Physics-guided Coverage Mapping (PCM): A New Methodology for Model Validation

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ABSTRACT

This manuscript deals with a fundamental question in model validation: given a body of available experiments and an envisaged domain of reactor operating conditions (referred to as reactor application), how can one develop a quantitative approach that measures the portion of the prior uncertainties of the reactor application that is covered by the available experiments? Coverage here means that the uncertainties of the reactor application are originating from and behaving in exactly the same way as those observed at the experimental conditions. This approach is valuable as it provides a scientifically defendable criterion by which experimentally measured biases can be credibly extrapolated (i.e., mapped) to biases for the reactor applications. This manuscript introduces a novel approach, referred to as physics-guided coverage mapping (PCM) which provides a natural solution to this problem by relying on high fidelity physics simulation. We discuss the potential advantages of PCM over the methods of similarity indices, data assimilation, and model calibration commonly employed in the nuclear community.

Key Words: Model validation, uncertainty quantification, criticality safety.

1. INTRODUCTION

Nuclear model validation practices measure the degree to which a given reactor model is a true representation of the real reactor behavior for the intended range of reactor operation. To deem a validation practice a success, one must be able to answer the following question with quantitative confidence: is there sufficient evidence in terms of analysis and experiments that the simulation predictions will be satisfactory for the intended reactor application? The criterion for satisfactory predictions is that the discrepancies between true and predicted future reactor responses can be bounded with high probability by preset margins. And the margins are to be estimated based on a proper account of all sources of uncertainties in the simulation plus some administrative margin to account for unknown sources of uncertainties. This type of question is important because a) building experiments for all possible reactor conditions of interest is impractical; b) no experiment can exactly duplicate reactor conditions, unless the reactor itself is used as the experiment; c) decisions for new reactor design with no operating experience have to be made solely based on available experiments; d) experiments are often built to understand the impact of separate effects of uncertainty sources; whose interaction is expected at reactor conditions. All these reasons necessitate a credible approach by which experimental biases can be reliably extrapolated (i.e., mapped) to the reactor conditions.

Accordingly two major tasks must be accomplished in order to perform model validation. The first task is experimental in nature; it involves the construction of experiments whose design is similar to the intended reactor application, with the primary goal of measuring the discrepancies (referred to hereinafter as experiment biases) between measured and model-predicted responses. The experiments are required for two reasons: a) no validation is credible without some level of comparison against reality; b) the prior uncertainties (computed based on a rigorous uncertainty propagation of all sources of uncertainties) for the reactor application of interest are typically too high to render an economical operation. It is therefore paramount for any useful validation practice to devise methods that employ experimental measurements (typically of low uncertainties) and analysis results to reduce the prior uncertainties of the reactor application down to the level that meets design and operation requirements. The second step of validation is computational, wherein model predictions at reactor conditions are employed in conjunction with the experiment biases to determine the application biases and their uncertainties; the application biases estimate the expected discrepancies between the true and predicted future responses for the reactor application. Ideally, if done correctly, the estimated application biases would be as close as possible to the true application biases which are observed when the real reactor is in operation. Biases represent systematic "mistakes" or "errors" in the modeling of both the experiment and the reactor application, which typically originate from modeling deficiencies and lack of knowledge about the "true" values for the physics parameters such as cross-sections.

The process of calculating application biases and their uncertainties is fundamental to any model validation as it provides the basis for the economical and safe operation of the reactor for the intended application. We will refer to this process as 'mapping' (sometimes referred to as 'scaling' by other practitioners). The mapping will describe the mathematical transformation employing experiment biases and analysis results of the experiments and the reactor application to determine the application biases and their uncertainties. If done correctly, the biases will help reduce the prior uncertainties for the reactor application. To measure the level of reduction of prior uncertainties due to inclusion of experimental results, we will introduce a quantitative term called 'coverage'. Great or high coverage implies that the experiments can be used to reduce the prior uncertainties of the reactor application, which is the case when the experiments are sufficiently similar to the reactor application. Poor or low coverage implies that the experiments are not sufficient to improve the predictions for the reactor application, which is the case when the experiment design is not similar or representative of the reactor application, or when the experimental measurements have high uncertainties. It is important to note that the notion of coverage or lack thereof has been employed before [1]. Another closely related term developed in the nuclear community is the 'similarity' (and sometimes referred to as 'representativity') which employs an inner product formula to measure the resemblance between an experiment and the reactor application of interest [2] (definition is given later in the text). Our goal here is to develop a new validation approach that can be used to develop more meaningful definitions for the coverage and/or similarity that can be used for general models, and that can be directly used to map the biases from the experimental to the application domain.

2. BACKGROUND ON PROPOSED METHODOLOGY

The objective of this paper is to introduce a new validation methodology, referred to hereinafter as physics-guided coverage mapping (PCM[1]), devised to map the biases from the experimental domain to the domain of reactor application by relying solely on the physics of the simulation while taking into account all sources of simulation uncertainties. The target of the PCM methodology is allow for a more natural definition of the 'coverage' which measures the portion of the prior uncertainty of the reactor application that can be explained, i.e., covered, by the experimental measurements. And, how the coverage is related to the mapping of biases.

To this end, we first introduce the mathematical nomenclature and notations required. Next, the conventional approach for mapping of biases is presented, which will set the stage for introducing our new PCM approach. Numerical results will follow to demonstrate applicability.

Mathematically, the physics model describing the **ex**periment is given by:

$$y_{\mathrm{ex}} = f_{ex}\left(x,u\right)$$

where x are basic physics parameters (such as cross-sections) and u are the experiment's control parameters (such as the experiment's materials, geometry, and composition specifications, etc.). One can abstractly describe the experimental design in terms of these control parameters, which are tuned to make sure the experiment is as similar as possible to reactor application. The y_{ex} are the responses of the experiment as predicted by the model. Let y_{ex}^{msr} be the experimental measurements corresponding to the model predictions y_{ex}.

Next, define the **r**ea**c**tor application using:

$$y_{rc} = f_{rc}\left(x,v\right)$$

where x are the same basic physics parameters employed earlier in the modeling of the experiment, and v are control parameters that describe the reactor design, e.g., size of the core, enrichment, etc. Notice that the experiments and the reactor conditions have different control parameters (i.e., u vs. v), and different functions (i.e., f_{ex} vs. f_{rc}); however, they both share x as part of their input data. Finally, the prior uncertainties for the basic physics parameters are typically described by probability distribution $p_{pri}(x)$ such that:

$$\int_{x_1}^{x_2} p_{pri}(x)dx$$

is the probability of finding x between x_1 and x_2. To simplify the discussion, we will employ Gaussian distributions to describe the prior parameter uncertainties. Generalization to non-Gaussian distribution is straightforward [3], but does not add significant insight considering the context of the current discussion. A multi-variable Gaussian distribution is fully described by a mean vector and a covariance matrix, denoted here by x_{pri} and $\mathbf{C}_{pri}$, respectively.

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[1] We recognize that pcm is a common unit for reactivity measurement; but since in our context PCM is a methodology, no confusion between the two terms is expected.

The conventional validation practice, depicted in Fig. 1, employs a calibration-based approach to the calculation of reactor application biases. This is done based on the assumption that the experiment biases originate from uncertainties in the basic physics parameters. A minimization search is formulated to calculate a posteriori estimate of physics parameters that minimizes the discrepancies between measured and predicted responses. Because the number of measured responses is often much lower than the number of physics parameters, the minimization problem is expected to have an infinite number of solutions. To render a well-posed search, prior information for the physics parameters are employed to regularize the search, described mathematically as follows (this approach is also referred to as Bayesian Estimation)[2]:

$$\min_{x}\left\{\left[y_{ex}^{msr}-f_{ex}\left(x,u\right)\right]^{T}\left[\mathbf{C}_{ex}^{msr}\right]^{-1}\left[y_{ex}^{msr}-f_{ex}\left(x,u\right)\right]+\left[x-x_{\infty}\right]^{T}\left[\mathbf{C}_{pri}\right]^{-1}\left[x-x_{\infty}\right]\right\}$$

where the first term is called the misfit term, measuring the discrepancy between measured and predicted experimental responses; the initial (i.e., prior to adjustment) value of this term is equal to the experiment bias. And the second term is called the regularization term, where x_{∞} represents the best guess for the physics parameters based on prior information, i.e., before the experimental measurements are collected. The confidence in the prior values of the parameters is described by the prior covariance matrix $\mathbf{C}_{pri}$, which is used as weight for the regularization term. This weighting ensures that parameters with very small uncertainties are hardly adjusted because they are accurately known, whereas parameters with high prior uncertainties are allowed to adjust within their prior uncertainty limits to better fit the measurements. The results of this minimization search are a set of adjusted parameter values, denoted by x_{pst}, and an updated covariance matrix, referred to as the posteriori covariance matrix, $\mathbf{C}_{pst}$. The minimizer x_{pst} is subsequently used to improve predictions at reactor conditions as follows: Let

$$y_{rc}^{pri}=f_{rc}\left(x_{pri},v\right)\text{ and }y_{rc}^{pst}=f_{rc}\left(x_{pst},v\right)$$

If measurements are available at the reactor level, let it be denoted by y_{rc}^{msr}, the premise of this approach is that:

$$\left\|y_{rc}^{msr}-y_{rc}^{pri}\right\|>\left\|y_{rc}^{msr}-y_{rc}^{pst}\right\|$$

which means that the adjusted predictions are closer to the measurements than the prior predictions. Using the posteriori parameter covariance matrix, the responses uncertainties calculated with the adjusted parameters can be estimated. The premise of data assimilation is that the posteriori responses uncertainties for the reactor application will be statistically consistent with the discrepancies between the measured and predicted future responses of the reactor application. This calibration-based approach faces several major challenges:

1) It relies on the adjustment of basic physics parameter, a practice that is frowned upon by many physicists who believe physics parameters are fundamental quantities that should never be calibrated. Interestingly, many practitioners refer to parameter calibration as “fudging” because there is a wide belief that any type of fitting against measurements runs the risk of

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[2] This formulation assumes Gaussian distribution for the prior parameter uncertainties, and Gaussian likelihood function for the responses [see any standard text on Bayesian theory, e.g., Ref. [4]].

cross-compensation for the error sources between the various parameters, especially when the number of adjustable parameters is much higher than the number of responses.

2) As a result of the adjustment procedure, physics parameters that are uncorrelated a priori become correlated post the adjustment. In criticality safety studies, for example, correlations appear a posteriori between the zirconium thermal absorption and the fast inelastic scattering cross-section of uranium isotopes[3] [5]. Although these correlations can be defended mathematically, they are meaningless to physicists and practitioners, since the original experiments used to measure the cross-sections for these isotopes are completely independent.
3) To solve the minimization problem, access to gradient information of the measured responses with respect to the physics parameters is typically needed. In the nuclear criticality community, derivatives with respect to cross-sections (representing model parameters) have been calculated using an adjoint variational approach which requires intrusive code modifications to calculate the adjoint function, a basic ingredient in the calculation of the derivatives.
4) Although the posteriori parameter uncertainties can be propagated to estimate the posteriori reactor responses uncertainties, there is no mathematical guarantee that these uncertainties will be realistic. This follows because no information about the reactor application is included in the adjustment procedure. To provide some measure of the relationship between the experiment and the application, the current validation practice relies on calculating a similarity index between the reactor application and the experiment defined by [2,5]:

$$s = \frac{g_{ex}^T \mathbf{C}_{pri} g_{rc}}{\sqrt{g_{rc}^T \mathbf{C}_{pri} g_{rc}} \sqrt{g_{ex}^T \mathbf{C}_{pri} g_{ex}}}$$

   where the vectors g_{ex} and g_{rc} are the gradients of a given response, e.g., critical eigenvalue, with respect to the basic physics parameters x as calculated from the experiment and the reactor application models, respectively. This metric takes on values between zero and one. When close to one, the analyst argues that the adjustments will likely work for the reactor application. When close to zero, the analyst becomes suspicious of the relevance of the experiment to the reactor application. In practice, this metric only serves as a qualitative metric that can be used to exclude experiments that are sufficiently different from the reactor application, when the adjustment procedure poorly fits the experimental data.

5) The basic assumption of the calibration-based approach is that observed biases are solely originating from physics parameters uncertainty. When other sources of uncertainty are prominently present, referred to as modeling uncertainty, the posteriori parameter values have to be over- or under-adjusted to account for modeling uncertainty. The impact of these over or under-adjustments may be significant enough to deteriorate the predictions for the reactor application. This is a challenging situation because it is difficult to hedge against the impact of modeling uncertainty even when it is carefully quantified prior to the adjustment search. Over the past ten years, the problem of model calibration under the influence of modeling uncertainty has occupied the attention of many practitioners, including applied mathematicians and statisticians, who have made several prominent proposals to account for the impact of

[3] The TSURFER module of SCALE represents an example of data adjustment techniques applied to criticality safety problems, Ref. [5]

modeling uncertainty on the adjusted parameters [6]. This problem however is arguably far from being solved.

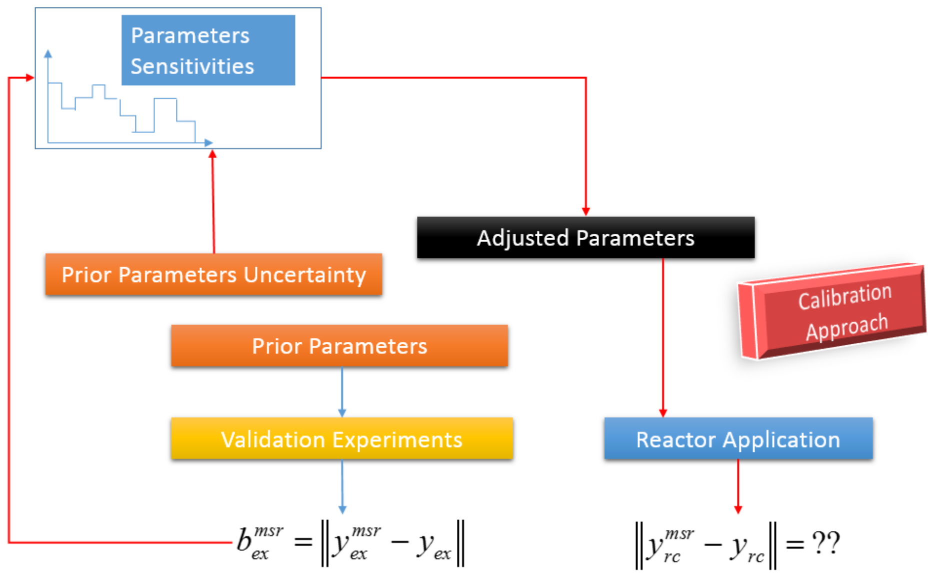


Fig 1. Calibration-based Approach for Reactor Application Bias Mapping

To address these challenges, PCM employs a different philosophy to map the biases. Instead of correcting for the various sources of uncertainties, via parameter adjustments, the physics models of the experiment and the reactor application are employed to find patterns between the experiment and the reactor responses directly, thus bypassing the need to calibrate the parameters. Depending on the quality of identified patterns, the experiment biases can be mapped to the reactor application domain. Fig. 2 depicts this situation for two different cases, one with high and the other moderate correlation between the reactor application eigenvalue k_{rc}^{eff} , and that of the experiment k_{ex}^{eff} .

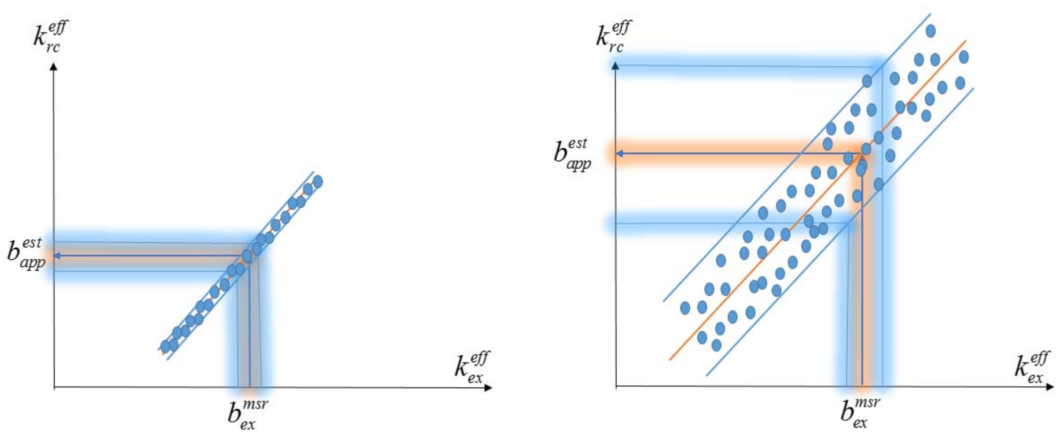


Fig 2. Basic Idea of PCM

In the left graph, the high correlation ensures that experiment bias b_{ex}^{msr} for the response can be mapped with low uncertainty to estimate the application bias b_{ex}^{est}. In the right figure however, the mapped application bias has higher uncertainty, which happens due to the presence of additional sources of uncertainties that are not common to both the experiment and the application, which results in reducing their mutual correlation. These scatter plots can be generated fairly easily employing the results of the two uncertainty analyses for the reactor application and the experiment (details will be provided in the next section). The following observations may be made:

a) The PCM methodology does not restrict the type of the relationship between the application and experiment responses to be linear. This is important in order to account for nonlinear relationships when sensitivity-based similarity indices would be no longer applicable.
b) To construct the scatter plots, PCM requires two uncertainty analyses, one done for the experiments model and one for the model of reactor application. These are straightforward non-intrusive analyses, wherein the number of model runs is independent of the number of model parameters or responses. Typically few hundred runs are sufficient for most problems.
c) Once the scatter plot is constructed, the application bias can be estimated using either parametric or nonparametric techniques. Parametric techniques such as response surface methods [6] can be employed, wherein the application bias is functionalized in terms of the experiment biases using a known polynomial (or generalized functions) with unknown coefficients, and the scatter plot is used to determine the unknown coefficients via least-squares fitting. The response surface predictions will denote the mapped bias and the residual of the fit will serve as a measure of the bias uncertainty. Nonparametric techniques such as kernel density estimators can also be used to map the application biases and their uncertainties [7].
d) PCM does not require access to derivative information, and hence the adjoint solver is no longer needed, implying that only black box code access is needed.

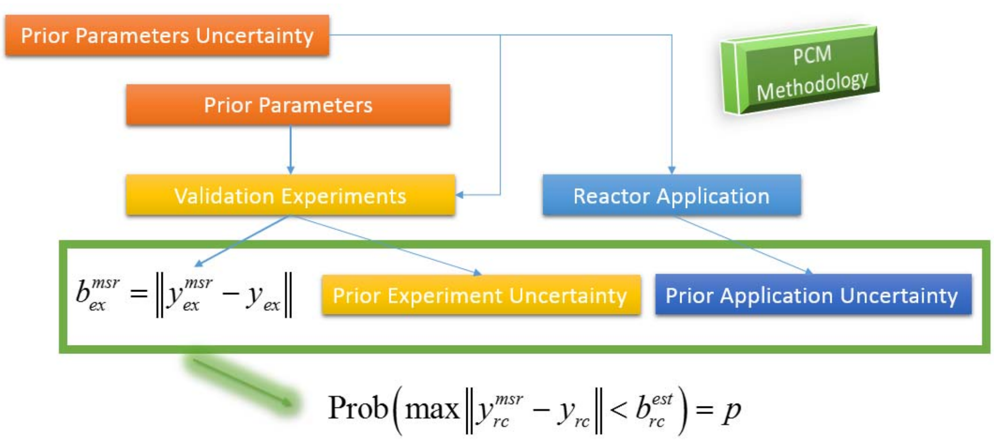


Fig 3. Proposed PCM Methodology for Reactor Application Bias Mapping

3. DESCRIPTION OF PCM ALGORITHM

A mathematical description of the steps required to implement PCM is given here. Assume that one is starting with M different experiments and a single application, where each experiment has a single response denoted as $y_{ex}^{msr,(i)}$, i =1, …, M. Also assume that computational models for the experiments and the application are available which are executed to obtain the reference values for the application response y_{rc}, and that of the experiments $y_{ex}^{(i)}$, i=1,2…, M. The goal is to employ the biases $y_{ex}^{(i)} - y_{ex}^{msr,(i)}$, i=1,…, M to determine a bias for the application response. The PCM algorithm proceeds as follows:

1. Identify all sources of uncertainties in the experiments and the application. Let x denote the common sources, while u_i refers to the sources unique to the i^{th} experiment, and v those of the application. For example, x can denote cross-sections, u_i the fuel to moderator ratio, geometry of the unit cell, etc., in the experiment, and v denotes uncertainties in one of the core parameters in the reactor application, e.g., the flow rate.
2. Generate N samples of x, u_i, and v according to their prior distributions. If x represents cross-sections, the prior covariance matrix should be used to ensure the x samples are statistically consistent with their prior uncertainties.
3. Execute the application and the M experiments computational models N times, each corresponding to one of the samples. This step is essentially an uncertainty analysis done for each experiment and the application.
4. Aggregate the N responses from the application and the i^{th} experiment into vectors y_{rc} and g_i both of length N, respectively, where i = 1,…, M
5. Find a relationship between the response of the application and the M experimental responses using the N training samples. Details on this are discussed below.
6. Based on relationship in 5, determine the application response, denoted by a vector y_{rc}^{proj} of N components. This variable is expected to be different from y_{rc}, because not all aspects of the application are captured by the experiments. The idea is to compare these two vectors to develop a useful definition for coverage.
7. Draw a scatter plot of the components of y_{rc} against those of y_{rc}^{proj}. If the experiments are indeed perfectly representative of the application, one would get a perfect contour that relates the two quantities. In reality, the scattered points will define a trend which describes the dependence of the application on the experiments, and the degree of the scatter will determine the uncertainty of this dependence.
8. Using the measured experimental biases as input to the relationship developed in 5, determine the estimated application bias, denoted, $y_{rc}^{est,proj}$
9. Using the scatter plot in 7, determine the possible values of the application bias that correspond to the value of $y_{rc}^{est,proj}$. This could be described as an interval or via a full PDF using kernel density estimation.

The relationship required in step #5 may be determined parametrically, i.e., using response surface methods, or via a large number of non-parametric statistical techniques, e.g., order statistics, kernel density estimators, projection pursuit techniques, etc. In this introductory presentation, we will use a simple parametric approach based on a linear surrogate model. Extension to other techniques will be part of future work.

Let the N samples of the application and experiments satisfy the following linear mapping in a Least-Squares sense:

$$y_{rc}^{(i)} = \alpha_1 g_1^{(i)} + \alpha_2 g_2^{(i)} + ... + \alpha_M g_M^{(i)}, \quad i = 1, ..., N$$

By minimizing the residual of these equations, one can determine the coefficients α_i. Following that, determine y_{rc}^{proj} according to step 6:

$$y_{rc}^{proj,(i)} = \alpha_1 g_1^{(i)} + \alpha_2 g_2^{(i)} + ... + \alpha_M g_M^{(i)}, \quad i = 1, ..., N$$

The N samples of y_{rc} and y_{rc}^{proj} are then graphed using a scatter plot on the x and y-axes, respectively, per step 7, example shown in Fig. 4 in the next section. Per step 8, the estimated application bias is given by:

$$y_{rc}^{est,proj} = \alpha_1 \left(y_{ex}^{msr,(1)} - y_{ex}^{(1)} \right) + ... + \alpha_M \left(y_{ex}^{msr,(M)} - y_{ex}^{msr,(M)} \right)$$

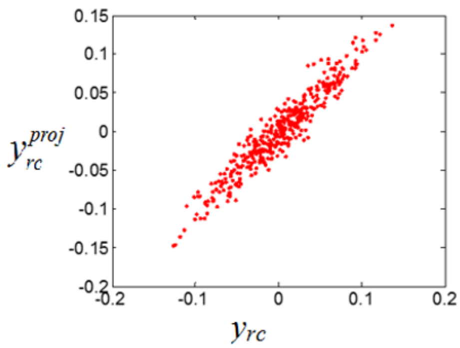
Using the scatter plot, determine the range of the application bias values that correspond to the estimated value. This range denotes the uncertainty in the estimated bias.

4. NUMERICAL EXPERIMENTS

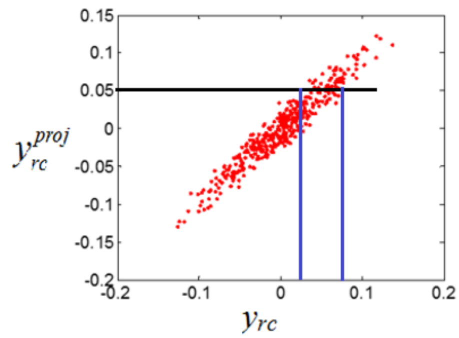
For this preliminary study, the sensitivity profiles for 25 critical experiments formed the pool of our analysis. The first K experiments (taken at K=10, K=15, and K=25) are grouped together to represent the experimental domain. Experiment #30 (see Appendix) is taken to represent the application of interest. In this work, we employ the new super-sequence CRANE [9], recently introduced into the SCALE code package to generate the N random samples for the cross-sections based on the SCALE 44-group covariance library (scale.rev05.44groupcov) [5]. The N samples are employed to generate N samples for the application experimental responses. Based on fitting to a linear model, the application responses estimated based on the experiments are scatter-plotted against the original application responses as done in Fig. 4. This figure may be used as follow: based on the M experiments biases, estimate the application bias $y_{rc}^{est,proj}$. Draw a horizontal line at this value on the y-axis (shown as black line in the right graph), and move horizontally to the scattered points, then move vertically towards the x-axis (shown as two blue lines) to determine the possible range of values for the application bias. In practice, this can be done analytically using kernel density estimators, but for the sake of this introductory presentation, a graphical description is provided. The value of this approach is that one can see clearly the relationship between the experiments and the application, and the impact of uncertainties on the mapped biases. If the bias uncertainty is small, the coverage is poor, and vice versa. Based on the insight learned from this application, one can develop more rigorous definitions for the coverage, which will be investigated in future work.

5. CONCLUSIONS

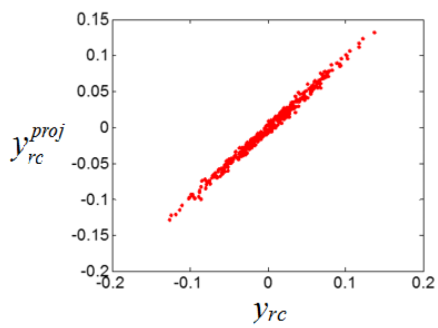
This manuscript has introduced a calibration-free methodology to support model validation. The new method simply relies on the physics of the experiments and the reactor operating conditions to map the experimental biases to the domain of reactor operation. The mapping is done based on a joint uncertainty analysis that is capable of quantifying the mutual information between the experimental and operational domains. This provides a unique ability to measure coverage, and map biases and biases uncertainties in a credible manner for general linear and nonlinear relationship, which avoids the calibration of model parameters, and the need for sensitivity coefficients.



Coverage with 10 Experiments



Coverage with 15 Experiments



Coverage with 25 Experiments

Fig. 4. Application Coverage by Experiments

6. ACKNOWLEDGEMENTS

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8. APPENDIX

Source: NEA/NSC/DOC(95)03/IV: EVALUATED EXPERIMENTS Critical and Subcritical Measurements Low Enriched Uranium Systems

LEU-COMP-THERM-001 Water-Moderated U(2.35)O2 Fuel Rods in 2.032-cm Square-Pitched Arrays

LEU-COMP-THERM-002 Water-Moderated U(4.31)O2 Fuel Rods in 2.54-cm Square-Pitched Arrays

LEU-COMP-THERM-003 Water-Moderated U(2.35)O2 Fuel Rods in 1.684-cm Square-Pitched Arrays (Gadolinium Water Impurity)

LEU-COMP-THERM-004 Water-Moderated U(4.31)O2 Fuel Rods in 1.892-cm Square-Pitched Arrays (Gadolinium Water Impurity)

LEU-COMP-THERM-005 Critical Experiments with Low-Enriched Uranium Dioxide Fuel Rods in Water Containing Dissolved Gadolinium

LEU-COMP-THERM-006 Critical Arrays of Low Enriched UO2 Fuel Rods with Water-to-Fuel Volume Ratios Ranging from 1.5 to 3.0

LEU-COMP-THERM-007 Water Reflected 4.738 Wt.% Enriched Uranium Dioxide Fuel Rod Arrays

LEU-COMP-THERM-008 Critical Lattice of UO2 Fuel Rods and Perturbing Rods in Borated Water

LEU-COMP-THERM-009 Water-Moderated Rectangular Clusters of U(4.31)O2 Fuel Rods (2.54-cm Pitch) Separated by Steel, Boral, Copper, Cadmium, Aluminum, or Zircalloy-4 Plates

LEU-COMP-THERM-010 Critical Arrays of Water-Moderated U(4.31)O2 Fuel Rods Reflected by Two Lead, Uranium, or Steel Walls

LEU-COMP-THERM-011 Critical Experiments Supporting Close Proximity Water Storage of Power Reactor Fuel, Part I - Absorber Rods

LEU-COMP-THERM-012 Water-Moderated Rectangular Clusters of U(2.35)O2 Fuel Rods (1.684-cm Pitch) Separated by Steel, Boral, Boroflex, Cadmium, or Copper Plates (Gadolinium Water Impurity)

LEU-COMP-THERM-013 Water-moderated Rectangular Clusters of U(4.31)O2 Fuel Rods (1.892-cm pitch) Separated by Steel, Boral, Boroflex, Cadmium, or Copper Plates, with Steel Reflecting Walls

LEU-COMP-THERM-014 Water-Reflected Arrays of U(4.31)O2 Fuel Rods (1.890-cm and 1.715-cm Square Pitch) in Borated Water

LEU-COMP-THERM-015 The VVER Experiments: Regular and Perturbed Hexagonal Lattices of Low-Enriched UO2 Fuel Rods in Light Water

LEU-COMP-THERM-016 Water-Moderated Rectangular Clusters of U(2.35) O2 Fuel Rods (2.032-cm Pitch) Separated by Steel, Boral, Copper, Cadmium, Aluminum, or Zircaloy-4 Plates

LEU-COMP-THERM-017 Critical Arrays of Water-Moderated U(2.35)O2 Fuel Rods Reflected by Two Lead, Uranium, or Steel Walls

LEU-COMP-THERM-018 Light Water Moderated and Reflected Low Enriched Uranium Dioxide (7 wt.%) Rod Lattice

LEU-COMP-THERM-019 Water-Moderated Hexagonally Pitched Lattices of U(5%)O2 Stainless Steel Clad Fuel Rods

LEU-COMP-THERM-020 Water-Moderated Hexagonally Pitched Partially Flooded Lattices of U(5%)O2 Zirconium Clad Fuel Rods

LEU-COMP-THERM-021 Hexagonally Pitched Partially Flooded Lattices of U(5%)O2 Zirconium Clad Fuel Rods Moderated by Water with Boric Acid

LEU-COMP-THERM-022 Uniform Water-Moderated Hexagonally Pitched Lattices of Rods with U(10%)O2 Fuel

LEU-COMP-THERM-023 Partially Flooded Uniform Lattices of Rods with U(10%)O2 Fuel

LEU-COMP-THERM-024 Water-Moderated Square-Pitched Uniform Lattices of Rods with U(10%)O2 Fuel

LEU-COMP-THERM-025 Water-Moderated Hexagonally Pitched Lattices of U(7.5%)O2 Stainless-Steel-Clad Fuel Rods

LEU-COMP-THERM-030 VVER Physics Experiments: Regular Hexagonal (1.27 cm Pitch) Lattices of Low-Enriched U(3.5 wt.% 235U)O2 Fuel Rods in Light Water at Different Core Critical Dimensions