THE CASUALIDAD METHOD FOR UNCERTAINTY EVALUATIONS OF BEST-ESTIMATE CODE CALCULATIONS

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Abstract

The description of the salient features of three independent approaches for estimating uncertainties associated with the predictions of complex system codes is provided together with the main drawbacks and advantages. The 1st approach is referred as Propagation of Input Uncertainties (PIU) and it is the "standard" one and the most used at the industrial level: it is based upon the selection of input uncertain parameters, on assigning related ranges of variations and, possibly, PDF (Probability Density Functions) and on performing a suitable number of code runs to get the combined effect of variation on the results. In the 2nd approach the uncertainty derives from the comparison between relevant measured data and results of corresponding code calculations and thus is referred as Output Error Propagation (OEP). The 3rd approach is based on the Bayesian inference technique and on the availability of experimental data by which computer model predictions can be improved and the ranges of variation of (in theory) 'all' input parameters can be characterized.

1. Introduction

Uncertainty analysis aims at characterizing the errors associated with experiments and predictions of computer codes, in contradistinction with sensitivity analysis, which aims at determining the rate of change (i.e., derivative) in the predictions of codes when one or more (typically uncertain) input parameters varies within its range of interest. Let's consider three relevant definitions, i.e., in alphabetic order, accuracy, sensitivity and uncertainty, as they are commonly accepted in the sector of deterministic accident analysis within the more general framework of nuclear reactor safety technology.

Accuracy is defined, [1], as "the known bias between a code prediction and the actual transient performance of a real facility". Therefore, the evaluation of accuracy implies the availability of a calculation result and of a measured value. Point values and continuous time trends shall be included in the definition. The experimental error is not part of the definition. However, in the majority of cases of practical interest in the area of accident analysis of nuclear power plants, the error that characterizes the thermal-hydraulic measurements is much lower of the error (i.e. the accuracy) that characterizes the comparison between measured and predicted values.

The sensitivity is, according to [2], "… the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation, and of how the given model depends upon the information fed into it.". Furthermore, "Sensitivity analysis studies the relationships between information flowing in and out of the model.". These definitions imply that the parameter values that characterize both (and only) the

boundary and initial conditions, e.g. representative of a system, and the numerical structure of a correlation embedded into the model (or code) constitute the typical objective of a Sensitivity Analysis (SA).

The uncertainty is the unknown error that characterizes the prediction of any code or model. The uncertainty analysis is, according to [1] and related to system thermal-hydraulic code predictions, "an analysis to estimate the uncertainties and error bounds of the quantities involved in, and the results from, the solution of a problem. Estimation of individual modeling or overall code uncertainties, representation (i.e. nodalization related) uncertainties, numerical inadequacies, user effects, computer compiler effects and plant data uncertainties for the analysis of an individual event". Furthermore, to conclude with a citation from [2], "… uncertainty is not an accident of the scientific method but its substance.". Within the present context, the uncertainty is the necessary supplement for a best-estimate thermal-hydraulic code prediction; see also [3].

The reason why an accuracy analysis (AA) is performed is mainly connected in the sector under investigation here (i.e. the deterministic accident analysis) with the demonstration of qualification for computer codes. The accuracy analysis implies the availability of relevant experimental data and of tools to characterize the resulting discrepancies from qualitative and quantitative viewpoints [4, 5]. The reasons why a sensitivity analysis (SA) is performed are strongly affected by the type and the objectives of the model and may range from verification purposes, to finding singular points (e.g. maximum and minimum) of an assigned output quantity, or the factors that mostly contribute to that output, or the correlation among input variables. It can be premised that needs for SA come from the fundamental principles of quality assurance. The reasons why an uncertainty analysis (UA) is performed come from nuclear safety principles and primarily from concepts like defense-in-depth. It must be ensured that the nominal result of a code prediction, 'best-estimate' in the present case, is supplemented by the uncertainty statement, that can be simplified as 'uncertainty bands', in such a way that connected safety margins are properly estimated.

The key result from AA is the demonstration of the qualification level of a code (i.e. a set of objective quantitative figures of merit which measure the discrepancy between experimental and code predictions) and the characterization of the range of parameters over which the code should be considered as qualified and applicable to situation of interest to nuclear reactor safety. The AA should also provide an answer to the scaling issue, [6]. The key result from SA is the influence of input parameters upon selected output quantities and the evaluation of the relative influence of input parameters, according to the definition given above. The key results from UA are error bands that bound the best-estimate predictions. Point value error bands can be distinguished from continuous error bands that bound one or several curves, as well as from three-dimensional graphic representations where instantaneous values for quantity-error (e.g. pressure) are reported together with time-error as a function of time, [7].

Therefore AA, SA and UA are closely linked, but important differences can be identified. All that is needed for a meaningful SA is the model and the input values, while UA attempts to estimate the actual error band value for an output; as a consequence, it needs a reference value typically not available (thus the definition of 'unknown' error). AA, on the other hand needs relevant experimental data. As an example, the check that an assigned model satisfies the first or the second principle of thermodynamics may not be the objective of SA, but it is the objective for UA and can be confirmed following AA. Furthermore, when performing SA, the values of the concerned input

parameters are varied arbitrarily around the initial (or nominal) value to a 'small' or to a 'large' extent depending upon the scope of the analysis; when performing the UA, whatever is the method adopted, a range of variation for the concerned input parameters must be assigned or available. SA may be a way to perform UA if input parameters are properly selected with proper ranges of variation. Specific aspects of thermal-hydraulic computer codes should be considered when performing either AA or SA or UA. One example is the impact of the nodalization upon the results: there are phenomena like Critical Heat Flux or Two-Phase Critical Flow characterized by explicit or implicit equations implemented in the codes and phenomena like Natural Circulation that depend upon both the equations implemented into the code and upon the structure and the parameters related to the nodalization. In this last case a significant (AA, or SA or UA) analysis must account for nodalization parameters like individual node length, equivalent diameters, node density (i.e. average node length) and relative position of nodes (i.e. thermal-hydraulic centers of nodes).

The present paper focuses on UA. The historical triggering for UA in the area of nuclear reactor thermal-hydraulics may be traced as the Regulatory Guides (e.g. RG 1-157 and, more recently 1-203, [3]) issued by US NRC to streamline the application of codes when demonstrating the compliance of reactor accident scenario calculation with the criteria in Appendix K of 10 CFR 50-46. However, an international code assessment project conducted within OECD/NEA/CSNI since the beginning of eighties also showed the exigency for UA.

The first framework for calculating the uncertainty was proposed by US NRC and denominated Code Scaling, Applicability, and Uncertainty (CSAU [8]). The application of the CSAU methodology resulted in the calculation of the Peak Cladding Temperature during a LBLOCA Design Basis Accident event for a Westinghouse 4-loop pressurized water reactor with the uncertainty to a 95% confidence level. The peak temperature was calculated using the TRAC thermal-hydraulic analysis code and was given as a single-valued number with uncertainty bands. In the meantime, a number of uncertainty methodologies were proposed in other countries, including the GRS, the UMAE and the AEA Technology methods, as summarized in [9] and [10]. These methods, although sharing a common goal with CSAU, use different techniques and procedures to obtain the uncertainties on key calculated quantities. More importantly, these methods have progressed far beyond the capabilities of the early CSAU analysis. Presently, uncertainty bands can be derived (both upper and lower) for any desired quantity throughout the transient of interest, not only point values like peak cladding temperature. For one case, the uncertainty method is coupled with the thermal-hydraulic code and is denominated CIAU (Code with capability of Internal Assessment of Uncertainty, [7, 11]) and discussed below in more detail. All these methods are described into detail in [12], including examples of applications to cases of industrial interest.

The purpose of the present paper is twofold: a) to identify the roadmaps for uncertainty evaluation adopted by the methods currently applied to the cases of industrial interest, making reference to the classification proposed in [12]; b) to propose an innovative method that might not suffer of the drawbacks identified for the current methods.

Namely the propagation of code input error and the propagation of the calculation output error constitute the key-words for identifying the methods of current interest for industrial applications; while the Adjoint Sensitivity Analysis Procedure and the Global Adjoint Sensitivity Analysis Procedure methods extended to performing uncertainty evaluation in conjunction with concepts

from Data Adjustment and Assimilation constitute the innovative method, whose fundaments are provided in [13] to [15].

2. Approaches for Uncertainty

The features of independent approaches for estimating uncertainties are reviewed below and highlighted in Figures from 1 to 3.

The propagation of code input errors (Fig. 1): this can be evaluated as being the most adopted procedure nowadays, endorsed by industry and regulators. It adopts the statistical combination of values from selected input uncertainty parameters (even though, in principle an unlimited number of input parameters can be used) to calculate the propagation of the errors throughout the code.

The propagation of code output errors (Fig. 2): this is the only demonstrated independent working alternative to the previous one and has also been used for industrial applications. It makes full and direct reference to the experimental data and to the results from the assessment process to derive uncertainty. In this case the uncertainty prediction is not propagated throughout the code.

The 'third' approach, (Fig. 3): this is an independent way, i.e. different from propagation of code input errors or from propagation of code output errors is based on Adjoint Sensitivity Analysis Procedure (ASAP), Global Adjoint Sensitivity Analysis Procedure (GASAP), [13] and [14] and Data Adjustment/Assimilation (DAA) methodology [15] by which experimental and calculated data, including the computation of sensitivities (derived from ASAP), are mathematically combined for the prediction of the uncertainty scenarios. The approach is reviewed hereafter as a deterministic method.

The first approach, reviewed as the prototype for propagation of code input errors, is the so-called "GRS method" [16], which includes the so-called "CSAU method" (Code Scaling, Applicability and Uncertainty) [8] and the majority of methods adopted by the nuclear industry. Although the entire set of the actual number of input parameters for a typical NPP (Nuclear Power Plant) input deck, ranging up to about 105 input parameters, could theoretically be considered as uncertainty sources by these methods, only a 'manageable' number (of the order of several tens) is actually taken into account in practice. Ranges of variations, together with suitable PDF (Probability Density Function) are then assigned for each of the uncertain input parameter actually considered in the analysis. The number of computations needed for obtaining the desired confidence in the results can be determined theoretically by the Wilks formula [17]. Subsequently, the identified computations (ca. 100) are performed using the code under investigation to propagate the uncertainties inside the code, from inputs to outputs (results). The logical steps of the approach are depicted in Fig. 1.

The main drawbacks of such methods are connected with: a) the need of engineering judgment for limiting (in any case) the number of the input uncertain parameters; b) the need of engineering judgment for fixing the range of variation and the PDF for each input uncertain parameter; c) the use of the code-nodalization for propagating the uncertainties: if the code-nodalization is wrong, not only the reference results are wrong but also the results of the uncertainty calculations; d) the process of selecting the (about) 100 code runs is demonstrably not convergent, and the investigation of results from two or more different sets of 100 calculations shows different values for uncertainty. A study performed by KAERI in the framework of the Phase III of BEMUSE project is summarized in [18].

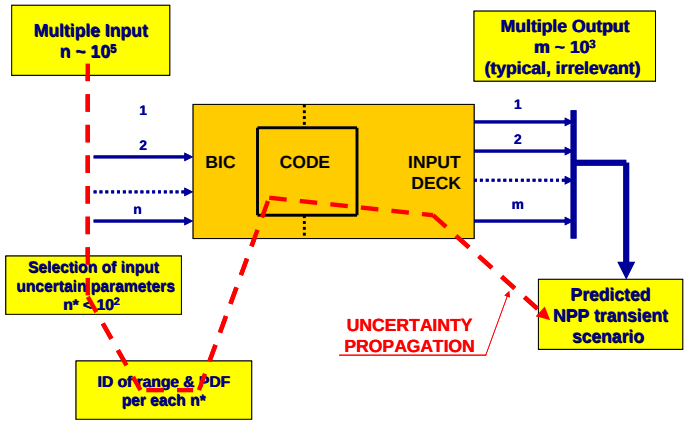

Figure 1 Uncertainty methods based upon propagation of input uncertainties (GRS method).

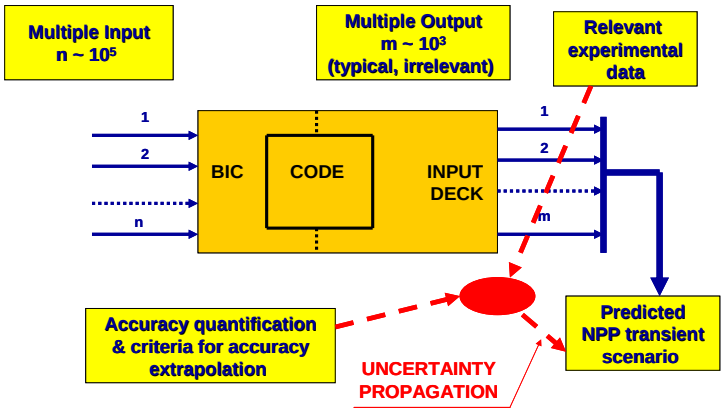

Figure 2 Uncertainty methods based upon propagation of output uncertainties (CIAU method).

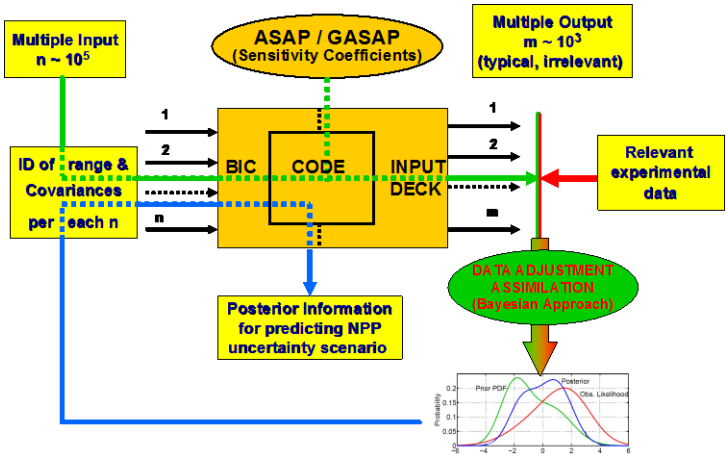

Figure 3 Uncertainty methodology based on Adjoint Sensitivity Analysis Procedure and Data Adjustment/Assimilation.

The second approach, reviewed as the propagation of code output errors, is representatively illustrated by the UMAE-CIAU (Uncertainty Method based upon Accuracy Extrapolation [19] 'embedded' into the Code with capability of Internal Assessment of Uncertainty [11, 7]). Note that this class of methods includes only a few applications from industry. The use of this method depends on the availability of 'relevant' experimental data, where here the word 'relevant' is connected with the specific NPP transient scenario under investigation for uncertainty evaluation. Assuming such availability of relevant data, which are typically Integral Test Facility (ITF) data, and assuming the code correctly simulates the experiments, it follows that the differences between code computations and the selected experimental data are due to errors. If these errors comply with a number of acceptability conditions [19], then the resulting (error) database is processed and the 'extrapolation' of the error takes place. Relevant conditions for the extrapolation are:

- Building up the NPP nodalization with the same criteria as was adopted for the ITF nodalizations;
- Performing a similarity analysis and demonstrating that NPP calculated data are "consistent" with the data measured in a qualified ITF experiment.

The main drawbacks of this method are as follows: (i) the method is not applicable in the absence of relevant experimental information; (ii) a considerable amount of resources is needed to establish a suitable error database, but this is a one-time effort, independent of subsequent applications of this method; (iii) the process of combining errors originating from different sources (e.g., stemming from different ITF or SETF (Separate Effect Test Facility), different but consistent nodalizations, different types of transient scenarios) is not based upon fundamental principles and requires detailed validation.

The third approach, depicted in Fig. 3, is based upon the powerful mathematical tools of ASAP, GASAP and DAA by which all parameters $\boldsymbol{\alpha}$ that affect any prediction, being part of either the code models or the input deck can be considered. The Adjoint Sensitivity Analysis Procedure (ASAP) [13, 14] is the most efficient deterministic method for computing local sensitivities **S** of large-scale systems, when the number of parameters and/or parameter variations exceeds the number of responses R of interest (that is the case of most problems of practical interest). In addition, also system's critical points **y** (i.e. bifurcations, turning points, saddle points, response extrema) can be considered and determined by the Global Adjoint Sensitivity Analysis Procedure (GASAP) [13, 14] in the combined phase-space formed by the parameters, forward state variables, and adjoint variables. Subsequently the local sensitivities of the responses R located at critical points **y** are analyzed by the ASAP. Once the sensitivity matrix **S** of the responses **R** respect to the parameters $\boldsymbol{\alpha}$ is available, the moment propagation equation is adopted to obtain the computed covariance matrix $\mathbf{C_R}$ of the responses starting from the covariance matrix $\mathbf{C}_{\boldsymbol{\alpha}}$ of the system parameters. The technique by which experimental observations are combined with code predictions and their respective errors to provide an improved estimate of the system state is known as Data Adjustment and Assimilation (DAA) [15] and it is based on a Bayesian inference process. The idea at the basis of DAA can be made more specific as follows: the computed results R and the respective statistical errors $\mathbf{C_R}$ predicted by mathematical models and based on 'prior' or 'first' guess PDF for the input parameters (i.e. $\mathbf{C}_{\boldsymbol{\alpha}}$) are combined with proper experimental observations M of the states of a system to generate 'adjusted' values for the system parameters ($\boldsymbol{\alpha}^{IE}$, where the suffix IE stays for improved estimate values) and the respective input covariance matrix ($\mathbf{C}_{\boldsymbol{\alpha}}^{\mathbf{IE}}$, or 'posterior' PDF). From this process,

which can be considered as improved estimate analysis of the system's states, the responses $\mathbf{R}^{IE}$ and the respective covariance matrix ($\mathbf{C_R}^{\mathbf{IE}}$) are finally derived.

The main drawbacks of this approach are as follows: (i) the method is not applicable in the absence of relevant experimental information; (ii) the adjoint model, needed for computing the sensitivity S, requires relatively modest additional resources to develop and implement if this is done simultaneously with the development of the original code; however if the adjoint model is constructed a posteriori, considerable skills may be required for its successful development and implementation; (iii) a considerable amount of resources is needed to establish a suitable database of improved estimates for the input parameters ($\boldsymbol{\alpha}^{IE}$) and for the respective input covariance matrix ($\mathbf{C}_{\boldsymbol{\alpha}}^{\mathbf{IE}}$), but this is a one-time effort, independent of subsequent applications of the method.

The maturity of the methods at the first two bullets may be considered as proved also based upon applications completed within the framework of initiatives of international institutions (OECD/NEA [9, 18] and IAEA [1]). The reason for the consideration of the approach at the third bullet derives from its potential to open an independent way (i.e. different from propagation of code input errors or from propagation of code output errors) for performing global uncertainty analysis. In this case, the method itself, as an uncertainty procedure, is not an established technology, but it constitutes an established idea and framework to pursue a mathematically based road to evaluate the uncertainty in system code predictions. In the following sections, the description of this pioneering method is provided.

3. A Pioneering Deterministic Method for Uncertainty Analysis

A large experimental database including SETFs and ITFs is available in the LWR safety area. The license applicants may select different sets of tests for assessing the uncertainty in the same or similar accident analysis. This results in a large variability in the uncertainty evaluations, due also to the inexistence of a uniform systematic method in identifying and reducing the number of important parameters and characterizing their uncertainty. In this context, it is important to note that the CSAU methodology was developed as a roadmap and supplies a comprehensive "basket of requirements" for developing applicable uncertainty methodologies, rather than being the standard uncertainty method.

The intent of this paper is to illustrate a comprehensive approach for utilizing quantified uncertainties arising from SETFs and ITFs in the process of calibrating complex computer models. The methodology proposed is capable of accommodating multiple SETFs and ITFs to learn as much as possible about uncertain parameters, allowing for the improvement of the computer model predictions based on the available experimental evidences. Notwithstanding a similar approach has been already adopted in other scientific fields like meteorology, the application to the nuclear safety area constitutes a real challenge due to the complexity (in terms of number of relevant phenomena and related parameters involved) of the system under consideration and its strong non-linearities. The pioneering approach can be considered also as an attempt to substitute the empiricism of the CIAU approach in relation to the statistical treatment of the accuracy for deriving the uncertainty values with a rigorous mathematical deterministic method.

The CASUALIDAD Method (Code with the capability of Adjoint Sensitivity and Uncertainty AnaLysis by Internal Data ADjustment and assimilation) [20] has been developed as a fully

deterministic method based on advanced mathematical tools for performing internally to the thermal-hydraulic system code the sensitivity and the uncertainty analysis. The availability of a suitable database of experiments (SETF and/or ITF) and related qualified code calculations constitutes a pre-requisite for the development and the following application of the methodology. Similarly to the CIAU method, the process to develop the database of improved estimations of input values and related covariance matrix is distinguished from the application process where the database is used to produce the improved uncertainty scenario. Thus the information embedded in the set of experiments and related qualified code calculations is elaborated and transformed in a suitable information for performing an improved (in the sense that is based on the experimental evidences) uncertainty evaluation. The general principles of the methodology together with brief descriptions of the main tools constituting the CASUALIDAD are given in §3.1, whereas the structure of the method (i.e. the development process and the application process) is discussed in detail in §3.2.

3.1 The bases of the CASUALIDAD method

Establishing the range and probability distribution function of parameters is fairly easy for parameters which describe the condition of the plant (such as initial power or geometrical data) or for parameters describing physical data (such as thermal conductivity of UO_2). It is more difficult for parameters relative to the constitutive relationships (such as interfacial friction) because they can not be directly measured in facilities. To this end, the results from SETF together with their experimental uncertainties are used for establishing or assessing these constitutive relationships. However, the models developed from SETF are applicable to a certain range of parameters, when often the correlations are extrapolated to more extreme conditions in correspondence of which the data is lacking. In order to justify these extrapolations, the ITF data, whose ranges of parameters are much close to typical conditions, are used.

The fundamental principle of the methodology here proposed can be derived from Fig. 4. The initial uncertainty distribution obtained from SETFs is shown coloured in red and represents the uncertainty in each model parameters (multiple model parameter distributions). After the application of the Bayesian Theorem (BT) and of the Principle of the Maximum Likelihood, the uncertainty distributions (in blue), that are prior estimations for the code-simulation, are obtained. The code simulations are further constrained (using BT & PML) against the ITF for taking into account both the system behaviour and the use of the correlations (developed starting from SETFs) outside their range of validity. At the end of this step, the key parameters, their contributions to the uncertainty of the system responses and the posterior parameter distributions (green curve in Fig. 4) are obtained. It shall be noted that if the extrapolation of the SETF conditions to the ITF conditions were not required and the various physics models employed in the ITF were truly uncoupled, then it is expected that the prior and posterior distributions results from the application of the BT & PML are very similar.

The framework for which the CASUALIDAD method has been developed deals with the dashed area in Fig. 4, or in other terms the methodology is system oriented having as main objective the improvements of the estimations of the system output responses and related covariance matrix (i.e. uncertainty) through the reduction and improvement of the input parameter values and covariance. Key elements for the methodology are:

1. Availability of a frozen qualified and internationally recognized thermal-hydraulic code;
2. Availability of a suitable large database of ITF containing different transient scenarios and different scales of facility (e.g. the set of tests belonging to the CIAU database [20]);
3. Use of a robust and rigorous procedure (i.e. set of acceptability criteria to be satisfied) for the qualification and acceptance of thermal-hydraulic system code calculations [20, 21]);
4. Availability of adequate (i.e. exact and efficient from CPU time point of view) methods for performing the local and global sensitivity analysis. The powerful tool based on ASAP is implemented for the derivation of the sensitivity matrix **S** containing the local derivatives of any response R_n (obtained by the code) respect to any parameter α_i ($\partial R_n/\partial \alpha_i$). For large-scale systems, in which the number of system parameters and/or parameter variations to be considered exceeds the number of responses of interest, the ASAP is, by far, the most advantageous method to employ. The global sensitivity analysis is performed through the implementation of the GASAP method aimed at determining all of the system's critical points and subsequently analyze them locally by ASAP. Thus, the strong non-linearities charactering the NPP system can be efficiently considered during the sensitivity analysis and taking into account for the following step dealing with the uncertainty evaluation;
5. Implementation of the DAA method based on the Bayesian Theorem and the Principle of the Maximum Likelihood for updating the "a priori" PDF of the input parameters $\boldsymbol{\alpha}$ and responses **R** with the available experiments $\boldsymbol{M}$ ("likelihood observations", see also element 2) for getting the "posterior" Improved Estimation (IE) of the input parameters $\boldsymbol{\alpha}^{IE}$, responses $\mathbf{R}^{IE}$ and related covariance matrixes;
6. Use of the concept of Status Approach for grouping together the "posterior" Improved Estimations of the input parameters $\boldsymbol{\alpha}^{IE}$, responses $\mathbf{R}^{IE}$ and related covariance matrixes deriving from similar transients, i.e. transients selecting the same path in the phase space of the selected driving quantities. A database of improved estimations is then generated and the appropriate information stored inside (i.e. the one in the phase space selected by the NPP calculation) can be used during the application process of the methodology (see Fig. 6).

The following are assumptions used in the methodology, notwithstanding criteria embedded in the methodology itself and evidences from other methods allow, respectively, detection when they are violated (and thus when the methodology can not be applied) or demonstration of their well-founded basis:

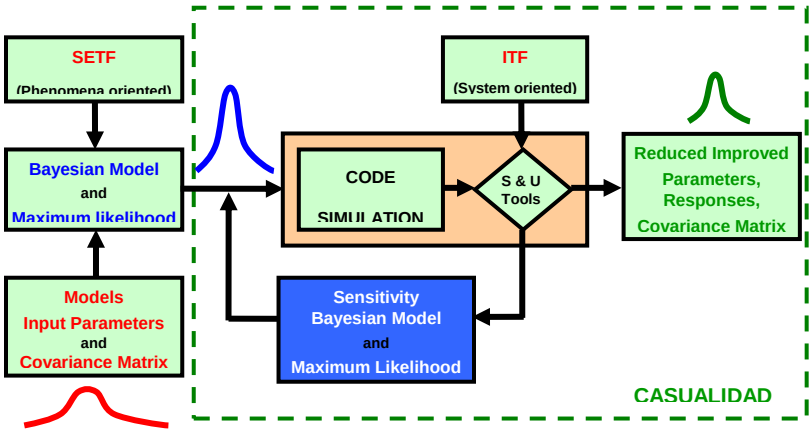

Figure 4 The basic idea and framework of the CASUALIDAD method.

a. Phenomena and transient scenarios in larger scale facilities are close enough to plant conditions (see also similar discussion for CIAU and UMAE). This assumption also supports the element 6 in the previous list;
b. Given that the discrepancy between measured and calculated responses (and not the absolute values) is used for evaluating the uncertainty, this difference is randomly dispersed around the zero value independent on the volume scaling factors of the facilities [6]. Moreover the methodology provides a consistent indicator χ^2 that quantifies the degree of consistency between calculation and experiment and allows neglecting the contributions of the related discrepancies to the uncertainty estimation when the consistency is poor (i.e. χ^2 far from unity value) [15];
c. The influence of user and nodalisation upon the uncertainty of the response is minimized in the methodology by the use of robust qualification procedures (see element 3 of the previous list). The consistent indicator χ^2 contributes to minimize this influence discharging the cases where the consistency between calculation and experiment are poor.

3.2 The structure of the method

The CASUALIDAD methodology has been structured in two main steps. The former has the aim to generate the database of improved estimations starting from the available set of experimental data and related qualified calculations; the latter is dealing with the use of the selected (from the obtained database) set of improved estimations for the uncertainty evaluation of the predicted NPP transient scenario. More details about the two processes are given hereafter.

CASUALIDAD, Development Process

The development of the method and related database implies the availability of qualified experimental ITF data (block a in Fig. 5), the use of UMAE as a robust and consistent procedure for qualifying thermalhydraulic system code calculations (block b) and the availability of a frozen version of system thermal-hydraulic code (block c). These requirements constitute pre-requisites to the development of the methodology itself. Moreover it is assumed that a Steady-State (SS) condition is achieved before running the transient scenario, and consequently the values of input parameters characterizing the Boundary and Initial Conditions (BIC) of the problem have to be considered after SS achievement. More precisely four different kinds of input parameters $\boldsymbol{\alpha}$ are distinguished:

1. The parameters $\boldsymbol{\alpha}_{SS}^{BIC}$ (block d) characterizing the Boundary, Initial Conditions (BIC), evaluated after SS achievement (e.g. the distribution of temperatures inside a core channel or the core inlet mass flow rates);
2. The parameters $\boldsymbol{\alpha}^{PP}$ (block e) used for specifying Properties and Phenomena (PP) of the system. They include for instance the geometrical properties of the system (e.g. the flow area of a pipe or the core active heat transfer surface area), the physical properties of the materials and also parameters characterizing complex thermal-hydraulic aspects, like the pressure drops distribution (e.g. the k form loss coefficients in RELAP). Respect with $\boldsymbol{\alpha}_{SS}^{BIC}$, the $\boldsymbol{\alpha}^{PP}$ parameters are those parameters whose values specified in the input deck do not change when the SS has achieved;
3. The parameters $\boldsymbol{\alpha}^{NOD}$ (block f) characterizing the nodalization (NOD), i.e. those parameters that constitute the discretization of the real system into an ensemble of control volumes and junctions

(e.g. in RELAP the pipes, branches, junctions, etc…) through the definition of the volume lengths and elevations, the volume flow areas, etc…;

4. The parameters $\boldsymbol{\alpha}^{CODE}$ (inside block c) that are part of the correlations inside the thermal-hydraulic system code. Respect with the other parameters, the $\boldsymbol{\alpha}^{CODE}$ are not directly accessible (modifiable) from the input deck.

The selection of the responses of interest (block g) among the thousands available from a code calculation implies an engineering judgment that must consider the relevance of the safety and licensing issues. No limitation exists when performing the sensitivity analysis (see blocks i and k). The situation is instead different when performing the uncertainty analysis (see blocks l and o), where practically the constrain derives from the available experimental responses (block a). Once the selected predicted responses $\boldsymbol{R}$ (block h) have been qualified through the application of the UMAE procedure, they are passed to the sensitivities tools (block i) together with all input parameters $\boldsymbol{\alpha}$. The sensitivity analysis is then performed locally and globally in order respectively to obtain exact local derivatives of any response R_n respect to any input parameter α_i ($\partial R_n/\partial \alpha_i$) and to determine all of the system's critical points $\mathbf{y}(\boldsymbol{\alpha})$ (bifurcations, turning points, saddle points, response extrema) and their derivatives ($\partial y_m/\partial \alpha_i$) respect to any input parameter α_i (block j). The local sensitivity analysis is performed by ASAP [13] or alternatively can be carried out by FSAP [13]. The global sensitivity analysis is performed by GASAP [14] and makes possible to efficiently deal with the non-linearities characterizing the NPP system. Finally, the sensitivity matrix $\mathbf{S}$ (block k) is obtained and can be used for the following purposes (among others):

a) To support the qualification process of the nodalization through quantitative parameters that allow the verification of possible 'unstable' behaviours of the developed nodalization, i.e. if a small change in any of the input parameters $\boldsymbol{\alpha}$ causes a large variation in the selected relevant responses $\mathbf{R}$. In fact, the matrix $\mathbf{S}$ supplying the derivatives of any selected response respect with any parameter (e.g. the derivative of the core PCT respect to the length of one node in the U-tube of the primary side of a steam generator) constitutes a large source of information for performing the qualification process;

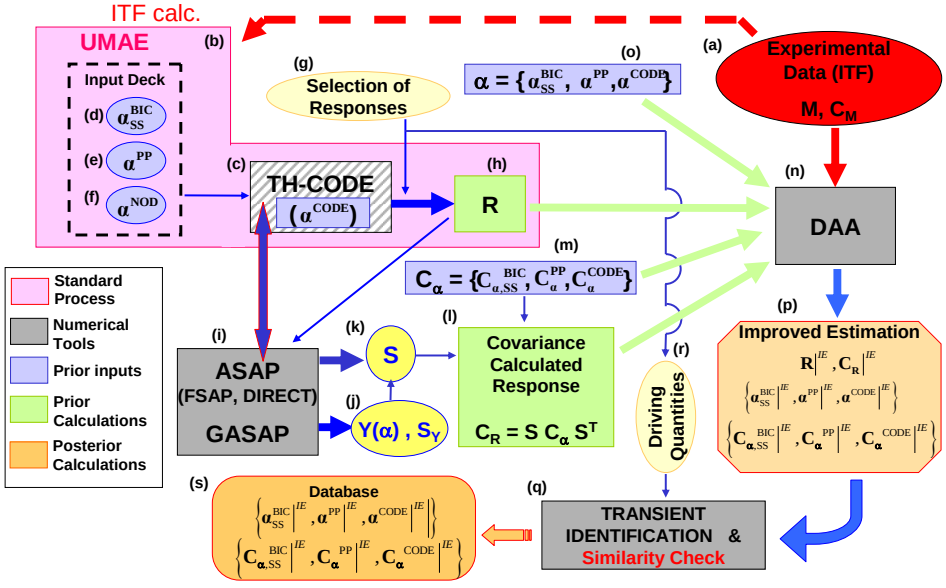

Figure 5 Flow-chart of CASUALIDAD method: development process.

b) To understand the system by highlighting important data and eliminating the unimportant ones;
c) To design and optimize the system (e.g., maximize availability/minimize maintenance);
d) To perform the uncertainty analysis (see blocks l and o).

While the sensitivity analysis is able to consider all parameters $\boldsymbol{\alpha}$, the uncertainty methodology here developed can not deal 'directly' with the $\boldsymbol{\alpha}^{NOD}$ parameters. The reason for such a limitation is due to the lack of information about the variance (and generally speaking about the covariance matrix) of the parameters characterizing the nodalization (e.g. the length of a volume in a PIPE component in RELAP). However, as deducible from the word 'directly', it shall be emphasized that this limitation does not imply that the input uncertainties arising from the development of the nodalization do not contribute to the final evaluation of the uncertainty of the responses. In fact, these input uncertainties affect the discrepancy between the experimentally measured values and the calculated values of the responses, and thus finally affect the uncertainty evaluation.

The a priori knowledge (block m) of the covariance matrixes $\boldsymbol{C}_{\alpha,SS}^{BIC}$, $\boldsymbol{C}_{\alpha}^{PP}$, $\boldsymbol{C}_{\alpha}^{CODE}$ (related to the BIC, PP and CODE parameters), deduced from SETFs, available literature or based on the engineering judgement, is then used in the "propagation of moments" equation to obtain the covariance matrix of the calculated responses (block l). The next step (block n) implies the use of the Data Adjustment and Assimilation (DAA) as a methodology based on the Bayesian Theorem and the Principle of the Maximum Likelihood for consistently incorporating observed information into a predicting model. More specifically, the "a priori" knowledge of the:

a) input parameters $\boldsymbol{\alpha}_{SS}^{BIC}$, $\boldsymbol{\alpha}^{PP}$, $\boldsymbol{\alpha}^{CODE}$ (block o),
b) input covariance matrixes (block m),
c) calculated response **R** (block h),
d) calculated covariance matrix $\boldsymbol{C}_{R}$ (block l),

is combined in a statistically optimal way with "likelihood observations" (block a), i.e.:

e) experimental measures of the responses **M**,
f) covariance matrix of the measured responses $\boldsymbol{C}_{M}$,

to consistently improve and reduce the posterior estimations (block p) of the:

g) input parameters $\boldsymbol{\alpha}_{SS}^{BIC}\Big|^{IE}, \boldsymbol{\alpha}^{PP}\Big|^{IE}, \boldsymbol{\alpha}^{CODE}\Big|^{IE}$,
h) calculated responses $\mathbf{R}\Big|^{IE}$,
i) input covariance matrixes $\boldsymbol{C}_{\alpha,SS}^{BIC}\Big|^{IE}, \boldsymbol{C}_{\alpha}^{PP}\Big|^{IE}, \boldsymbol{C}_{\alpha}^{CODE}\Big|^{IE}$,
j) calculated covariance matrix $\boldsymbol{C}_{R}\Big|^{IE}$.

Moreover, the DAA method supplies a quantitative measure of the consistency of the process through an indicator factor that allows to quantify the degree of the discrepancy between calculation and experiment and to not perform the adjustment and assimilation when the consistency is poor. As the CASUALIDAD methodology is system oriented, the experimental data to be incorporated in the DAA process are mostly dealing with ITF. However, SETF and also NPP operational transients can be processed if available.

The final step of the methodology has the goal to generate a database of (posterior) improved and reduced estimations of the input parameters and related covariance matrixes. For each of the

experimental test, the process up to the block p has to be repeated for generating the posterior estimations. Considering that different kinds of scenario can be processed and that the same phenomena can occur in different transients, the concept of the "Status Approach" is adopted for generating the database. The idea here is to group together the improved estimations of the input parameters $\boldsymbol{\alpha}^{IE}$ and related covariance matrixes $\boldsymbol{C}_{\alpha}^{IE}$ deriving from 'similar' transients, i.e. transients selecting the same path in the phases space of the selected driving quantities. It is assumed that similar tests generate similar values of the percentages of parameters' variation between "posterior" and 'prior' estimations (similarity check). Thus, if the similarity check is positively passed, the transient identification process (block q), based on the phases space subdivision characterized by the selected driving quantities (block r), allows identifying the location inside the database where to store the obtained posterior estimations (block s).

CASUALIDAD, Application Process

The application of the CASUALIDAD is straightforward once the database of the improved estimations of the input parameters and related covariance matrixes is available.

The UMAE method (block a, in Fig. 6) is used to qualify the NPP transient calculation obtained by a qualified thermal-hydraulic system code (block b). The input parameters characterizing both the input-deck (i.e. $\boldsymbol{\alpha}_{SS}^{BIC}$, $\boldsymbol{\alpha}^{PP}$, $\boldsymbol{\alpha}^{NOD}$, block c) and the correlations inside the code ($\boldsymbol{\alpha}^{CODE}$, inside block b) are evaluated on the 'first-guess' basis (i.e. values deduced from SETFs, available literature or based on the engineering judgment). The relevant responses from safety and licensing point of view must be selected (d) in agreement with the selection process performed during the development of the methodology. The transient evolution, defined by the quantities **R** (block e), selects a path in the phase space subdivision (characterized by the selected driving quantities, f), that can be identified through the transient identification process (block g). Thus, the posterior estimations of the input parameters (block h) and input covariance matrixes (block i) associated to the selected path in the phases space can be drawn out from the database.

A second thermal-hydraulic system code calculation is set up using the posterior estimations of the parameters characterizing both the input deck ($\boldsymbol{\alpha}_{SS}^{BIC}\big|^{IE}$, $\boldsymbol{\alpha}^{PP}\big|^{IE}$) and the correlations inside the code ($\boldsymbol{\alpha}^{CODE}\big|^{IE}$) in order to get an improved estimation of the responses $\mathbf{R}^{IE}$ (block j). The UMAE method is still used to qualify the improved NPP transient calculation.

The sensitivity analysis is then performed locally and globally (block k) to derive the system's critical points (block l) and the derivatives of any response respect with any input parameter (block m). The sensitivity matrix **S** can be used for the same purposes stated in the development process of the methodology having as reference the NPP instead of the ITF system. Lastly, the uncertainty evaluation is performed by the "propagation of moments" equation where the improved estimations of the input covariance matrixes $\boldsymbol{C}_{\alpha,SS}^{BIC}\big|^{IE}$, $\boldsymbol{C}_{\alpha}^{PP}\big|^{IE}$, $\boldsymbol{C}_{\alpha}^{CODE}\big|^{IE}$ are used (block i) to obtain the improved estimation of the covariance matrix of the calculated responses $\boldsymbol{C}_{R}^{IE}$ (block n). The diagonal values on the $\boldsymbol{C}_{R}^{IE}$ matrix allow finally the generation of continuos uncertainty bands for the selected responses $\mathbf{R}^{IE}$ (block o). An example of the application of CASUALID is given in [22].

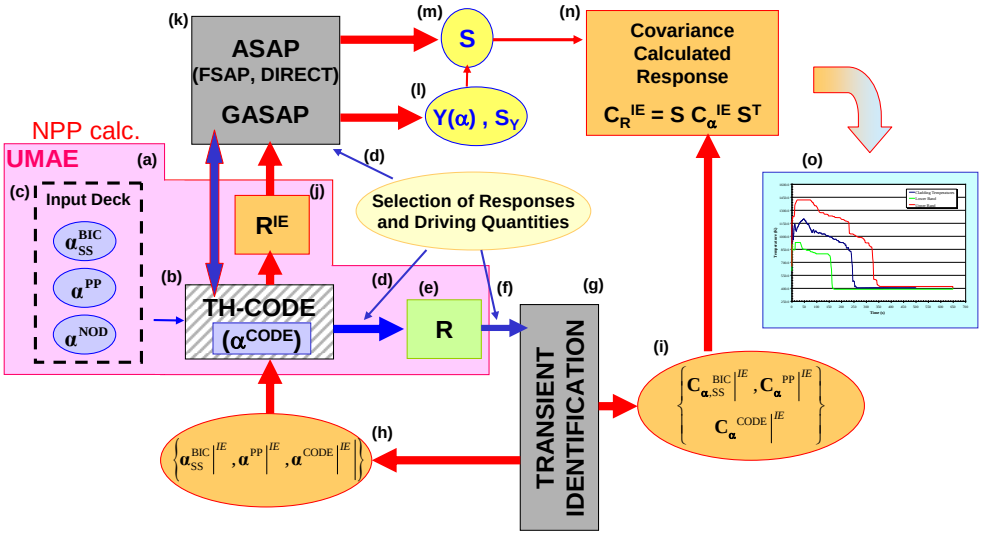

Figure 6 Flow-chart of CASUALIDAD method: application process.

4. Conclusions

The uncertainty evaluation constitutes the ending necessary step for the application of a system thermal-hydraulic code to the nuclear technology. Therefore, any application of a best estimate code without the uncertainty evaluation is meaningless because an error is unavoidable for any prediction. The differences between accuracy, uncertainty and sensitivity have been emphasized and the origins of, or the reasons for, uncertainty should be clearly in mind when developing an uncertainty approach.

Three main independent ways have been described in the paper to evaluate the uncertainty even though focus has been given to the innovative approach, named CASUALIDAD, based on the Bayesian inference technique and on the availability of experimental data by which computer model predictions can be improved and the ranges of variation of input parameters can be characterized.

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