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Leakage-Corrected Discontinuity Factors for PHWR Lattices – A simple Test

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Abstract

Due to the heterogeneity present in the core, solving for the flux in the full core is not practical without applying homogenization. Theoretically, by using exact discontinuity factors and node-homogenized cross sections, the node flux and power for the node-homogenized core equal the ones of the heterogeneous core. However, exact node homogenized parameters can only be obtained if exact heterogeneous node boundary conditions are known but, of course, exact node boundary conditions are determined by the full-core heterogeneous flux which is not known a priori for any real practical case. However, near-exact node-boundary conditions can be obtained by performing global-local iterations between lattice and core calculations, a process known as leakage correction.

Leakage-corrected discontinuity factors and homogenized cross sections have shown very good gain in accuracy for PT-SCWRs. This work investigates the application of leakage-corrected cross-sections and discontinuity factors via global-local iterations to a simple, one-dimensional, model consisting of several PHWR-type nodes.

Keywords: Reactor and Radiation Physics, Computer Codes and Modeling, Discontinuity Factors, Leakage Correction, Generalized Equivalence Theory

1. Introduction

Every nuclear reactor core is heterogeneous by design due to the different materials existence in the core and the fuel enrichment arrangements to obtain a uniform power production while maintaining neutron economy.

PHWR uses high-pressure tubes for the fuel and coolant, and a low-pressure calandria to contain the moderator. The heavy water moderator is contained in unpressurized calandria, which passes through the high-pressure tubes that contains fuel and coolant. Fuel is in the form of bundles. The coolant flows in pressurized tubes and the flow through the adjacent pressure tubes is in opposite direction. The only PHWR type that has found wide commercial applications is the CANDU reactor. For a CANDU 6 reactor, there are 380 fuel channels with lattice pitch of 28.6cm. Each fuel channel includes 12 fuel bundles and each fuel bundle is a concentric-ring type arrangement including 37-fuel elements.

The core heterogeneity increases with burn-up. Burn-up changes the fuel isotopic composition due to isotopic depletion or build-up process. The major causes of increased heterogeneity of the rector core

with burn-up are fuel composition changes and reactivity device configuration changes. Moreover for CANDU reactors the on-line refueling is also another important factor increasing heterogeneity.

Accurate neutron flux is very important for safety analysis and the operation of nuclear power plant and so power distributions must be known throughout the reactor cycle. To define power adequately, fuel pin level flux detail is required.

The neutron behavior is most accurately defined by neutron-transport equation:

$$\nabla \bullet \left[\Omega \Psi_g(\Omega,\vec{r})\right] + \Sigma_{tg}(\vec{r})\Psi_g(\Omega,\vec{r}) - \sum_{g'}\int_{\Omega'} \Sigma_{sg'\to g,\Omega'\to\Omega}(\vec{r})\Psi_{g'}(\Omega',\vec{r})d\Omega'$$
$$= \frac{1}{k_{eff}}\frac{1}{4\pi}\chi_g \sum_{g'}\int_{\Omega'} \nu\Sigma_{fg'}(\vec{r})\Psi_{g'}(\Omega',\vec{r})d\Omega \qquad (1)$$

Using the transport equation to solve the neutron flux problem for the entire core is too complicated and computationally not manageable. If the flux is assumed to be linearly anisotropic, the P_1 approximation of the transport equation is obtained, which can be subsequently simplified to the diffusion equation:

$$\nabla \bullet \left[D_g(\vec{r})\Psi_g(\vec{r})\right] + \Sigma_{tg}(\vec{r})\Psi_g(\vec{r}) - \sum_{g'}\Sigma_{sg'\to g}(\vec{r})\Psi_{g'}(\vec{r})$$
$$= \frac{1}{k_{eff}}\chi_g \sum_{g}\nu\Sigma_{fg}(\vec{r})\Psi_g(\vec{r}) \qquad (2)$$

Considering the high heterogeneity of the nuclear reactor core and wide energy range of the neutrons, even though the fuel pins, local coolant and moderator is taken as homogeneous, the reactor core still remains highly heterogeneous due to inner-assembly and intra-assembly fuel composition variations. The diffusion approximation of this kind of heterogeneous core is still computationally very expensive and not feasible due to the size. Therefore, the geometry and energy group dependency of the problem needs to be simplified further. To overcome the calculation difficulty, spatial simplification and group condensation is used.

Homogenization is a widely used spatial simplification, which yields only assembly-level heterogeneity. The basic homogenization method is called simple homogenization. The homogenized macroscopic neutron cross sections are calculated as the flux-weighted cross section averages over the assemblies (also called nodes). Standard Homogenization does not assure that node-integrated reaction rates, surface leakage, and eigenvalue of the whole core are preserved. For this purpose, *discontinuity factors* was introduced through generalized equivalence theory (GET) [1, 2]. In generalized equivalence theory, homogeneous multi-group flux is taken only piecewise continuous (continuous in each homogeneous node, but discontinuous at node boundaries) so by the use of discontinuity factors at node boundaries node-integrated reaction rates are preserved. Most modern nodal methods are based on the generalized equivalence theory. Homogenized parameters are obtained by single node detailed fine-mesh lattice calculations. In theory, by using exact discontinuity factors and node-homogenized cross

sections, the node flux and power for the node-homogenized core becomes equal the ones of the heterogeneous core. Since exact node boundary conditions is not known a priori for any real practical case, lattice calculations are performed with reflective boundary conditions, which assume zero leakage. Especially for the nodes where leakage is large, this approximation can result in large errors. Near-exact node-boundary conditions can be obtained by performing *global-local iterations* between lattice and core calculations [3] , a process known as *leakage correction* [4]. This work investigates the application of leakage-corrected cross-sections and discontinuity factors to a simple, one-dimensional, model consisting of several PHWR-type nodes.

2. Approach

Generalized equivalence theory introduces additional homogenization parameters called *discontinuity factors*, to obtain node average neutron flux, reaction rates and leakage equal to those in the detailed flux heterogeneous model.

Homogenization requires node average homogeneous flux $\Phi_g(\vec{r})$, heterogeneous flux $\Psi_g(\vec{r})$ are equal to each other for any node *i:*

$$\int_{V_i} \Phi_g(\vec{r})\,dV = \int_{V_i} \Psi_g(\vec{r})\,dV \tag{3}$$

Equality of node averaged reaction rates expressed as:

$$\hat{\Sigma}_{gi} \int_{V_i} \Phi_g(\vec{r})\,d\vec{r} = \int_{V_i} \Sigma_g(\vec{r})\Psi_g(\vec{r})\,d\vec{r} \tag{4}$$

By using Eqs. (3) and (4), the node homogeneous cross-section is defined as:

$$\hat{\Sigma}_{gi} = \frac{\int_{V_i} \Sigma_g(\vec{r})\Psi_g(\vec{r})\,dV}{\int_{V_i} \Phi_g(\vec{r})\,d\vec{r}} \tag{5}$$

The generalized equivalence theory permits the surface-averaged fluxes to be discontinuous across each nodal surface. At the interface surface *s* between two nodes *i* and *j*, this discontinuity condition is imposed as:

$$\bar{\Phi}^i_{sg} f^i_{sg} = \bar{\Phi}^j_{sg} f^j_{sg} \tag{6}$$

Where, $\bar{\Phi}^{i}_{sg}$ is group-g homogeneous flux in node i and $\bar{\Phi}^{j}_{sg}$ is group-g homogeneous flux in node j averaged over the node *i* and node *j* surface *s*. The coefficients of the equation f^{i}_{sg} and f^{j}_{sg} are called discontinuity factors.

$$f_{sg} = \frac{f^{j}_{sg}}{f^{i}_{sg}} = \frac{\bar{\Phi}^{i}_{sg}}{\bar{\Phi}^{j}_{sg}} \tag{7}$$

If the discontinuity factors were taken as unity, Eq. (7) reduces to the usual flux-continuity condition so the method reduces to standard homogenization.

The difficulty arises while implementing the discontinuity factors for the heterogeneous neutron flux $\Psi_{g}(\vec{r})$ since in practice the reference heterogeneous solution is not known. Therefore, the heterogeneous flux is approximated by single node fine-mesh lattice calculations by using reflective boundary conditions. The assumption of "no net neutron leakage" may result in large errors depending on the real leakage of the node. The discontinuity factors obtained by using reflective boundary conditions are called assembly discontinuity factors, f^{0}_{sg}, which are the ratio of the face-averaged heterogeneous flux, $\bar{\Psi}^{0}_{Sg}$, to the node-averaged heterogeneous flux, $\bar{\Psi}^{0}_{Vg}$, [1, 2]:

$$f^{0}_{sg} = \frac{\bar{\Psi}^{0}_{Sg}}{\bar{\Psi}^{0}_{Vg}} \tag{8}$$

Instead of using assembly discontinuity factors, if the non-reflective 'real' boundary conditions can be obtained, the node flux and power for the node-homogenized core become equal the ones of the heterogeneous core. To find near-exact node-boundary conditions and so enhance the homogenization accuracy an iterative process has been developed called *global-local iterations,* which uses homogeneous boundary conditions expressed in the form of albedos. The starting point of the method is reflective boundary condition. The homogenization is performed initially by using assembly discontinuity factors. The resulting homogenized core is solved to obtain new boundary conditions and this new set of non-reflective boundary conditions are used in lattice calculations which yields more accurate discontinuity factor each time. The method is and iterative process and converges in less than 10 iterations. When the method converges, not only the accurate discontinuity factors are found, the near exact boundary conditions are also obtained so the resulting boundary conditions can be also used to find the pin-power detail.

3. Calculations and Results

The lattice calculations are performed with DRAGON [5] using two-group collision probabilities individually for every node. Heterogeneous two-group transport cross-sections are obtained from single node transport calculation by using 69-groups macroscopic cross-section from WIMSD4 [7] library. Homogeneous calculations are performed in two-group diffusion theory using a mesh-centered finite difference code developed at UOIT [6]. The global-local iteration FORTRAN code interfaces with the

transport code DRAGON and the diffusion solver. The program extracts cross sections for each node calculated by DRAGON using currently available boundary conditions. These parameters then passed to the diffusion code where full core diffusion calculation is performed. By using the diffusion calculation results, the boundary conditions are calculated and they are fed to DRAGON for the next iteration.

To test the method, a configuration consisting of 11 fuel nodes and a reflector node was analyzed. Such a configuration corresponds roughly to a set of nodes extending radially from the center of the reactor to its periphery. The studied configuration is represented in Fig. 1.

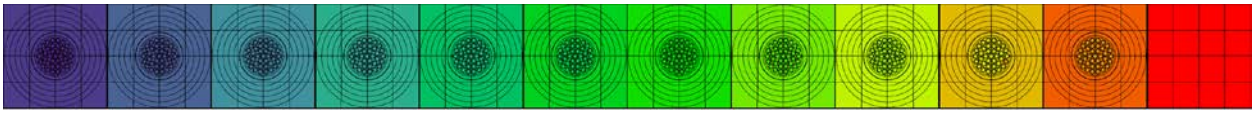

Fig. 1 12 bundle 1-D DRAGON Model

To test the capability of the method, a highly heterogeneous case is selected. The first 9 nodes (counting from the left) contain discharge-burnup fuel, whereas the last two nodes, neighboring the reflector, contain fresh fuel. Two sets of boundary conditions are studied. The first set uses reflective BC on all boundaries and the second set uses reflective BC in all but the right reflector boundary, where vacuum BC are imposed. The reflector node used in the test case contains only moderator heavy water and no fuel. Because of lack of neutron sources, eigenvalue problem cannot be defined. Therefore the transport calculation with homogeneous boundary conditions cannot be employed, the discontinuity factors for the reflector nodes are approximate to be equal to 1.

The results for the reflective BC model are shown in Table 1 and Fig. 2, and those for the vacuum BC model are shown in Table 2 and Fig. 3. The node fission rates are normalized to an average of 1. Table 1 and 2 also show the node-by-node percent error as well as the root mean square (RMS) percent error.

Table 1: Normalized Fission Rates for the Case of Vacuum B.C.

Node	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**RMS % err**
DRAGON	0.22	0.24	0.28	0.36	0.47	0.62	0.84	1.15	1.58	2.32	2.93	
Standard Hom.	0.08	0.10	0.14	0.20	0.30	0.45	0.69	1.07	1.66	2.78	3.53	
err % (std. hom.)	-61.2	-58.0	-52.3	-45.0	-36.7	-27.7	-17.8	-6.9	5.3	20.0	20.7	37.2
Iterated solution	0.21	0.24	0.28	0.36	0.47	0.62	0.85	1.16	1.59	2.41	2.82	
err % (iterated)	-0.6	-0.5	-0.4	-0.3	-0.1	0.1	0.3	0.4	0.6	3.9	-3.5	1.6

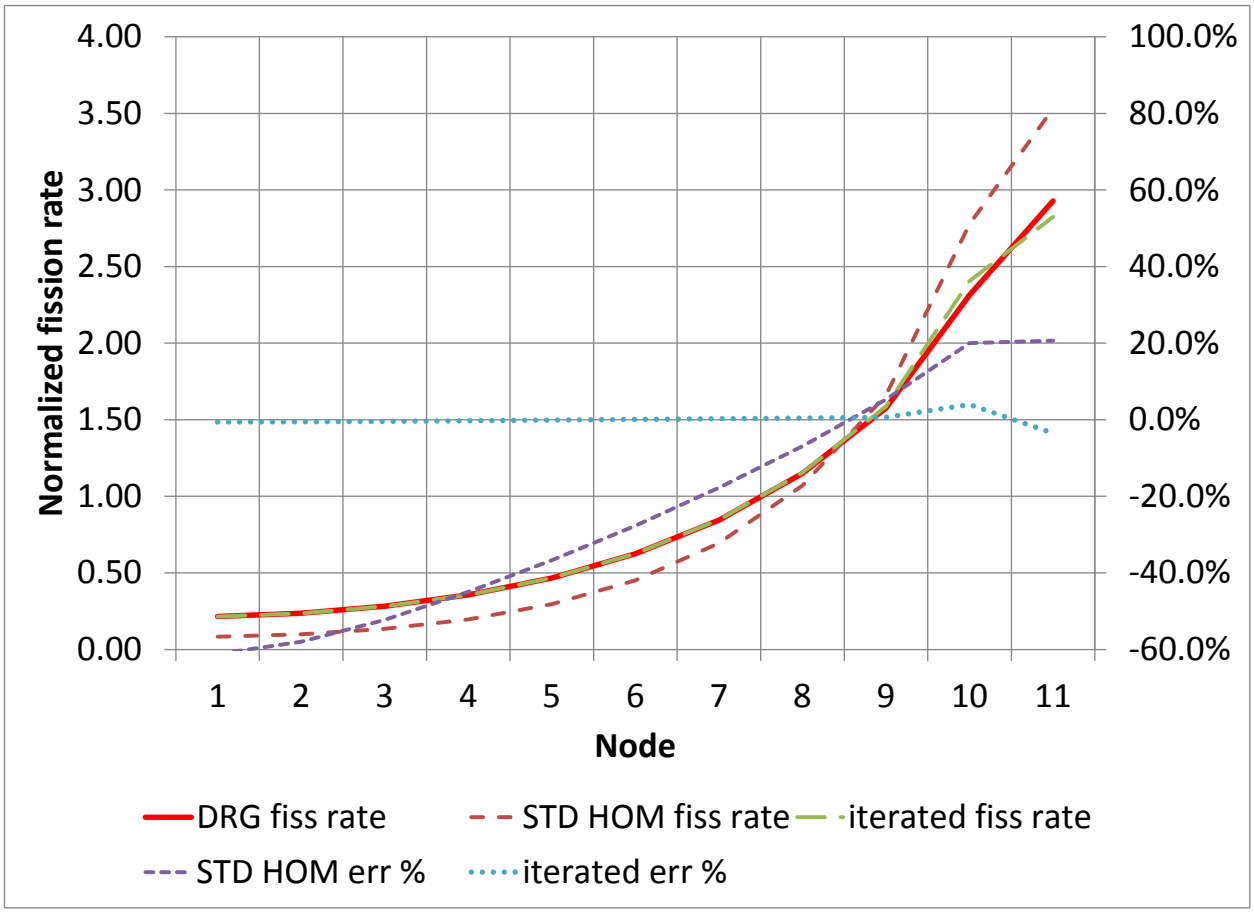

Fig. 2 Normalized Fission Rates for the Case of Vacuum B.C.

Table 2: Normalized Fission Rates for the Case of Vacuum Reflector B.C.

Node	1	2	3	4	5	6	7	8	9	10	11	RMS % err
DRAGON	1.19	1.18	1.16	1.13	1.09	1.04	0.99	0.93	0.86	0.84	0.61	
Standard Hom.	0.79	0.80	0.82	0.85	0.90	0.95	1.02	1.11	1.21	1.43	1.13	
err % (std. hom.)	-33.6	-32.2	-29.2	-24.4	-17.8	-8.7	3.4	19.5	41.0	71.0	84.9	40.8
Iterated solution	0.93	0.93	0.94	0.95	0.97	0.98	1.00	1.03	1.06	1.21	1.00	
err % (iterated)	-21.6	-20.5	-18.7	-15.7	-11.5	-6.0	1.4	10.9	23.5	44.5	64.0	27.6

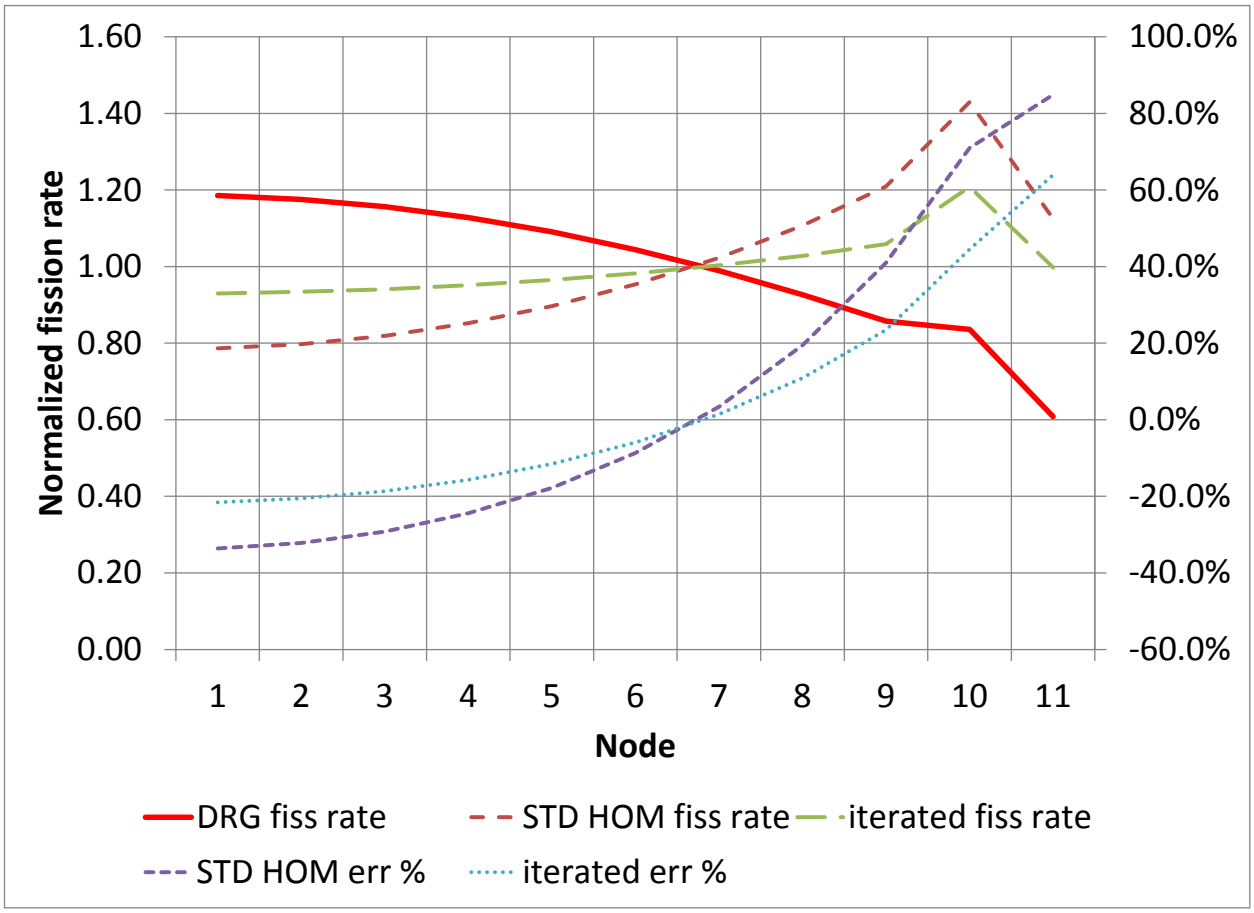

Fig. 3 Normalized Fission Rates for the Case of Vacuum Reflector B.C.

It can be seen from Table 1 and Fig. 2 that applying the global-local iterative method improves the accuracy significantly compared to the case of standard homogenization. Indeed, the RMS % error is reduced from 37% to 1.6%. At the same time, the accuracy improvement is much more modest for the case of vacuum BC, as illustrated in Table 2 and Fig. 3. It can be seen that the application of the iterative method only reduces the RMS error from 41% to 28%. It is surmised that the modest reduction is due to the approximate discontinuity factors used in the reflector. It is probable that the reflector discontinuity factors are much closer to 1 in the case of reflective BC than in the case of vacuum BC.

4. Conclusion and Future work

Leakage-corrected discontinuity factors appear to improve the accuracy of neutronics calculations for PHWR lattices even when they are only applied to fuel nodes. However, the magnitude of the improvement is strongly dependent on the amount of neutron leakage coming out of the reflector. For situations with large leakage, leakage-corrected discontinuity factors, other than unity, will have to also be applied to reflector nodes.

5. References

[1] K.S. SMITH, “Spatial Homogenization Methods for Light Water Reactor Analysis” Ph.D. thesis, MIT, (1980)

[2] K.S. SMITH, “Assembly Homogenization Techniques for Light Water Reactor Analysis”, Progr. Nucl. Energy, 17, 303-335 (1986).

[3] K. CARISSE and E. Nichita “Leakage-Corrected Discontinuity Factors for a Second-Generation Th-Pu Pressure-Tube SCWR”, Trans. Am. Nucl. Soc. 110, (2014)

[4] F. RAHNEMA and E. NICHITA, “Leakage Corrected Spatial (Assembly) Homogenization Technique”, Ann. Nucl. Energy 24, 477 (1997)

[5] G. MARLEAU, A. HEBERT, R. ROY, *“A User Guide for DRAGON, Release 3.06L”*, Ecole Polytechnique de Montreal, (2013)

[6] E. Nichita, K. Zabienski and M. Gravel “A Three-Dimensional Two-Energy-Group Finite-Difference Neutron-Diffusion Code with Discontinuity Factors”, Proc. 28th Annual Conference of the Canadian Nuclear Society, Saint John NB June 3-6, (2007)

[7] F. Leszczynski, D. Lopez Aldama, A. Trkov, editors, WIMS-D library update, IAEA, Vienna, Austria, (2007)