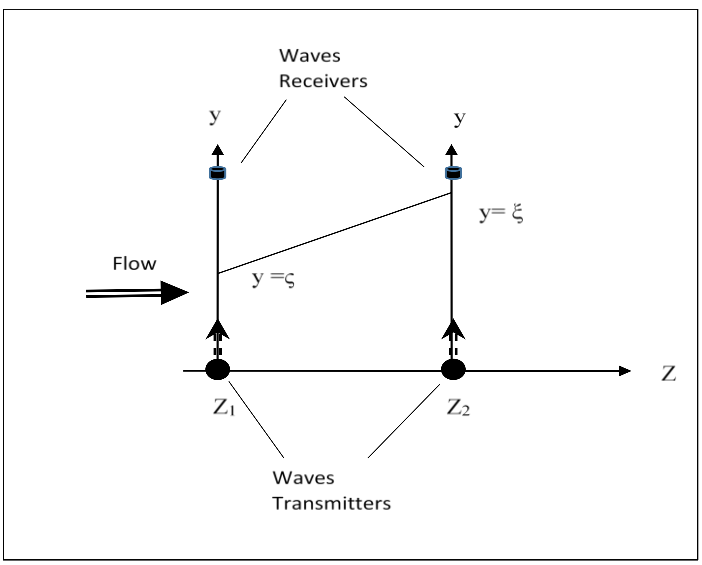


Figure 3.2: Schematics of cross correlation flow measurement. Function $R_0(\varsigma,\xi,\tau)$ is a two-point correlation along the flow, taken are the points $(y,z)=(\varsigma,z_1)$ and $(y,z)=(\xi,z_2)$

Function $R_0(\varsigma,\xi,\tau)$ is a correlation of turbulent velocity component v_t in two points located along the direction of propagation of the ultrasonic waves at two pipe cross-sections $z = z_1$; $y = \varsigma$ and $z = z_2$; $y = \xi$. From (3.4), it follows that velocity v_m, defined by (3.3), is a weighted average of sweeping velocity defined from two-point correlations $R_0(\varsigma,\xi,\tau)$. As a result of the presence of a time-averaged velocity gradient in pipe flow, v_m is not equal to cross-section average axial flow velocity U. The relation between U and v_m is critically important in cross correlation flow measurement technology, not only for obtaining accurate results, but also in establishing traceability and uncertainty of flow measurements.

4. Mathematical model and laboratory testing

This section will explain the purpose of the mathematical model, and its main principles. It will then explain the laboratory tests, and present a comparison of laboratory test results to simulated results.

Mathematical modeling of cross correlation flow measurement was conducted, and the results of mathematical modeling were compared with results of a laboratory test. Laboratory test and mathematical modeling were conducted on a pipe flow on different distances downstream of a 90-degree elbow.

The modeling included the following steps:

- Generation of inlet turbulent velocity field at pipe cross-section $z = z_1$.
- Numerical simulation of time-average velocity profile using $k-\varepsilon$ model.
- Calculation of turbulence velocity field in cross-section $z = z_2$ using local Taylor approximation.
- Calculation of measured flow velocity v_m using equations (3.1), (3.2), and (3.3).

The spectrum of signal $\phi(t)$ from equation (3.1), measured by the flow meter in the laboratory tests, was used in the model to generate the inlet turbulent velocity field. It was assumed that the turbulence spectrum is constant across the pipe cross-section area, except a small area near the pipe wall. The ratio of cross-section average flow velocity U, to measured velocity v_m, was obtained by the mathematical model. The same ratio was obtained in laboratory testing, using the cross correlation flow meter CROSSFLOW to obtain v_m, and using laboratory weigh tank instrumentation to obtain U. The two ratios, one from the model and the other from laboratory results, were compared to validate the model.

Schematic of the tests set-up is shown in Figure 4.1. The laboratory test was conducted at the Utah Water Research Laboratory in Utah State University. Ultrasonic cross correlation flow measurement was conducted using the CROSSFLOW flow meter, developed and manufactured by AMAG.

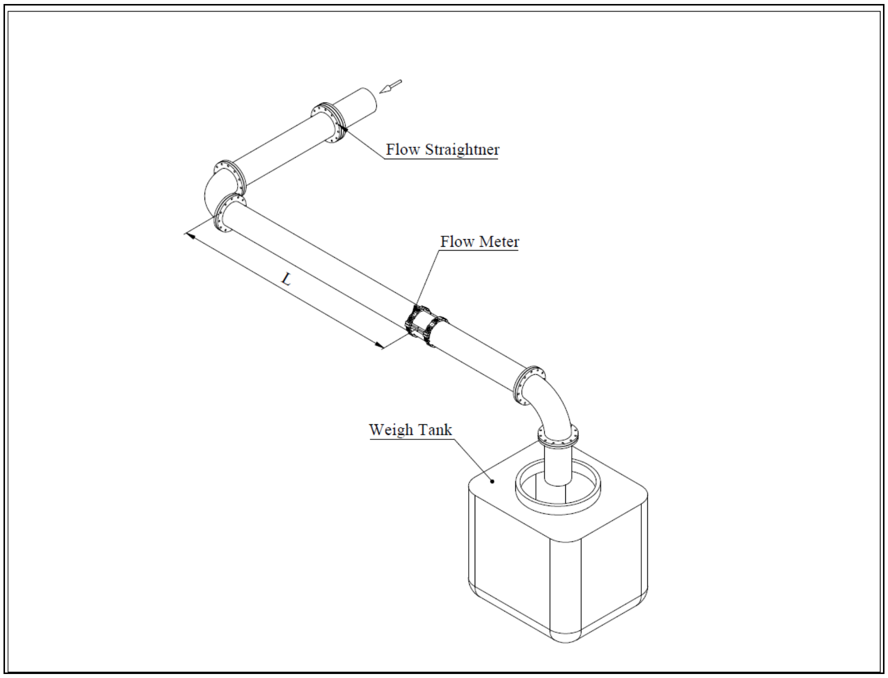


Figure 4.1: Schematics of the Test Setup. Test Section consists of 12-inch plastic pipe with carbon steel 90-degree elbow and flow conditioner.

Good agreement between laboratory test results and modeling results were observed, not only in the trend of the ratio U/v_m, but in its absolute value as well. These results are presented in Figure 4.2, where the ratio is presented as a function of distance from an upstream elbow where measurement, and for which simulations based on the mathematical model, were conducted. Distance from the upstream elbow is normalized to pipe diameter. No adjustments of the modeling parameters were made in the mathematical model of the flow meter.

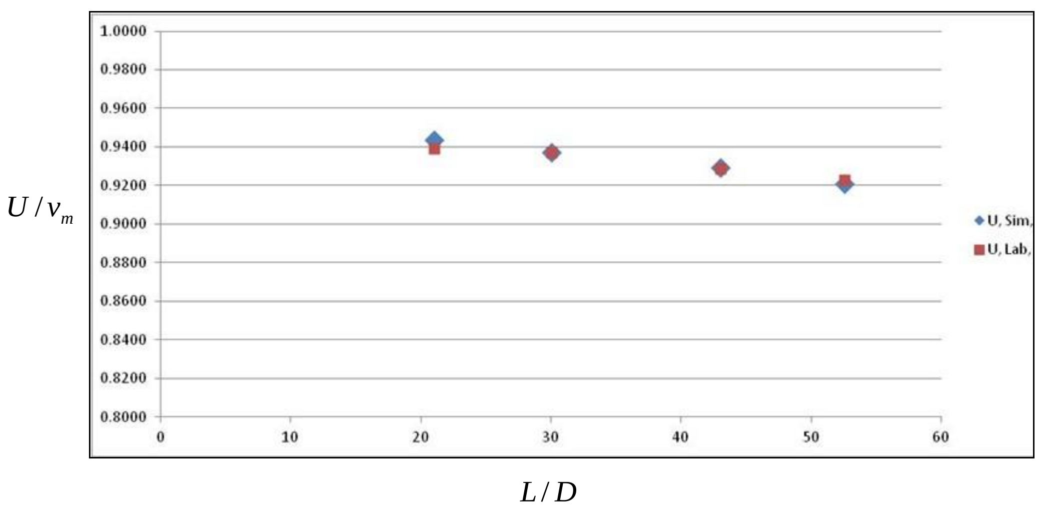
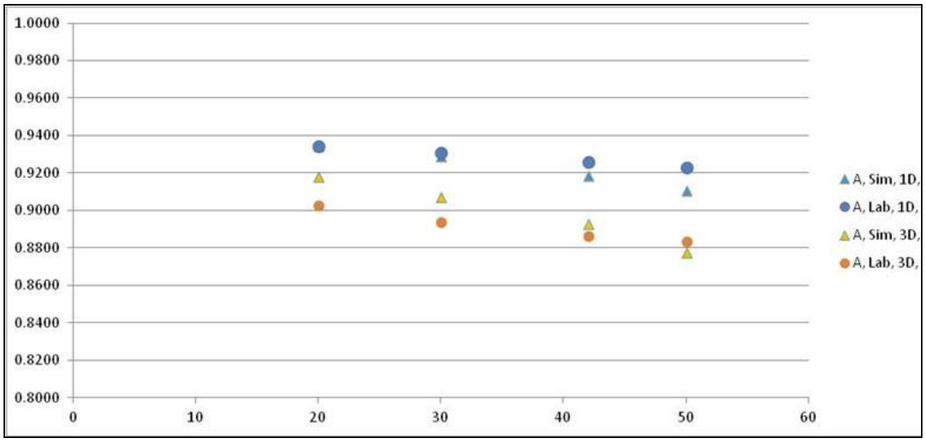


Figure 4.2: Comparison of the ratio U/v_m predicted by mathematical modeling (blue diamonds) and obtained in laboratory testing (red squares). The ratio is along the vertical axis. Distance from upstream elbow (L) normalized to pipe diameter (D), is along the horizontal axis.

The dependence of the ratio U / v_m on the spacing between ultrasonic beams, $(z_2 - z_1)$, is shown in Figure 4.3. Test data presented in Figure 4.3 was obtained at the AMAG Flow Laboratory in Mississauga, Ontario. Measurements were conducted downstream of a 90-degree elbow, on a 4-inch diameter plastic pipe.

U / v_m



L / D

Figure 4.3: Comparison of the ratio U / v_m predicted by mathematical modeling (triangles) and obtained in laboratory testing (circles). Figure shows dependence of ratio on spacing between ultrasonic beams. Spacing shown is 1D (blue) and 3D (orange), where D is the pipe diameter. The ratio is along the vertical axis. Distance from upstream elbow (L) normalized to pipe diameter (D), is along the horizontal axis.

The mathematical model correctly predicts a reduction of the U / V_m value with increasing distance between ultrasonic beams. For the transducer spacing of 3D (3 pipe diameters), the difference between modeled results, and test results, increases as it approaches the upstream 90-degree elbow. The fact that the difference is gradually increasing when approaching the elbow at 3D spacing, may be a result of complicated flow patterns existing close to the elbow being incompatible with approximations made in the model, at a sufficient distance between ultrasonic beams. For both spacing values, the simulated results drop at a greater rate than the laboratory test results, as one moves right along the horizontal axis, effectively observing conditions farther from the upstream elbow. This is likely the result of how the axial flow profiles were determined, when conducting simulations. Despite these potential effects, the difference between simulated and laboratory test results, is within 2%.

5. Conclusion

In conclusion, the mathematical model of cross correlation flow measurement agrees well with laboratory test results. Mathematical modeling can therefore be used as a tool in determining the effect of turbulent conditions on flow meter output, and designing and optimizing laboratory testing, in order to assist in ensuring desired traceability and uncertainty.

6. References

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