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A Reformulation of the Transport-Transport SPH Equivalence Technique

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Abstract

The superhomogénéisation (SPH) equivalence technique is a correction procedure based on equivalence factors. These equivalence factors are computed in such a way that a macro calculation made over a macro region and a coarse energy group with a simplified transport operator leads to the same leakage and reaction rates as a reference calculation performed without homogenization and with a fine group discretization. The situation where the macro calculation is performed with diffusion theory is a well understood and a common application of the technique. However, the case where the macro calculation is performed in transport theory is more complex and the SPH technique was reformulated in order to take into account the angular parity of the flux moments and cross sections. We found that the general rule to multiply all cross sections by a SPH factor and to divide all flux moment by the same factor is not valid. A new correction strategy is proposed to deal with transport-theory macro calculations. The strategy is slightly different whether or not the macro calculation is performed with a spherical harmonics (PN or SPN) discretization. The new approach was implemented in the DRAGON lattice code. Numerical results are comparing the classical and reformulated techniques.

Keywords: Superhomogénéisation (SPH), Reactor physics, Lattice calculation, Dragon code

1. Introduction

The equivalence procedure is an important step of a lattice calculation in reactor physics. As depicted in Fig. 1, the neutron flux is obtained using a deterministic approach so that the reaction rates could be homogenized over macro regions and condensed over macro groups. An equivalence procedure will follow to force a macro-calculation made over these macro-regions and macro groups to preserve the initial reaction rates. A well-known class of equivalence procedures is based on the introduction of *discontinuity factors* on internal surfaces of the macro-regions.[1] Another class of procedures, known as *superhomogénéisation* (SPH) equivalence techniques, is based on the correction of cross sections and diffusion coefficients using *equivalence factors*.[2] The SPH approach underwent many improvements over years, mainly related to the normalization of the correction factors in each macro-group.[3] A noticeable contribution was made by Yamamoto who found that the Selengut normalization can be applied *a posteriori*, after convergence of the correction factors.[4]

In the classical implementation of the SPH technique, all cross sections and diffusion coefficients are multiplied by the equivalence factors. In order to preserve the reference reaction rates, the corresponding fluxes are divided by the same factors. This approach is consistent with a macro-calculation based on the diffusion theory or, more generally, with the P_1 form of the transport equation. In any other case, this straightforward correction introduces inconsistencies in the higher-angular moments of the cross sections. The incompatibility of the classical SPH technique with the simplified P_n method was first reported by Guérin et al. in 2011 (see Ref. [5]). However, we claim that this incompatibility extends to all macro-solutions made with the transport equation. This paper is an attempt to correct these inconsistencies.

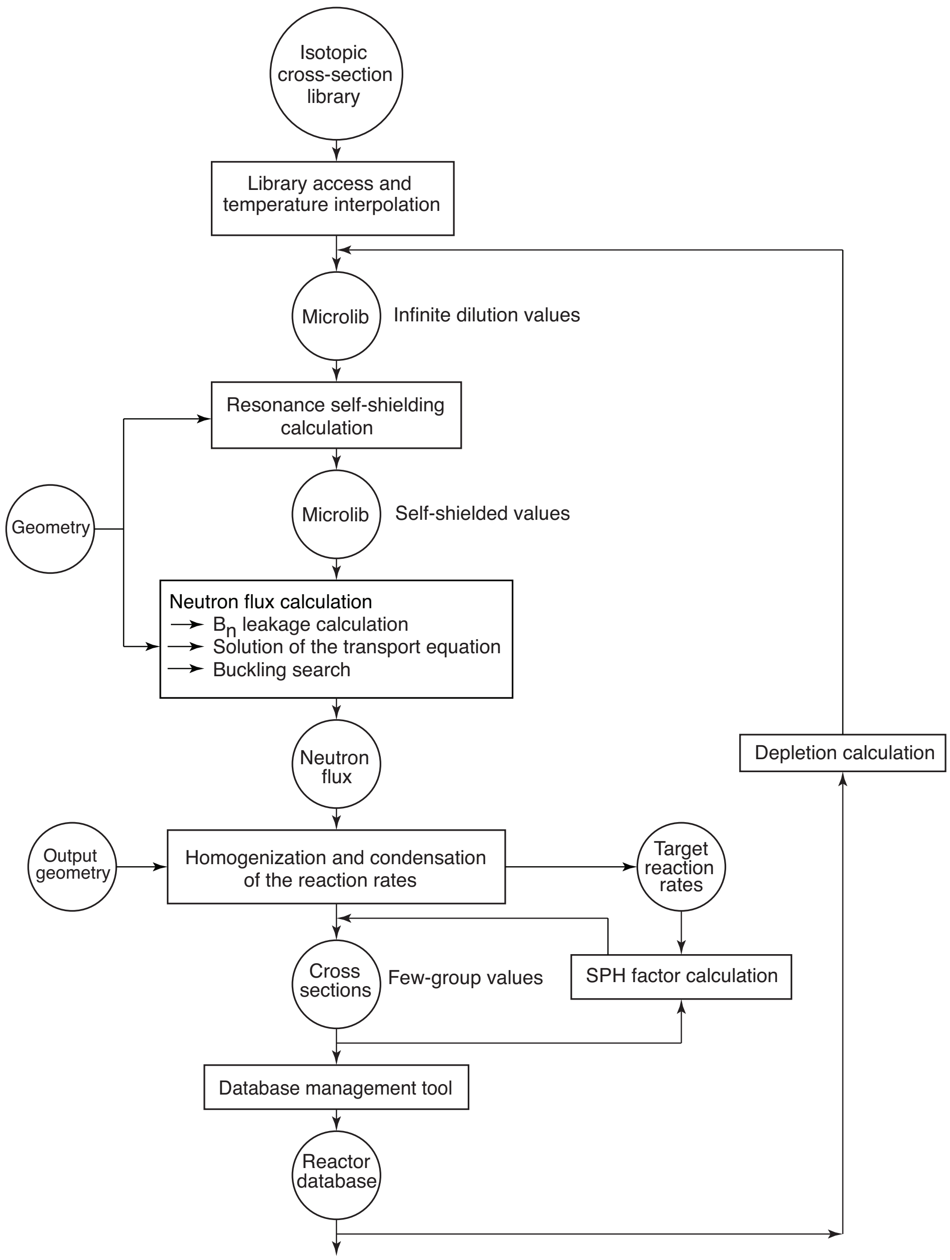

Figure 1 Dataflow of a lattice calculation

2. Theory

The SPH equivalence technique is based on the calculation of a set of *equivalence factors* $\{\mu_{m,k}, m \in C_m \text{ and } k \in M_k\}$, where C_m and M_k is a macro region and a coarse energy group of a full-core or macro calculation (see Sect. 4.4 of Ref. [6]). These equivalence factors are computed in such a way that a macro calculation made over C_m and M_k with a simplified transport operator leads to the same leakage and reaction rates as a reference calculation performed without homogenization and with a fine group discretization.

The SPH correction is applied differently, depending on the type of macro-calculation:

1. In the case where the macro-calculation is done with the diffusion theory, neutron balance is satisfied if the SPH correction is applied as follows:

$$\nabla \cdot J_g(r) + \mu_g \Sigma_g(r) \frac{\phi_g(r)}{\mu_g} = \frac{\chi_g}{k_{\text{eff}}} \sum_{h=1}^{G} \mu_h \nu \Sigma_{\text{f},h}(r) \frac{\phi_h(r)}{\mu_h} + \sum_{h=1}^{G} \mu_h \Sigma_{\text{s0},g\leftarrow h}(r) \frac{\phi_h(r)}{\mu_h}$$

   and

$$J_g(r) = -\mu_g D_g(r) \frac{\nabla \phi_g(r)}{\mu_g}.$$

   In conclusion:

   - Diffusion coefficients and all P_0 cross sections (including the total cross section must be multiplied by μ_g.
   - Scattering matrix terms $\Sigma_{\text{s0},g\leftarrow h}(r)$ must be multiplied by μ_h.
   - Averaged and volume-integrated fluxes must be divided by μ_g.

2. In the case where the macro-calculation is done with the simplified P_n method, the neutron balance is satisfied if the SPH correction is applied on even parity equations as follows: [5]

$$\mu_g \Sigma_{0,g}(r) \frac{\phi_{0,g}(r)}{\mu_g} + \nabla \cdot \phi_{1,g}(r) = \frac{\chi_g}{k_{\text{eff}}} \sum_{h=1}^{G} \mu_h \nu \Sigma_{\text{f},h}(r) \frac{\phi_{0,h}(r)}{\mu_h} + \sum_{h=1}^{G} \mu_h \Sigma_{\text{s0},g\leftarrow h}(r) \frac{\phi_{0,h}(r)}{\mu_h}$$

$$\frac{2\ell}{4\ell+1} \nabla \cdot \phi_{2\ell-1,g}(r) + \mu_g \Sigma_{0,g}(r) \frac{\phi_{2\ell,g}(r)}{\mu_g} + \frac{2\ell+1}{4\ell+1} \nabla \cdot \phi_{2\ell+1,g}(r) = \sum_{h=1}^{G} \mu_h \Sigma_{\text{s}2\ell,g\leftarrow h}(r) \frac{\phi_{2\ell,h}(r)}{\mu_h}$$

   and on odd-parity equations as follows:

$$\frac{2\ell+1}{4\ell+3}\nabla\frac{\phi_{2\ell,g}(r)}{\mu_g}+\frac{\Sigma_{1,g}(r)}{\mu_g}\phi_{2\ell+1,g}(r)+\frac{2\ell+2}{4\ell+3}\nabla\frac{\phi_{2\ell+2,g}(r)}{\mu_g}=\sum_{h=1}^{G}\frac{\Sigma_{\mathrm{s}2\ell+1,g\leftarrow h}(r)}{\mu_g}\phi_{2\ell+1,h}(r)$$

where $\ell\geq 1$.

In conclusion:

- All P_0 cross sections (including the total cross section in the even-parity equations) must be multiplied by μ_g.
- The total cross section in the odd-parity equations must be divided by μ_g.
- Scattering matrix terms $\Sigma_{\mathrm{s}\ell,g\leftarrow h}(r)$ with ℓ even must be multiplied by μ_h.
- Scattering matrix terms $\Sigma_{\mathrm{s}\ell,g\leftarrow h}(r)$ with ℓ odd must be divided by μ_g.
- Even parity fluxes (averaged and volume-integrated) must be divided by μ_g.
- Odd parity fluxes (averaged and volume-integrated) are not modified.

3. In the case where the macro-calculation is done in transport theory, but not with a P_n–type method, the macroscopic total cross section *is not modified*, and the even-odd corrections consistent with the simplified P_n method are reported to the macroscopic within-group scattering cross sections. They are now corrected as

$$\tilde{\Sigma}_{\mathrm{s}2\ell,g\leftarrow g}(r)=\mu_g\Sigma_{\mathrm{s}2\ell,g\leftarrow g}(r)+(1-\mu_g)\Sigma_{0,g}(r)$$

and

$$\tilde{\Sigma}_{\mathrm{s}2\ell+1,g\leftarrow g}(r)=\frac{\Sigma_{\mathrm{s}2\ell+1,g\leftarrow g}(r)}{\mu_g}+\left(1-\frac{1}{\mu_g}\right)\Sigma_{1,g}(r)$$

where $\ell\geq 0$.

Other cross sections and scattering matrix terms are corrected the same way as for the simplified P_n method.

3. Numerical results

The main objective of the reformulated SPH algorithm is to provide a consistent correction to the high-order Legendre moments of the cross sections. This improved consistency should not introduce any regression in the correction accuracy for zeroth-order moment cross sections, as provided by the classical technique. In the case of a transport-diffusion SPH correction, both classical and reformulated techniques are identical.

In the case of a transport-PIJ correction, the classical and reformulated form of the SPH algorithm are different. In the classical form, the total cross section is multiplied by the SPH factor, so that the collision probabilities of the macro-calculation muls be recalculated at each SPH iteration. With the reformulated method, the total cross sections are constant and the collision probabilities are computed only once. We shoud therefore verify that the reformulated algorithm does't introduce any regression with respect to the classical approach.

We will consider the one-group non-regression test `TCM32.c2m` of the standard DRAGON5 distribution.[7] The geometry of this case is depict in Fig. 2 and the cross sections are presented in Table 1.

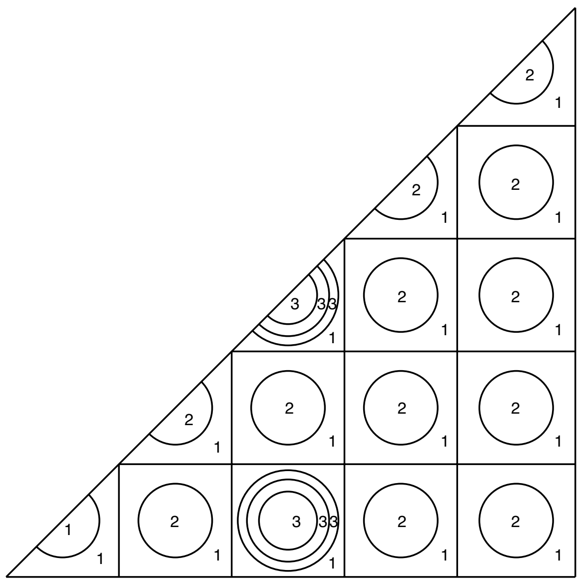

Figure 2 Geometry of test TCM32.c2m

	Σ (cm^{-1})	Σ_s (cm^{-1})	$\nu\Sigma_f$ (cm^{-1})
1	0.3683	0.3661	0
2	0.36522	0.3234	0.0964
3	0.8453	0.5216	0

Table 1 Cross sections of test TCM32.c2m

The test case represents a 9 x 9 heterogeneous assembly solved using the interface current (IC) approach of the SYBILT: module of DRAGON5. Next, a cell-by-cell homogenization is performed to obtain a 9 x 9 distribution of reaction rates. Homogenized cross sections can be obtained from these reaction rates without or with SPH equivalence. Finally, a collision probability calculation is performed using the NXT: module of DRAGON5 over the 9 x 9 homogenized assembly and the effective multiplication factor is compared to the original one. The overall process is depicted in Fig. 3.

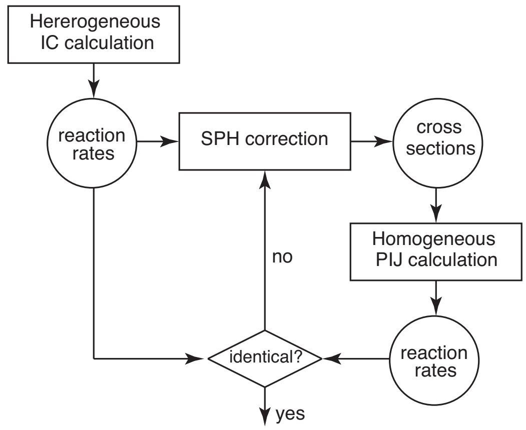

Figure 3 SPH algorithm

If the cell-by-cell homogeneous cross sections are obtained directly from the reaction rates without SPH correction, the homogeneous PIJ calculation leads to an effective multiplicative factor (K_{eff}) in error by 2581 pcm. Both classical and reformulated SPH algorithms behave similarly and converge in 5 iterations, as depicted in Fig. 4. All homogenization error is corrected by the SPH algorithm and the reformulated algorithm introduces no regression with respect to the classical one.

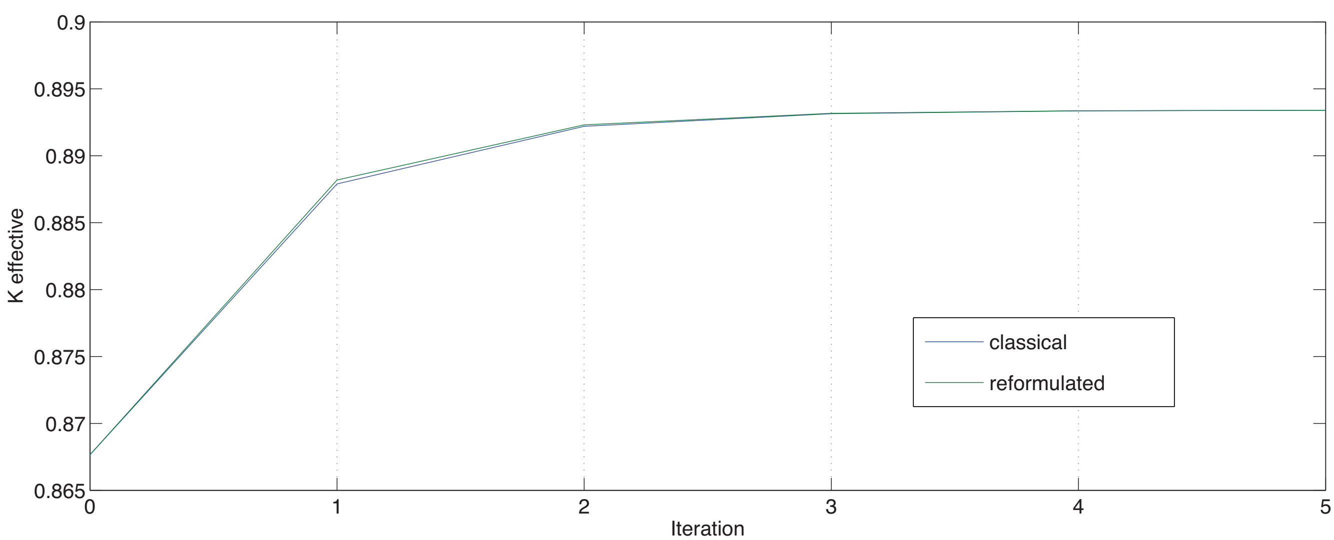

Figure 4 Convergence of classical and reformulated SPH algorithms

4. Conclusion

We have reformulated the SPH algorithm in a way that make possible the consistent correction of Legendre moments of the cross sections, without introducing regressions in the more conventional cases where only zeroth moments cross sections are required. Both classical anf reformulated SPH algorithms were shown to behave similarly on a transport-PIJ equivalence non-regression testcase picked in the standard DRAGON5 distribution. However we are gaining more consistency in future applications where scattering anisotropy of the neutron source is important.

5. References

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