7th International Conference on Modelling and Simulation in Nuclear Science and Engineering (7ICMSNSE)
Ottawa Marriott Hotel, Ottawa, Ontario, Canada, October 18-21, 2015

7ICMSNSE-100

An Approach for Modelling CANDU Fuel String Vibration Induced by Unsteady Flow

M. Fadaee and S. D. Yu
Department of Mechanical and Industrial Engineering, Ryerson University,
Toronto, Ontario, M5B 2K3
mofadaee.ryerson.ca, syu@ryerson.ca

Abstract

A comprehensive dynamical model is presented in this paper to handle vibration a string of 12 CANDU6 fuel bundles inside a pressure tube under operating conditions. A finite element based computer program is developed at Ryerson University in collaboration with Candu Energy Inc. to simulate fuel string vibration and vibration induced wear in the pressure tube.

The focus of this paper is dynamic frictional contact among fuel elements via spacer pads, between fuel elements and the pressure tube via bearing pads, and between neighboring fuel bundles via endplates. The types of deformations are bending, torsion and axial for the fuel elements, and in-plane and out-of-plane bending for the endplates. The system equations of motion are discretized in the time-domain by means of the Bozzak-Newmark scheme; the contact problem is handled using an iterative LCP algorithm. The unsteady flow and flow induced excitations are obtained using the FLUENT-LES.

Keywords: Candu fuel, unsteady flow, vibration

1. Introduction

In a CANDU fuel channel, the heavy water coolant flows through a string of fuel bundles and brings out the heat generated by fuel elements for steam production. To understand the complex behaviors of flow-induced vibration of the fuel string, accurate and reliable models must be developed to capture the elasto-rigid motions of fuel elements subjected to non-smooth unilateral frictional contact constraints at multiple locations. The friction between the outer fuel element and the pressure tube is two-dimensional. An individual fuel element moves with the fuel bundle in a rigid body manner in the closely packed spaces of a fuel channel, and deforms as a slender structure in the form of bending, axial and torsional deformations.

Past and recent nuclear station condition reports (e.g., Norsworthy and Ditschun[1]) indicate that CANDU fuel bundles vibrate during operations. Vibrations has caused and continue to cause excessive wear of the pressure tubes, undesired amount of debris in the coolant, and even loss of bundle integrity due to failures of components such as cracking of the endplates[2]. Acoustic pressure pulsation and flow-induced vibrations (FIV) are known to be the main causes of fuel bundle vibrations [3]-[5].Yetisir and Fisher [6] studiedthe effect of turbulence excitation on fretting wear between bearing pads and pressure tube. Hassan and Rogers [7] investigate vibration of a fuel element by applying several frictional models to understand the effect of tube-support clearance and preload.Xu et al. [8] examined bending vibration of a single fuel element subjected to frictionless contact by means of beam finite element method and the Wilson-θ method. Recently Yu and Fadaee [9]presented a finite element model for bending, axial, and torsional free vibrations of a straight beam using three-node higher-order mixed finite element.

Flow-induced vibration of fuel bundles is a very challenging problem. One of the challenges is the multiple unilateral frictional contact (MUFC) constraints between the fuel elements and pressure tube, between the neighbouring fuel elements and between neighbouring bundles.Finite element method is used to model the complex geometry of a fuel element and the non-smooth constraints. Fuel elements experience small rigid displacements and small elastic displacements due to the limited available spaces inside a fuel channel. This allows for the use of linear theories (linear relationships between stresses and strains, and linear relationships between strains and displacement gradients), and more importantly consideration of rigid body displacements within the framework of the structural finite elements. This makes it possible to develop a feasible fuel string vibration model for simulating fuel string fretting and fretting induced component wear in a fuel channel.

An implicit incremental displacement Bozzak-Newmark scheme is then employed to seek a numerical solution in the time domain for vibrations of two fuel bundles subjected to 2D friction and unilateral contact constraints. To be able to effectively handle the two types of the non-smooth constraints - 2D friction and unilateral contact, the equations of system states are formulated in terms of the incremental displacements. In handling the multiple unilateral frictional constraints at a time step, the sub-structuring method is used to eliminate all interior DOF's[10]. The coupled gap equations in the directions of all probable contact points and the their associated frictional forces in the two tangential directions (axial and circumferential) are reduced, through variable transformations and an auxiliary incremental displacement variable, to a linear complementarity problem (LCP) for which a solution can be obtained using the Lemke algorithm. At each time step, the incremental displacement vectors are resolved into the tangential and normal directions of motion. Base on Coulomb’s law of friction, the frictional force acts in the direction opposite to the true direction of motion or tendency of motion. In the proposed approach, the direction angle of the frictional force is estimated based on the velocities at the end of the previous time step. For small time steps, the proposed scheme yields satisfactory results without iterations. The contact forces in the radial direction and the frictional force in the tangential direction along with the sliding velocities are computed for eachpaired contact. These parameters can be used to assess the material loss of the pressure tube.

2. Dynamic equations of an unconstrained fuel string

Contact/impact and sliding motion between different components of CANDU fuel string cause, and accelerate, the wear of the contacting components. The wear damage is directly related to the contact/impact forces and accumulated sliding distances.

UFC among a string of fuel bundles are classified into three types (I) contact between fuel elements via spacer pads within a fuel bundle (internal contact nodes), (II) contact between neighboring bundles via endplates, and (III) contact between the bundles and the pressure tube via the bearing pads (external contact nodes). In this paper, a node-to-node (NTN) contact scheme will be used to determine the normal forces and sliding velocities for type I and II of contacts. A node-to-surface (NTS) contact scheme will be used to determine the actual contact/impact regions, normal forces, and sliding velocities for type II and III contacts.

In the case of contact between fuel elements via spacer pads, a single node will be arranged at the middle of each spacer as a contact node. In the case of contact between the bearing pads and the pressure tube, three contact nodes are arranged axially along each bearing pad. For contact between two fuel bundles, same number of nodes as fuel elements will be considered at both contacting bundles.

The Coulomb frictional law will be employed to formulate the interactions between two contacting surfaces. The proposed procedure for handling NTN and NTS contacts is based on the Bozzak-Newmark integration scheme, introduction of auxiliary incremental variables, and the LCP scheme. The use of the LCP approach at a time step eliminates the need for iterations for multiple UFC constraints.

Frictional sliding between two-dimensional contacting surfaces is known to dissipate energy and cause damage in many engineering systems, including fretting wear of the supporting pressure tube of CANDU fuel bundles in a fuel channel. The non-smooth characteristics of frictional forces represented by the equality equations for non-zero sliding velocities and the in-equality equations for zero sliding velocity make it very difficult to accurately model the behaviour of a stick-slip vibrational system subjected to dry friction. For a vibrational system, the beginning and ending of each state (stiction or slip) cannot be known a priori. Although, for some very simple vibration systems, it may be possible to determine the precise moment of occurrence of each state, it is impossible to obtain a solution on an individual basis without a systematic approach for a large-scale dynamical system with hundreds or thousands of UFC constraints such as CANDU fuel string vibration in a pressure tube. In addition, for realistic values of dry frictional coefficients, the well-known stable integration schemes, such as the Runge-Kutta method, would become unstable and fail to yield credible results. To overcome the challenges, a new incremental displacement based numerical procedure is developed in conjunction with the Bozzak-Newmark integration and the LCP approach. Case studies conducted to date show that the proposed procedure is accurate and robust, and can capture the precise occurrence of the slip-stick motion of a vibration system. It is proposed here that the new scheme is extended to a large- scale vibration system and implemented into the FSFIV model.

Equations of motion of a fuel bundle at time $\mathrm{t} = \mathrm{t}_{\mathrm{i}+1}$ in terms of incremental displacement after sub structuring interior nodes may be written as

$$ {}^{j}k^{*}\{{}^{j}\Delta q\}_{i+1} = \{{}^{j}Q^{*}\}_{i+1} - \{{}^{j}Q_{c}\}_{i+1} + \{{}^{j}Q_{f}\}_{i+1} \tag{1} $$

where incremental displacement are sorted as

$$ \{{}^{j}\Delta q\} = \begin{Bmatrix} \Delta q_{L} \\ \Delta q_{R} \\ \Delta q_{a} \\ \Delta q_{b} \end{Bmatrix} $$

${}^{j}Q_{c}$ are the contact forces; ${}^{j}Q_{f}$ are the friction forces; Subscripts L, R, a and b represent left, right, external and internal contact nodes respectively; Left superscripts is the bundle number, $j = 1{:}\,m$ where m is the maximum number of fuel bundles. Eq. (1) may be assembled for all bundles to form the fuel string,

$$ \begin{bmatrix} {}^{1}k^{*5} & 0 & 0 & 0 \\ 0 & {}^{2}k^{*5} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & {}^{m}k^{*5} \end{bmatrix} \begin{Bmatrix} {}^{1}\Delta q \\ {}^{2}\Delta q \\ \vdots \\ {}^{m}\Delta q \end{Bmatrix}_{str} = \begin{Bmatrix} {}^{1}Q^{*} \\ {}^{2}Q^{*} \\ \vdots \\ {}^{m}Q^{*} \end{Bmatrix} - \begin{Bmatrix} {}^{1}Q_{c} \\ {}^{2}Q_{c} \\ \vdots \\ {}^{m}Q_{c} \end{Bmatrix} + \begin{Bmatrix} {}^{1}Q_{f} \\ {}^{2}Q_{f} \\ \vdots \\ {}^{m}Q_{f} \end{Bmatrix} \tag{2} $$

For each interface incremental displacement may be written in terms of relative displacement to be able to use Columbus friction law and in order to capture actual amount of sliding.

For the interface number *l*, right nodes of bundle *j*, ${}^{j}\Delta q_R$, are in contact with left nodes of bundle *j+1*, ${}^{j+1}\Delta q_L$. In this study for any interface *l*,right nodes of bundle *j*, ${}^{j}\Delta q_R$, are considered as contactors and the corresponding ones on the left side of bundle *j+1*, ${}^{j+1}\Delta q_L$are the targets. Directions of gaps are defined from contactors to targets.

Interface incremental displacement may be related to relative displacement using matrix transformation. For one interface this transformation looks like

$$\begin{Bmatrix} {}^{j}\Delta q_R \\ {}^{j+1}\Delta q_L \end{Bmatrix} = \begin{bmatrix} I_i & 0 \\ I_i & I_i \end{bmatrix} \begin{Bmatrix} {}^{j}\Delta q_R \\ {}^{l}\Delta q_{L/R} \end{Bmatrix} \tag{3}$$

Where ${}^{l}\Delta q_{L/R}$ is the relative incremental displacement of targets with respect to the contactors, at interface *l*. Transformation matrixin (3) may be written for the whole fuel string as

$$\begin{Bmatrix} {}^{1}\Delta q_R \\ {}^{2}\Delta q_L \\ {}^{2}\Delta q_R \\ {}^{3}\Delta q_L \\ {}^{3}\Delta q_R \\ \vdots \end{Bmatrix} = \begin{bmatrix} I_i & 0 & & & 0 \\ I_i & I_i & & & \\ & & I_i & 0 & \\ & & I_i & I_i & \\ sym & & & & \ddots \end{bmatrix} \begin{Bmatrix} {}^{1}\Delta q_R \\ {}^{1}\Delta q_{L/R} \\ {}^{2}\Delta q_R \\ {}^{2}\Delta q_{L/R} \\ {}^{3}\Delta q_R \\ \vdots \end{Bmatrix} = T_I \begin{Bmatrix} {}^{1}\Delta q_R \\ {}^{1}\Delta q_{L/R} \\ {}^{2}\Delta q_R \\ {}^{2}\Delta q_{L/R} \\ {}^{3}\Delta q_R \\ \vdots \end{Bmatrix} \tag{4}$$

Lets only look the first interface nodes, Eq. (1) may be rewritten as

$$k^* \begin{Bmatrix} {}^{1}\Delta q_R \\ {}^{2}\Delta q_L \\ \vdots \end{Bmatrix}_{str} = \begin{Bmatrix} {}^{1}Q_R^* \\ {}^{2}Q_L^* \\ \vdots \end{Bmatrix} - \begin{Bmatrix} {}^{1}Q_{c\,R} \\ {}^{2}Q_{c\,L} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} {}^{1}Q_{f\,R} \\ {}^{2}Q_{f\,L} \\ \vdots \end{Bmatrix} \tag{5}$$

Substitute Eq. (4) into (5) and pre multiply the so obtain equation by T_I^T, one may obtain

$$T_I^T k^* T_I \begin{Bmatrix} {}^{1}\Delta q_R \\ {}^{1}\Delta q_{L/R} \\ \vdots \end{Bmatrix}_{str} = \begin{Bmatrix} {}^{1}Q_R^* + {}^{2}Q_L^* \\ {}^{2}Q_L^* \\ \vdots \end{Bmatrix} - \begin{Bmatrix} {}^{1}Q_{c\,R} + {}^{2}Q_{c\,L} \\ {}^{2}Q_{c\,L} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} {}^{1}Q_{f\,R} + {}^{2}Q_{f\,L} \\ {}^{2}Q_{f\,L} \\ \vdots \end{Bmatrix} \tag{6}$$

According to Newton's third law we may notice that friction and contact forces acting on each bundle have the same magnitude and opposite direction of those acting on the neighboring bundle at the same interface. This can be stated as following for the first interface,

$${}^{1}Q_{c\,R} = -\,{}^{2}Q_{c\,L}, \qquad {}^{1}Q_{f\,R} = -\,{}^{2}Q_{f\,L} \tag{7}$$

More generally for interface *l*, Eq. (7) may be written as

$${}^{j}Q_{c\,R} = -{}^{j+1}Q_{c\,L}, \qquad {}^{j}Q_{f\,R} = -{}^{j+1}Q_{f\,L} \tag{8}$$

Substituting Eq. (8) into (6)

$$T_I^T k^* T_I \begin{Bmatrix} {}^{1}\Delta q_R \\ {}^{1}\Delta q_{L/R} \\ \vdots \end{Bmatrix}_{str} = \begin{Bmatrix} {}^{1}Q_R^* + {}^{2}Q_L^* \\ {}^{2}Q_L^* \\ \vdots \end{Bmatrix} - \begin{Bmatrix} 0 \\ {}^{2}Q_{c\,L} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} 0 \\ {}^{2}Q_{f\,L} \\ \vdots \end{Bmatrix} \tag{9}$$

Now sub-structuring scheme may be used in order to reduce the size of the problem that should be solved using a LCP solver. Interior nodes are not involved explicitly in the frictional contact formulation and therefore contact or frictional forces on these nodes are zero. This may be written as

$$\begin{bmatrix} k_{II}^{*} & k_{Io}^{*} \\ k_{oI}^{*} & k_{oo}^{*} \end{bmatrix} \begin{Bmatrix} q_I \\ q_o \end{Bmatrix}_{i+1} = \begin{Bmatrix} Q^{*}{}_{I} \\ Q^{*}{}_{o} \end{Bmatrix}_{i+1} - \begin{Bmatrix} Q_c \\ 0 \end{Bmatrix}_{i+1} + \begin{Bmatrix} Q_f \\ 0 \end{Bmatrix}_{i+1} \tag{10}$$

Where subscript I refer to interfacial nodes (have at least one type of contact); subscript O refer to interior (nodes without contact). Interior nodes can be eliminate from Eq. (10) and reduced equation of equilibrium may be written as

$$k^{**}\{q_I\}_{i+1} = \{Q^{**}\}_{i+1} + \{Q_f\}_{i+1} - \{Q_c\}_{i+1} \tag{11}$$

where, $k^{**} = k_{II}^{*} - k_{Io}^{*} {k_{oo}^{*}}^{-1} k_{oI}^{*}$
$\{Q^{**}\}_{i+1} = -k_{Io}^{*} {k_{oo}^{*}}^{-1} \{Q^{*}{}_{o}\}_{i+1} + \{Q^{*}{}_{I}\}_{i+1}$

2.1 **Formulating gaps**

2.1.1 Bundle-to-bundle gap

For the interface l, between bundles j and $j+1$, one may define the gap as

$$\{ {}^{l}g_{lr} \}_{i+1} = \{ {}^{l}g_{lr} \}_{i} + \{ {}^{j+1}\Delta u_{n_L} \}_{i+1} - \{ {}^{j}\Delta u_{n_R} \}_{i+1}$$

whereΔu_n is the displacement of the contacting nodes in direction of gap. This can be represented in terms of relative displacement

$$\{ {}^{l}g_{lr} \}_{i+1} = \{ {}^{l}g_{lr} \}_{i} + \{ {}^{l}\Delta u_{n_{L/R}} \}_{i+1} \tag{12}$$

Note that

$${}^{l}\Delta u_{n_{L/R}} = {}^{j+1}\Delta u_{n_L} - {}^{j}\Delta u_{n_R}$$

for each interface right (${}^{j}\Delta u_R$) is the contactor and left (${}^{j+1}\Delta u_L$) is the target. ${}^{l}\Delta u_{L/R}$is displacement of the target relative to the contactor. Positive direction of gap is from right to the left.
For all bundles Eq. (12) can be written as,

$$\begin{Bmatrix} {}^{1}g_{lr} \\ {}^{2}g_{lr} \\ \vdots \\ {}^{n-1}g_{lr} \end{Bmatrix}_{i+1} = \begin{Bmatrix} {}^{1}g_{lr} \\ {}^{2}g_{lr} \\ \vdots \\ {}^{n-1}g_{lr} \end{Bmatrix}_{i} + \begin{Bmatrix} {}^{1}\Delta u_n \\ {}^{2}\Delta u_n \\ \vdots \\ {}^{n-1}\Delta u_n \end{Bmatrix}_{L/R_{i+1}} \tag{13}$$

2.1.2 External gap formulation

The contact force from pressure tube acting on fuel element is modelled as a gap activated spring. The contact force will present only when the initial gap is consumed. An auxiliary coordinate y introduced to represent the position of the spring.
For bundle *j* one can write the following relation between incremental displacement and gap,

$$\{{}^{j}y\}_{i+1} = \{{}^{j}g_a\}_{i+1} - \{{}^{j}g_a\}_i + \{{}^{j}\Delta u_{ar}\}_{i+1} \tag{14}$$

If the stiffness of the gap-activated spring is K (for bundle j), equation of equilibrium at time t_{i+1} may be written as

$$[{}^{j}K]\{{}^{j}y\}_{i+1} = \left\{{}^{j}F_{c_a}\right\}_{i+1} \tag{15}$$

Substitute Eq. (14) into (15)

$$[{}^{j}K]\left(\{{}^{j}g_a\}_{i+1} - \{{}^{j}g_0\} + \{{}^{j}u_{ar}\}_i + \{{}^{j}\Delta u_{ar}\}_{i+1}\right) = \left\{{}^{j}F_{c_a}\right\}_{i+1} \tag{16}$$

For the whole string of bundles we may obtain,

$$[K]\left(\begin{Bmatrix} {}^{1}g_a \\ {}^{2}g_a \\ \vdots \\ {}^{n}g_a \end{Bmatrix}_{i+1} - \begin{Bmatrix} {}^{1}g_a \\ {}^{2}g_a \\ \vdots \\ {}^{n}g_a \end{Bmatrix}_{i} + \begin{Bmatrix} {}^{1}\Delta u_{ar} \\ {}^{2}\Delta u_{ar} \\ \vdots \\ {}^{n}\Delta u_{ar} \end{Bmatrix}_{i+1}\right) = \begin{Bmatrix} {}^{1}F_{c_a} \\ {}^{2}F_{c_a} \\ \vdots \\ {}^{n}F_{c_a} \end{Bmatrix}_{i+1}$$

where

$$[K] = \begin{bmatrix} {}^{1}K & 0 & 0 & 0 \\ 0 & {}^{2}K & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & {}^{n}K \end{bmatrix}$$

2.1.3 Internal gap formulation

Internal gaps are defined as distance between spacer pads on neighbouring fuel rods at the mid-plane. For each bundle internal incremental displacement is related to internal gap as

$${}^{j}g_{b_{i+1}} = {}^{j}g_{b_i} + \left[{}^{j}T_{bb}\right]_i {}^{j}\Delta u_{b_{i+1}} \tag{17}$$

where ${}^{j}T_{bb}$ is the transformation matrix between global coordinate and the local internal gap coordinate.For the whole fuel string one may write

$$\begin{Bmatrix} {}^{1}g_b \\ {}^{2}g_b \\ \vdots \\ {}^{n}g_b \end{Bmatrix}_{i+1} = \begin{Bmatrix} {}^{1}g_b \\ {}^{2}g_b \\ \vdots \\ {}^{n}g_b \end{Bmatrix}_{i} + \begin{bmatrix} {}^{1}T_{bb} & 0 & 0 & 0 \\ 0 & {}^{2}T_{bb} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & {}^{n}T_{bb} \end{bmatrix}_i \begin{Bmatrix} {}^{1}\Delta u_b \\ {}^{2}\Delta u_b \\ \vdots \\ {}^{n}\Delta u_b \end{Bmatrix}_{i+1}$$

For all gaps following complementarity conditions holds true all the time,

$$g \geq 0, \qquad F_c \geq 0, \qquad g.F_c = 0 \tag{18}$$

2.2 Handling Friction

Relativeincremental displacement $\Delta q_{L/R}$, at each node on the interface, may be projected into tangent and normal directions, Δu_t and Δu_n. At each interface between two bundles, in the direction normal to the contacting surface, the following equation of equilibrium is assumed to hold at all times,

$$P_{i+1} = N_{i+1} \tag{19}$$

where$P_{i+1} > 0$ is the net external normal force applied on each nodes of contact at t_{i+1} and N_{i+1} is the normal force acting on the same node. Multiplying Eq. (19) by the coefficient of friction, μ, we obtain the following alternative form of equation of equilibrium

$$\mu N_{i+1} - \mu P_{i+1} = 0 \tag{20}$$

In this research, the kinetic and static coefficients of friction are considered equal. Four different scenarios are possible for the relativedisplacement$\Delta \mathrm{q}_{\mathrm{L/R}}$of the two contacting nodes (target with respect to contactor) in the tangent direction of motion: forward slip, stiction with tendency to move forward, stiction with tendency to move backward and backward slip. In each possible state, the frictional force is written as follow

1) Forward slip: $(\Delta u_t)_{i+1} > 0,\ \ (F_t)_{i+1} = -(\mu N)_{i+1}$
2) Forward stiction: $(\Delta u_t)_{i+1} = 0,\ -(\mu N)_{i+1} \le (F_t)_{i+1} \le 0$
3) Backward stiction: $(\Delta u_t)_{i+1} = 0,\ 0 \le (F_t)_{i+1} \le (\mu N)_{i+1}$
4) Backward slip: $(\Delta u_t)_{i+1} < 0,\ \ (F_t)_{i+1} = (\mu N)_{i+1}$

(21)

States 1 and 2 in Eq. (21) represent the motion or tendency of motion in the positive tangent direction where $(\Delta u_t)_{i+1} \ge 0$, and states 3 and 4 represent the motion or tendency of motion in the negative tangent direction where $(\Delta u_t)_{i+1} \le 0$. For the forward slip and forward stiction, the Coulomb's law of friction may be written as

$$(\mu N)_{i+1} + (F_t)_{i+1} \ge 0 \tag{22}$$

For states 1 and 2, we wish to introduce the following two new variables,

$$(\Delta \hat{u}_t)_{i+1} = \sup((\Delta u_t)_{i+1}, 0) \tag{23}$$

$$(\hat{s})_{i+1} = (\mu N)_{i+1} + (F_t)_{i+1} \tag{24}$$

where sup is the supremum of a set of variables; $(\Delta \hat{u}_t)_{i+1}$ is the supremum value of the incremental displacement; $(\hat{s})_{i+1}$ is the slack force. It can be verified that the so-defined variables, $(\Delta \hat{u}_t)_{i+1}$ and $(\hat{s})_{i+1}$, are non-negative, and satisfy the complementary condition. These conditions may be written as

$$(\Delta \hat{u}_t)_{i+1} \ge 0, \qquad (\hat{s})_{i+1} \ge 0, \qquad (\Delta \hat{u}_t)_{i+1} \cdot (\hat{s})_{i+1} = 0 \tag{25}$$

For the backward slip and tendency to move backward, $(\Delta u_t)_{i+1} \le 0$, the friction force points toward the positive tangential direction. The Coulomb's law of friction may be written as

$$(\mu N)_{i+1} - (F_t)_{i+1} \ge 0 \tag{26}$$

We introduce following two new variables,

$$(\Delta \check{u}_t)_{i+1} = \sup(-(\Delta u_t)_{i+1}, 0) \tag{27}$$

$$(\check{s})_{i+1} = (\mu N)_{i+1} - (F_t)_{i+1} \tag{28}$$

Again $(\Delta\breve{u}_t)_{i+1}$ and $(\breve{s})_{i+1}$ are non-negative and complementary to each other. The frictional interaction for states 3 and 4 may be written as

$$(\Delta\breve{u}_t)_{i+1} \geq 0, \qquad (\breve{s})_{i+1} \geq 0, \qquad (\Delta\breve{u}_t)_{i+1} \cdot (\breve{s})_{i+1} = 0 \tag{29}$$

It can be verified that the incremental displacement in the tangent direction is related to the two the supremum displacement variables as follows

$$(\Delta u_t)_{i+1} = (\Delta\hat{u}_t)_{i+1} - (\Delta\breve{u}_t)_{i+1} \tag{30}$$

2.3 Formulate the final LCP

After sub structuring the interior nodes and relate incremental displacements with three kinds of gaps (Eq. (12), (16) and (17)) one may obtain the following LCP equation

$$\bar{K}\begin{Bmatrix} \widehat{\Delta u} \\ \widetilde{\Delta u} \\ F_c \end{Bmatrix}_{i+1} = \bar{Q}_{i+1} + \begin{Bmatrix} \hat{S} \\ \breve{S} \\ g \end{Bmatrix}_{i+1} - \begin{Bmatrix} 0 \\ 0 \\ g \end{Bmatrix}_i \tag{31}$$

Eq. (31) can be solved using direct or iterative schemes. In this paper LCP is solved using Lemke Algorithm.

2.4 Numerical results and discussion

In this section an example is solved using the proposed method and numerical results are presented.Fuel string in this example is consist of two candu7 bundles as shown in Figure 1. Each bundle is made of 37 fuel elements and two endplates. Global coordinates are shown in Figure 1 and Figure 2. Fix boundary condition is applied to the right hand side of the second bundle. Initial gap between the two bundles is set to be zero. Five axial forces, each equal to 20 N, is applied to five fuel rods (number 1, 5, 9, 14 and 37, see Figure 2) to hold the two bundles together.

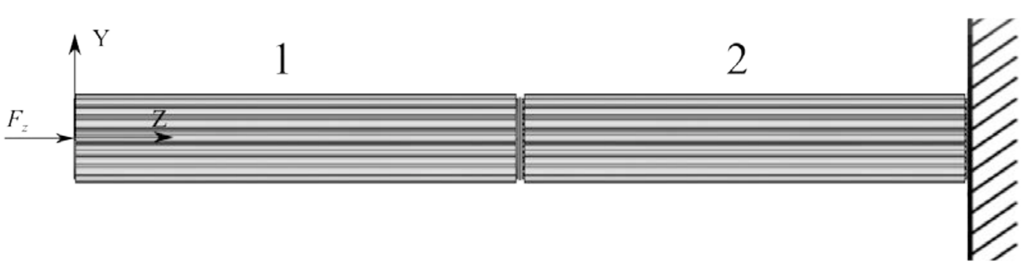

Figure 1Fuel string consist of two bundles

Four harmonic excitations are applied to the left side of bundle 1, at the location of rods number 1, 5, 9 and 14 as it is shown in Figure 2.

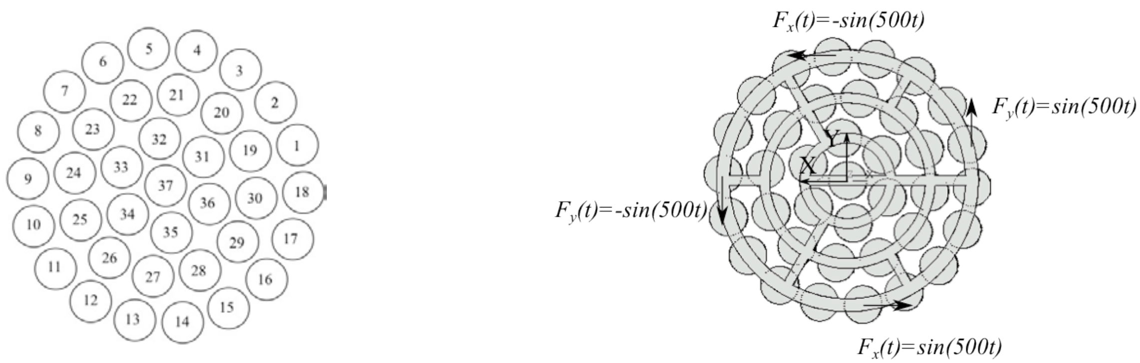

Figure 2 Harmonic excitations applied to the left endplate of the first bundle

Geometrical and material properties of the bundles are presented inTable 1.

Rods inner diameter (mm)	12.31
Rods outer diameter (mm)	13.1
Rods length (mm)	492.3
Endplate thickness (mm)	1.5
Zircaloy's density (sheath, kg/m3)	7700
UO_2density(kg/m3)	10600
Zircaloy's Young's modulus (Gpa)	180
UO_2Young's modulus (Gpa)	97

Table 1, Geometrical and material properties of the bundle

Static and dynamic coefficient of friction is 0.2 in this example. Problem is solved for 0.25 seconds with a time step size of 0.001 second. Fuel rods on the outer ring are subjected to contact with the pressure tube at 2 locations along each rod. 78 internal gaps are set for internal contact between the fuel elements. However no external or internal contact is occurred in this example. The main focus of this problem is the bundle-to-bundle contact.

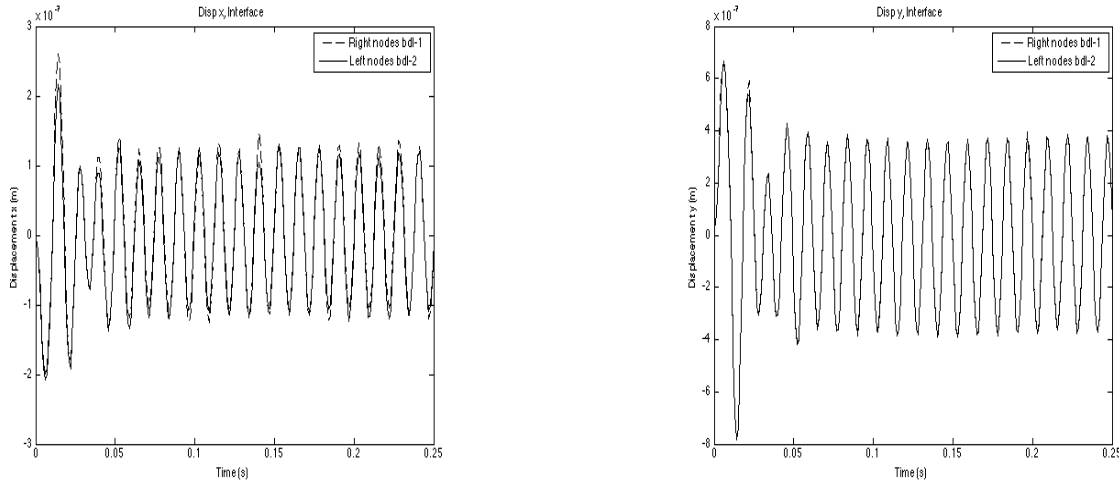

Figure 3 X and Y Displacement of rod number 1 at bundle-bundle interface(right endplate of bundle 1 and left endplate of bundle 2)

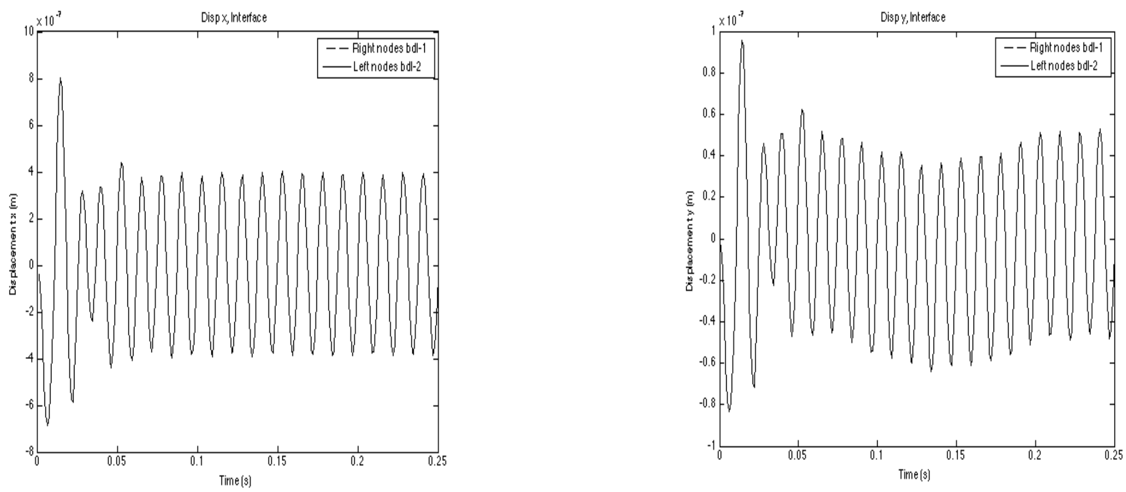

Figure 4 X and Y Displacement of rod number 5 at bundle-bundle interface (right endplate of bundle 1 and left endplate of bundle 2)

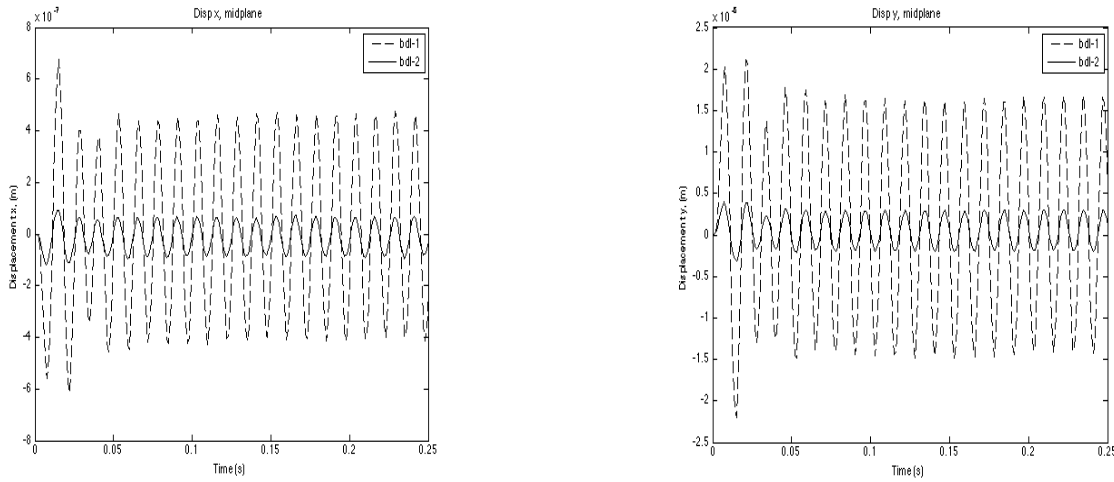

Figure 5 X and Y Displacement of rod number 1 at mid-plane of bundle 1 and 2

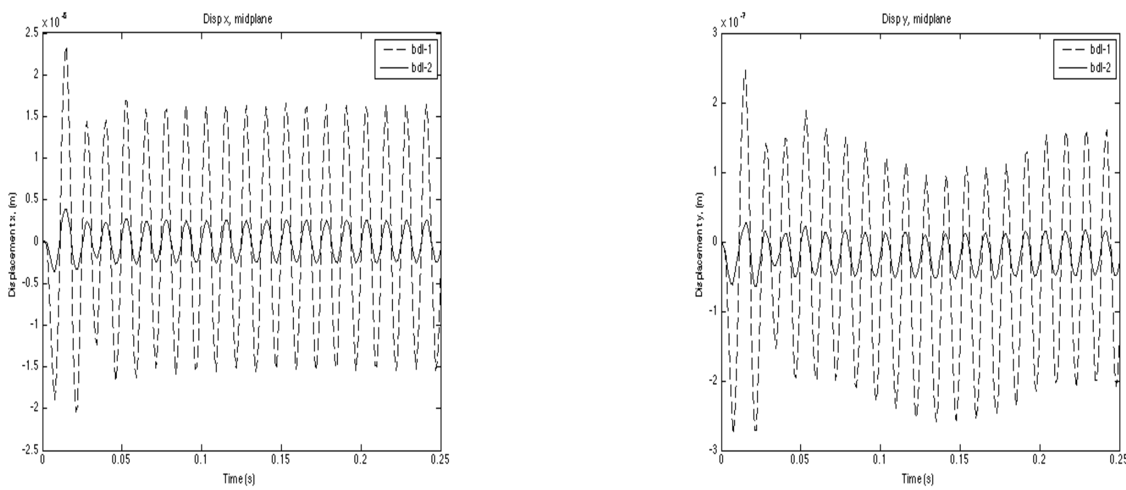

Figure 6 X and Y Displacement of rod number 5 at mid-plane of bundle 1 and 2

As it is plotted in Figure 3 and Figure 4 both sliding and stiction occurs at the interface between bundle 1 and 2 at location of rod number 1 and 5. Displacement of rods 1 and 5 at the mid-plane is presented in Figure 5 andFigure 6. Displacement of rod 1 and 5 at mid-plane in bundle 1 is approximately 5 times greater that displacement of the same node on bundle 2.

3. Summery

In this paper a comprehensive numerical model for handling unilateral frictional contact in a CANDU fuel string is presented. Coulomb's law of friction is employed to handle two-dimensional friction between bundles and pressure tube through bearing pads and between the neighbouring bundles. Linear complementarity problem is formulated and solved using lemke's algorithm. At the end numerical results obtained from the presented scheme for a fuel string consist of two bundles is presented.

4. Acknowledgement

The authors would like to acknowledgment with appreciation the financial support from Natural Science and Engineering Research Council of Canada (NSERC) Candu Energy Inc., and the technical support from Dr. Zhen Xu and Dr. Masoud Shams of Candu Energy Inc.

5. References

[1] Norsworthy, A.G., Ditschun, A., 1995. Fuel bundle to pressure tube fretting in Bruce and Darlington. Proceedings-Annual Conference, Canadian Nuclear Association 2, 16.

[2] Misra A. et al. 1994. Acoustic modeling in support of fuel failure investigation in a CANDU nuclear generating station, ASME PVP Division Publication 279, 99-118.

[3] Judah, J., 1992. Overview of fuel inspections at the Darlington nuclear generatingstation. In: Annual International Conference – Canadian Nuclear Association,pp. 3.1–3.22.

[4] Norsworthy, A.G., Field, G.J., Meysner, A., Dalton, K., and Crandell, A., 1994. Fuel Bundle to Pressure Tube Fretting in Bruce and Darlington Reactors. Canadian Nuclear Society, 15th Annual Conference Proceedings. 2, 2.

[5] Stewart. W.B., 1992. Darlington NGS Unit 2 Fuel Damage Investigation. Proceedings of the 13th Annual CNS Conference, Saint John, June 7-10.

[6] Yetisir, M., Fisher, N.J., “Prediction of pressure tube fretting-wear damage due to fuel vibration” Nuclear Engineering and Design. 1997. 176, 261–271.

[7] Hassan, M., Rogers, R., “Friction modelling of preloaded tube contact dynamics. Nuclear Engineering and Design” 2005. 235, 2349–2357.

[8] Xu S., Kim. Y. andXu Z. “Modeling of Transient Dynamic Bundle Deormation Using Time Integration Scheme”11th International Conference on CANDU Fuel, Niagra falls, Ontario, Canada, 2010 October 17-20.

[9] S. D. Yu and M. Fadaee, “Comprehensive Dynamic Model for Axial, Flexural and Torsional Vibration of a CANDU Fuel Element”Civil-Comp Press/Proceedings of the 11th International Conference on Engineering Computational Technology, paper 297, 2012.

[10] Yu S. D. and Hojatie. H. “Modeling lateral contact constraint among CANDU fuel rods”Nuclear Engineering and Design,2012, 254 (2013) 16– 22.