7th International Conference on Modelling and Simulation in Nuclear Science and Engineering (7ICMSNSE)
Ottawa Marriott Hotel, Ottawa, Ontario, Canada, October 18-21, 2015

7ICMSNSE-002

Code Development of Total Sensitivity and Uncertainty Analysis for Reactor Physics Calculations

Chenghui Wan[1], Liangzhi Cao[1], Hongchun Wu[1], Tiejun Zu[1], Wei Shen[1,2]
[1] School of Nuclear Science and Technology, Xi'an Jiaotong University, Xi'an, China
[2] Canadian Nuclear Safety Commission, Ottawa, Ontario, Canada
chenghuiwan@stu.xjtu.edu.cn, caolz@mail.xjtu.edu.cn, hongchun@mail.xjtu.edu.cn, tiejun@mail.xjtu.edu.cn, Wei.Shen@cnsc-ccsn.gc.ca

Abstract

Sensitivity and uncertainty analysis are essential parts for reactor system to perform risk and policy analysis. In this study, total sensitivity and corresponding uncertainty analysis for responses of neutronics calculations have been accomplished and developed the S&U analysis code named UNICORN. The UNICORN code can consider the implicit effects of multigroup cross sections on the responses. The UNICORN code has been applied to typical pin-cell case in this paper, and can be proved correct by comparison the results with those of the TSUNAMI-1D code.

Keywords: sensitivity analysis, uncertainty analysis, neutronics

1. Introduction

In recent years, there has been an increasing demand from nuclear research, industry, safety and regulation for best-estimate predictions to be provided with their confidence bounds [1]. Uncertainty quantification can satisfy this demand to determine the appropriate design margins for the nuclear system. As neutronics calculation is the prerequisite for predictions of the nuclear system, the uncertainties introduced in neutronics calculations would impact the prediction results of the nuclear system. Therefore, uncertainty analysis has been focused on the neutronics calculations and the imprecisions of cross sections have been treated as one of the most significant sources of uncertainties [2] recently. According to the previous researches, the uncertainties of neutronics calculation responses are non-ignorable, with relative standard deviations of eigenvalue up to be about 0.55% for the Peach Bottom 2 (PB-2) pin-cell [3] and 0.43% for the Three Mile Island Unit-1 (TMI-1) core [4]. In this context, it's necessary and significant to perform uncertainty analysis to neutronics calculations to obtain much more confident prediction results.

There exit two categories of methods widely applied to perform uncertainty analysis, aimed at propagating cross-section uncertainties to the responses of neutronics calculations: the deterministic method and the statistical sampling method. For the deterministic method, sensitivity analysis is essential and necessary to obtain the relative sensitivity coefficients of responses with respect to cross sections firstly. And then the "sandwich rule" is applied to calculate the uncertainties of responses by combining the relative sensitivity coefficients and relative variance-covariance matrix (VCM). In order to obtain the relative sensitivity coefficients, the perturbation theory (PT) [5] and the direct numerical perturbation method (DNP) [6] are available. For the statistical sampling method, the samples of cross sections are generated according to the distributions of them firstly. And then each cross-section sample is used as input parameters to carry out the neutronics calculations to obtain corresponding responses. Finally, the statistical calculation is applied to obtain the uncertainties of responses.

In this paper, the UNICORN code has been developed, with capabilities of performing sensitivity and uncertainty analysis for responses of neutronics calculations with respect to multigroup cross sections. The DNP method and statistical sampling method have been chosen and accomplished in the UNICORN code to perform sensitivity and uncertainty analysis respectively. For uncertainty analysis, the statistical sampling method has the obvious advantages, including no approximation and no limit to the number of responses, compared with the deterministic method. However, the relative sensitivity coefficients, which can be used to perform similarity analysis [7] and cross-section adjustment [8], are beyond the capability of the statistical sampling method. Therefore, the sensitivity analysis function has been developed in UNICORN, and the DNP method is chosen to obtain the relative sensitivity coefficients, because the DNP method is convenient and no rely on the models of neutronics calculations, compared with the PT method. In addition, the lattice code DRAGON 4.0 [9] is used to carry out the resonance self-shielding and neutron-transport calculations with application of the WIMSD-4 format multigroup cross-section library.

In section 2, an overview is given firstly and then the theories and methods applied are introduced. In section 3, numerical results and analysis are given and explained. Finally, conclusions are summarized in section 4.

2. Theories and Methods

2.1 Overall calculation flow

The flowchart of UNICORN is as shown in Fig. 1. There are there parts of main works included in the UNICORN code. Firstly, the essential nuclear data should be obtained. In UNICORN, both the integral cross sections including σ_{t}, σ_{s}, and σ_{a}, and the basic cross sections including $\sigma_{(\mathrm{n,elas})}$, $\sigma_{(\mathrm{n,inel})}$, $\sigma_{(\mathrm{n,2n})}$, $\sigma_{(\mathrm{n,3n})}$, $\sigma_{(\mathrm{n,f})}$, $\sigma_{(\mathrm{n},\gamma)}$, $\sigma_{(\mathrm{n,p})}$, $\sigma_{(\mathrm{n,D})}$, $\sigma_{(\mathrm{n,T})}$, $\sigma_{(\mathrm{n,He})}$, $\sigma_{(\mathrm{n},\alpha)}$ and ν can be analyzed. All these cross sections can be obtained by combining the cross-section information included in WIMSD-4 format multigroup cross-section library and those in NJOY [10] output files. Secondly, the multigroup cross-section perturbation model should be established. In UNICORN, both the DNP method and the statistical sampling method would perform perturbations to multigroup cross sections. Therefore, the cross-section perturbation model is established to perform perturbations to multigroup cross sections and keep them consistency and balance. Thirdly, the DNP method and the statistical sampling method would be accomplished and developed in UNICORN. For DNP method, the relative sensitivity coefficients and relative uncertainties of responses with respect to the multigroup cross sections are obtained. And for the statistical sampling method, the uncertainties of responses are given.

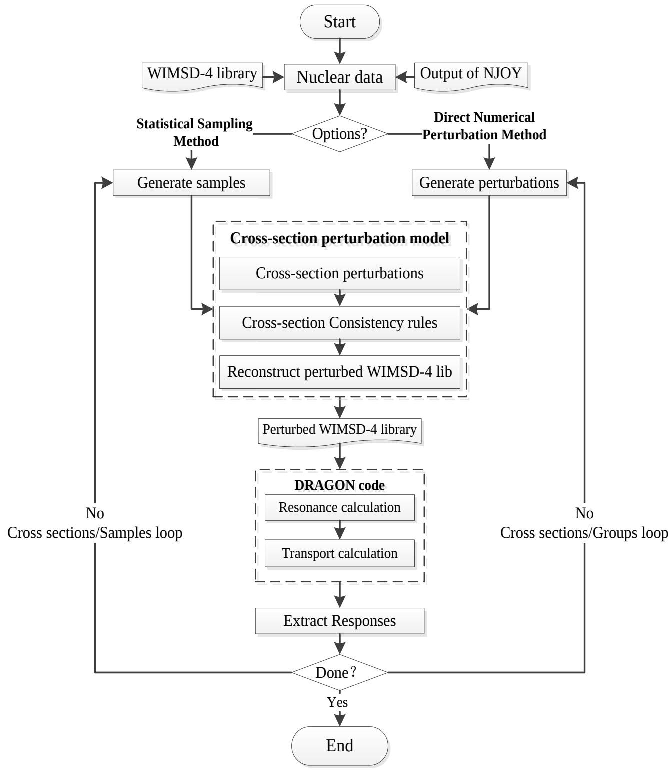

Figure 1. Flowchart of the UNICORN code

In the following context, the multigroup cross-section perturbation, the statistical sampling method and the DNP method are introduced in detailed in subsections.

2.2 Multigroup cross-section perturbation model

As mentioned above, the cross-section perturbation model should be established for both the DNP method and the statistical sampling method. In this section, the perturbation propagations and cross-section consistency rules are introduced in detail.

Firstly, the perturbations of multigroup cross sections should be established through the perturbations added to the point-wise cross sections. According to the deterministic method, multigroup cross sections are needed for resonance self-shielding and neutronics calculations. And the multigroup cross sections are generated by the point-wise ones with application of weighting flux as shown in Eq. (1).

$$\sigma_{x,g}(T,\sigma_0)=\frac{\int_{\Delta E_g}\sigma_x(E,T)\phi(E,\sigma_0)dE}{\int_{\Delta E_g}\phi(E,\sigma_0)dE} \tag{1}$$

where σ_0 is the Bondarenko parameter, T is the temperature; $\sigma_x(E,T)$ presents the point-wise cross section for type of x, and $\sigma_{x,g}(T, \sigma_0)$ stands for the gth group cross-section of type x; $\phi(E,\sigma_0)$ is the weighting flux. The perturbations of multigroup cross sections should be consistency with the perturbations propagated from the point-wise ones. In this paper, it is assumed that the perturbation for the *gth* group of type x is performed by uniform relative perturbation added to the point-wise cross section within the energy range of the *gth* group, as shown in Eq. (2).

$$\sigma_x^{'}(E,T)=(1+\delta_{x,g})\sigma_x(E,T) \quad E_{g-1}\le E\le E_g \tag{2}$$

where E_{g-1} and E_g present the lower and upper energy boundaries of the *gth* group; $\sigma_x^{'}(E,T)$ stands for the perturbed point-wise cross section of type *x*, and $\delta_{x,g}$ is the relative perturbation factor added to the cross section of type *x*. The perturbation propagations from point-wise cross sections to the multigroup ones are different to different types of cross sections. For the cross sections without resonance, the weighting flux is selected or input by user and independent of the point-wise cross sections, which is the function of energy *E*. Therefore, the perturbation propagations are linear and can be presented shown as in Eq. (3).

$$\sigma_{x,g}^{'}(T)=\frac{\int_{\Delta E_g}\sigma_x^{'}(E,T)\phi(E)dE}{\int_{\Delta E_g}\phi(E)dE}=(1+\delta_{x,g})\frac{\int_{\Delta E_g}\sigma_x(E,T)\phi(E)dE}{\int_{\Delta E_g}\phi(E)dE}=(1+\delta_{x,g})\sigma_{x,g}(T) \tag{3}$$

However, for the resonant cross sections, the perturbations are non-linear. Because the weighting flux within resonance-energy regions are obtained by solving neutron slowing-down equation and cross sections perturbations would result in perturbations to the weighting flux. And the solution of slowing-down equation can be presented as shown in Eq. (4), according to the narrow resonance (NR) approximation.

$$\phi(E,\sigma_0)=\frac{\sigma_p^r+\sigma_0}{\sigma_t(E)+\sigma_0}\Psi(E) \tag{4}$$

where σ_p^r and $\sigma_t(E)$ present the potential scattering cross section and total cross section of the resonant nuclide correspondingly, and $\psi(E)$ stands for the 1/*E* shape. Therefore, the perturbation propagations for the resonant cross sections within resonance groups can be presented as shown in Eq. (5).

$$\begin{aligned}\sigma_{x,g}^{'}(T,\sigma_0)&=\frac{\int_{\Delta E_g}\sigma_x^{'}(E,T)\phi(E,\sigma_0)dE}{\int_{\Delta E_g}\phi(E,\sigma_0)dE}=(1+\delta_{x,g})\frac{\int_{\Delta E_\mu}\sigma_x(\mu,T)\dfrac{\sigma_p^r+\sigma_0}{\sigma_t(\mu,T)+\delta_{x,g}\sigma_x(\mu,T)+\sigma_0}d\mu}{\int_{\Delta E_\mu}\dfrac{\sigma_p^r+\sigma_0}{\sigma_t(\mu,T)+\delta_{x,g}\sigma_x(\mu,T)+\sigma_0}d\mu}\\&=(1+\delta_{x,g})\frac{\int_{\Delta E_\mu}\sigma_x(\mu,T)\dfrac{\sigma_p^r+\sigma_0}{(1+\delta_{t,g})\sigma_t(\mu,T)+\sigma_0}d\mu}{\int_{\Delta E_\mu}\dfrac{\sigma_p^r+\sigma_0}{(1+\delta_{t,g})\sigma_t(\mu,T)+\sigma_0}d\mu}=(1+\delta_{x,g})\sigma_{x,g}(T,\sigma_0^{'})\end{aligned} \tag{5}$$

By combining Eq. (3) and Eq. (5), the multigroup cross-section perturbation model is established.

Secondly, consistency rules should be established to keep cross sections balance. As mentioned above, the UNICORN code can perform analysis not only to integral cross sections, but also to basic cross sections, which are absent from the neutron-transport equation and the WIMSD-4 library. The perturbations of basic cross sections should be presented to the lumped ones contained in WIMSD-4. Therefore, the consistency rules are established. According to the WLUP [11] project, the consistency rules for the WIMSD-4 library can be presented as shown in Eq. (6), (7) and (8).

$$\sigma_{s,g\to h}=\sigma_{(\mathrm{n,elas}),g\to h}+\sigma_{(\mathrm{n,inel}),g\to h}+2\sigma_{(\mathrm{n,2n}),g\to h}+3\sigma_{(\mathrm{n,3n}),g\to h} \tag{6}$$

$$\sigma_{a,g}=\sigma_{(n,f)}+\sigma_{(n,g)}+\sigma_{(n,\alpha)}+\sigma_{(n,2\alpha)}+\sigma_{(n,\mathrm{p})}+\sigma_{(n,\mathrm{D})}+\sigma_{(n,\mathrm{T})}+\sigma_{(n,\mathrm{He3})}-\sigma_{(n,2n),g}-2\sigma_{(n,3n),g} \tag{7}$$

$$\sigma_{t,g}=\sigma_{a,g}+\sigma_{s,g} \tag{8}$$

According to these consistency rules, the perturbations of basic cross sections can be presented in the lumped integral cross sections included in WIMSD-4 and thus effect the responses of neutronics calculations.

2.3 Statistical sampling method

For any system, the relationship between input parameters and responses can be briefly characterized as shown in Eq. (9).

$$\boldsymbol{R} = \boldsymbol{f}(\boldsymbol{X}) \tag{9}$$

where $\boldsymbol{X}$ presents the multi-input vector and can be characterized as $\boldsymbol{X}=[x_1,x_2,\dots,x_{\mathrm{nx}}]^T$; $\boldsymbol{R}$ presents the multi-response vector and can be characterized as $\boldsymbol{R}=[R_1,R_2,\dots,R_{\mathrm{nR}}]^T$.

The procedures for uncertainty estimation by statistical sampling method can be summarized into four main steps [12].

Firstly, cross-section distribution regions are required. The group-wise VCM can be generated by application of the NJOY code. In VCM, the diagonal elements present the variances or uncertainties of cross sections, and the off-diagonal elements are covariance between them.

Secondly, it's essential to generate the cross-section samples. The cross-section samples can be generated by using Eq. (10).

$$\boldsymbol{X}_S = \boldsymbol{\Sigma}^{1/2}\boldsymbol{Y}_S + \boldsymbol{\mu} \tag{10}$$

where $\boldsymbol{X}_S$ and $\boldsymbol{Y}_S$ represent the samples for cross sections and independent parameters respectively; $\boldsymbol{\mu}=[\mu_1, \mu_2,\dots, \mu_{\mathrm{nx}}]^T$ is the expectation value vector of cross sections; $\boldsymbol{\Sigma}$ is the group-wise VCM of cross sections.

Thirdly, the input-responses mapping can be obtained by carrying out the target code to perform neutronics calculations with use of cross-section samples. And the mapping can be presented as $[\boldsymbol{X}_{S,i}, \boldsymbol{R}_i]$ (i=1,2,…,nS, where nS is the number of samples).

Finally, statistical calculation is used to estimate the uncertainties of responses according to the mappings obtained by the third step. And the standard deviation of the kth response can be obtained as shown in Eq. (11).

$$\sigma(R_k) = \sqrt{\frac{1}{nS-1}\sum_{i=1}^{nS}(R_{k,i} - R_{k,0})^2} \tag{11}$$

where $\sigma(R_k)$ is the standard deviation of the kth response and $R_{k,0}$ present the expectation value which can be characterized as shown in Eq. (12).

$$R_{k,0} = \frac{1}{nS}\sum_{i=1}^{nS} R_{k,i} \tag{12}$$

By application of the four steps above, the uncertainties of responses due to multigroup cross sections can be determined and obtained.

In this paper, the bootstrap method [13] has been accomplished for determining the confidence intervals. The confidence intervals are obtained by re-sampling method and can be calculated as shown in Eq. (13).

$$\Delta\sigma(R_k) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\sigma(R_k)_i - \sigma(R_k)_0)^2} \tag{13}$$

where $\Delta\sigma(R_k)$ presents the deviation of the uncertainty results by application of the N re-samples; $\sigma(R_k)_i$ is the uncertainty result of the ith re-samples, and $\sigma(R_k)_0$ presents the expectation value of the N uncertainty results.

2.4 Direct numerical method

The DNP method is a straightforward method to calculate the relative sensitivity coefficients. And the relative sensitivity coefficients of responses with respect to multigroup cross sections can be presented briefly as shown in Eq. (14).

$$\begin{aligned} S_{R_k,\sigma_{x,g}} &= \frac{\sigma_{x,g}}{R_k}\frac{\partial R_k}{\partial \sigma_{x,g}} \approx \frac{\sigma_{x,g}}{R_k}\frac{R_k((1+\delta^+_{x,g})\sigma_{x,g}) - R_k((1+\delta^-_{x,g})\sigma_{x,g})}{(\delta^+_{x,g}-\delta^-_{x,g})\sigma_{x,g}} \\ &= \frac{1}{R_k}\frac{R_k((1+\delta^+_{x,g})\sigma_{x,g}) - R_k((1+\delta^-_{x,g})\sigma_{x,g})}{\delta^+_{x,g}-\delta^-_{x,g}} \end{aligned} \tag{14}$$

where $\sigma^+_{x,g}$ and $\sigma^-_{x,g}$ represent the positive and negative relative perturbations for the *g*th group's cross section with type of *x* respectively; $\sigma_{x,g}$ and R_k stand for the un-perturbed cross section and the *kth* response respectively.

Uncertainty analysis can be performed by combining the relative sensitivity coefficients with relative covariance matrix based on the "sandwich rule". The "sandwich rule" for uncertainty quantification of eigenvalue to the multigroup cross sections can be presented as shown in Eq. (15).

$$\frac{\sigma^2(R_k)}{R_k^{\ 2}} = \mathbf{S}_{R_k,\alpha_i}\mathbf{\Sigma}_{\alpha_i\alpha_j}\mathbf{S}^T_{R_k,\alpha_j} \tag{15}$$

where $\sigma^2(R_k)$ is the variance of responses due to uncertainties of multigroup cross sections; $\mathbf{\Sigma}_{\alpha_i\alpha_j}$ presents the relative covariance data for cross sections α_i and α_j.

3. Numerical Results and Analysis

3.1 Verification of UNICORN

For verification of UNICORN, a 238-group WIMSD-4 format cross-section library has been generated based on ENDF/B-VII.0 with application of NJOY in this paper. With the 238-group library, NECP-RB31 benchmark case has been analyzed by both UNICORN and TSUNAMI-1D to verify the cross-section perturbation model and development of UNICORN. The eight most significant types of cross sections and the total relative sensitivity coefficients with respect to them are shown in **Table 1**.

Table 1. The eight most significant types of cross section to k_∞ of NECP-RB31

Nuclides	**Cross section**	**Relative sensitivity coefficients**	
		TSUNAMI-1D	UNICORN
^{235}U	v	9.52E-01	9.51E-01
^{235}U	$\sigma_{(n,f)}$	2.50E-01	2.48E-01
^{238}U	$\sigma_{(n,\gamma)}$	-1.84E-01	-1.80E-01
^{235}U	$\sigma_{(n,\gamma)}$	-1.50E-01	-1.49E-01
^{1}H	$\sigma_{(n,elas)}$	1.50E-01	1.48E-01
^{1}H	$\sigma_{(n,\gamma)}$	-5.04E-02	-5.03E-02
^{238}U	v	4.83E-02	4.91E-02
^{238}U	$\sigma_{(n,f)}$	2.21E-02	2.24E-02

It can be observed that the total relative sensitivity coefficients calculated by UNICORN code can agree well with those calculated by the TSUNAMI-1D code. In addition, for much more detailed comparison of total relative sensitivity coefficients of UNICORN code and TSUNAMI-

1D code, the group-wise total relative sensitivity coefficients of the eight most significant cross sections are compared and as shown in **Fig**. 2, 3, 4 and 5.

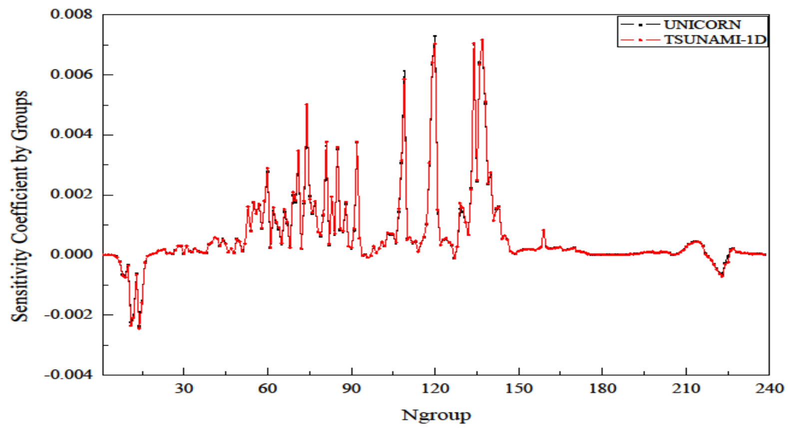

Figure 2. Group-wise relative sensitivity coefficients for $\sigma_{(n,elas)}$ of ^{1}H

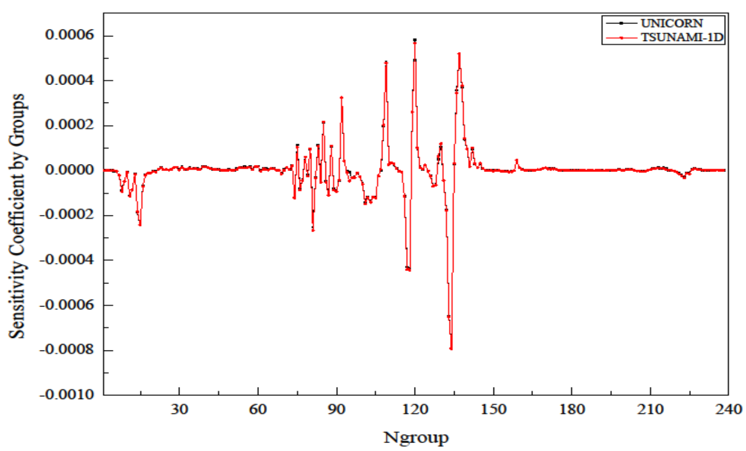

Figure 3. Group-wise relative sensitivity coefficients for $\sigma_{(n,elas)}$ of ^{16}O

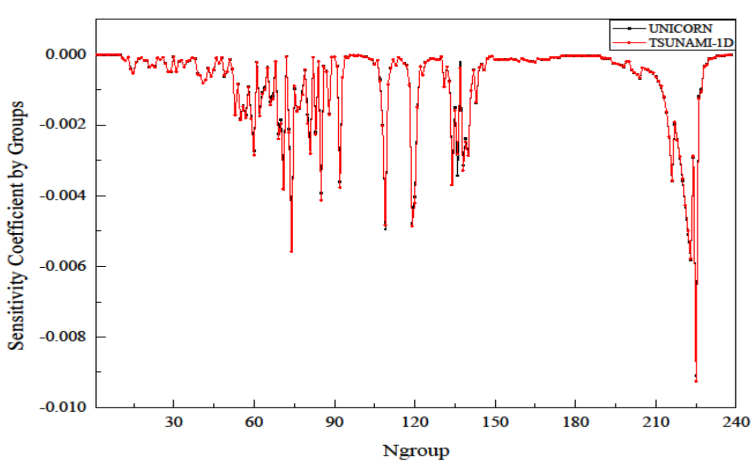

Figure 4. Group-wise relative sensitivity coefficients for $\sigma_{(n,\gamma)}$ of ^{238}U

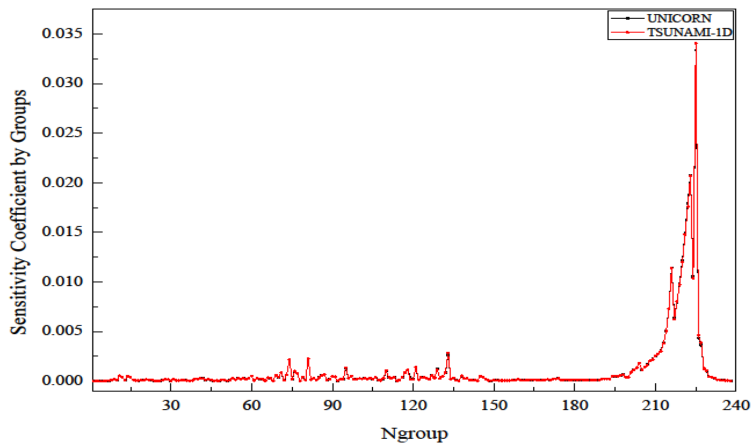

Figure 5. Group-wise relative sensitivity coefficients for $\sigma_{(n,f)}$ of ^{235}U

It can be observed that the group-wise relative sensitivity coefficients of eigenvalue with respect to cross sections by UNICORN are agreed well with those of TSUNAMI-1D. It should be noted that the DNP method is a straightforward method to calculate the relative sensitivity coefficients, applying the difference quotient to estimate the partial derivative, therefore the numerical differences are inevitably existed for the relative sensitivity coefficient results of UNICORN and TSUNAMI-1D. And for the relative sensitivity coefficients, whose value is almost zero, the relative errors between the results of UNICORN and TSUNAMI-1D could be up to 50% or even larger, while for the others, the relative errors are within 5%. These relative errors are within the acceptable ranges for the relative sensitivity coefficients using the DNP method. In this context, it can be proved that the multigroup cross-section perturbation model established in this paper is correct and the development of UNICORN is also correct.

3.2 Application of UNICORN

For application of UNICORN, the NECP-RB31 benchmark [14] has been analyzed by both DNP method and statistical sampling method with 69-group WIMSD-4 library. In this application, the eigenvalue and 2-group few-group macroscopic cross sections are treated as the responses under analysis. And the relative covariance matrixes of these responses, which are caused by $^{235}U(\sigma_{(n,elas)}, \sigma_{(n,inel)}, \sigma_{(n,2n)}, \sigma_{(n,f)}, \sigma_{(n,\gamma)}$ and $\nu)$, $^{238}U(\sigma_{(n,elas)}, \sigma_{(n,inel)}, \sigma_{(n,2n)}, \sigma_{(n,f)}, \sigma_{(n,\gamma)}$ and $\nu)$, $^{16}O(\sigma_{(n,elas)}, \sigma_{(n,inel)}, \sigma_{(n,\gamma)}$ and $\sigma_{(n,\alpha)})$ and $^{1}H(\sigma_{(n,elas)}$ and $\sigma_{(n,\gamma)})$, are as shown in **Table 2**, **3** and **4**.

Table 2 presents the relative covariance matrix for responses by applying the DNP method, and the σ(R)/R term means the relative standard deviation of corresponding responses. **Table 2** and **3** present the relative covariance matrix obtained by statistical sampling method, and **Table 2** shows the expectation values of relative covariance matrix by 20 different re-samples with *nS* =100. And Table 3 contains the standard deviations for the relative covariance matrixes obtained by the different 20 re-samples.

Table 2. Relative covariance matrix of few-group cross sections by DNP method

Res./Res.	k_{eff}	$\Sigma_{t,1}$	$\Sigma_{t,2}$	$\nu\Sigma_{f,1}$	$\nu\Sigma_{f,2}$	$\Sigma_{s,1,1}$	$\Sigma_{s,1,2}$	$\Sigma_{s,2,1}$	$\Sigma_{s,2,2}$

k_{eff}	5.69E-05	-4.79E-06	6.70E-07	2.74E-05	5.12E-05	-4.60E-06	1.56E-06	-1.92E-06	1.05E-06
$\Sigma_{t,1}$	-4.79E-06	6.99E-05	8.54E-06	1.13E-06	1.31E-06	6.99E-05	8.29E-05	1.03E-05	9.16E-06
$\Sigma_{t,2}$	6.70E-07	8.54E-06	6.50E-06	2.80E-06	1.40E-06	8.35E-06	1.51E-05	3.86E-06	6.96E-06
$\nu\Sigma_{f,1}$	2.74E-05	1.13E-06	2.80E-06	4.51E-05	2.27E-05	9.97E-07	7.06E-06	1.34E-06	2.87E-06
$\nu\Sigma_{f,2}$	5.12E-05	1.31E-06	1.40E-06	2.27E-05	6.07E-05	1.21E-06	4.15E-06	1.69E-06	1.30E-06
$\Sigma_{s,1,1}$	-4.60E-06	6.99E-05	8.35E-06	9.97E-07	1.21E-06	6.99E-05	8.16E-05	1.10E-05	8.96E-06
$\Sigma_{s,1,2}$	1.56E-06	8.29E-05	1.51E-05	7.06E-06	4.15E-06	8.16E-05	1.34E-04	-3.12E-06	1.62E-05
$\Sigma_{s,2,1}$	-1.92E-06	1.03E-05	3.86E-06	1.34E-06	1.69E-06	1.10E-05	-3.12E-06	2.32E-05	3.88E-06
$\Sigma_{s,2,2}$	1.05E-06	9.16E-06	6.96E-06	2.87E-06	1.30E-06	8.96E-06	1.62E-05	3.88E-06	7.50E-06
σ(R)/R|%	7.54E-01	8.36E-01	2.55E-01	6.72E-01	7.79E-01	8.36E-01	1.16E+00	4.81E-01	2.74E-01

Table 3. Relative covariance matrix of few-group cross sections by statistical sampling method

Res./Res.	k_{eff}	$\Sigma_{t,1}$	$\Sigma_{t,2}$	$\nu\Sigma_{f,1}$	$\nu\Sigma_{f,2}$	$\Sigma_{s,1,1}$	$\Sigma_{s,1,2}$	$\Sigma_{s,2,1}$	$\Sigma_{s,2,2}$
k_{eff}	5.12E-05	-4.65E-06	6.06E-07	2.51E-05	4.45E-05	-4.59E-06	1.36E-06	-4.24E-06	1.08E-06
$\Sigma_{t,1}$	-4.65E-06	8.59E-05	1.36E-05	2.81E-06	2.22E-06	8.58E-05	9.65E-05	9.10E-06	1.48E-05
$\Sigma_{t,2}$	6.06E-07	1.36E-05	9.40E-06	3.56E-06	1.83E-06	1.32E-05	1.90E-05	3.31E-06	1.02E-05
$\nu\Sigma_{f,1}$	2.51E-05	2.81E-06	3.56E-06	4.59E-05	2.05E-05	2.63E-06	6.70E-06	1.54E-06	3.73E-06
$\nu\Sigma_{f,2}$	4.45E-05	2.22E-06	1.83E-06	2.05E-05	5.35E-05	2.11E-06	4.09E-06	2.07E-06	1.77E-06
$\Sigma_{s,1,1}$	-4.59E-06	8.58E-05	1.32E-05	2.63E-06	2.11E-06	8.58E-05	9.53E-05	9.77E-06	1.44E-05
$\Sigma_{s,1,2}$	1.36E-06	9.65E-05	1.90E-05	6.70E-06	4.09E-06	9.53E-05	1.31E-04	-3.98E-06	2.08E-05
$\Sigma_{s,2,1}$	5.12E-05	-4.65E-06	6.06E-07	2.51E-05	4.45E-05	-4.59E-06	1.36E-06	-4.24E-06	1.08E-06
$\Sigma_{s,2,2}$	-4.65E-06	8.59E-05	1.36E-05	2.81E-06	2.22E-06	8.58E-05	9.65E-05	9.10E-06	1.48E-05
σ(R)/R|%	7.15E-01	9.27E-01	3.07E-01	6.78E-01	7.32E-01	9.26E-01	1.14E+00	4.95E-01	3.35E-01

Table 4. Standard deviations of relative covariance matrix by statistical sampling method

S.d./S.d.	k_{eff}	$\Sigma_{t,1}$	$\Sigma_{t,2}$	$\nu\Sigma_{f,1}$	$\nu\Sigma_{f,2}$	$\Sigma_{s,1,1}$	$\Sigma_{s,1,2}$	$\Sigma_{s,2,1}$	$\Sigma_{s,2,2}$
k_{eff}	1.08E-06	4.94E-08	2.01E-08	7.18E-07	9.54E-07	5.27E-08	2.14E-07	1.77E-07	2.92E-08
$\Sigma_{t,1}$	4.94E-08	4.67E-07	1.87E-07	7.35E-08	2.80E-08	4.71E-07	4.15E-07	3.02E-07	2.04E-07
$\Sigma_{t,2}$	2.01E-08	1.87E-07	1.17E-07	4.49E-08	2.29E-08	1.86E-07	2.02E-07	1.48E-07	1.28E-07
$\nu\Sigma_{f,1}$	7.18E-07	7.35E-08	4.49E-08	6.56E-07	7.53E-07	7.36E-08	1.64E-07	1.41E-07	5.19E-08
$\nu\Sigma_{f,2}$	9.54E-07	2.80E-08	2.29E-08	7.53E-07	1.18E-06	2.77E-08	8.08E-08	1.42E-07	3.05E-08
$\Sigma_{s,1,1}$	5.27E-08	4.71E-07	1.86E-07	7.36E-08	2.77E-08	4.76E-07	4.06E-07	3.13E-07	2.02E-07
$\Sigma_{s,1,2}$	2.14E-07	4.15E-07	2.02E-07	1.64E-07	8.08E-08	4.06E-07	7.05E-07	1.72E-07	2.22E-07
$\Sigma_{s,2,1}$	1.77E-07	3.02E-07	1.48E-07	1.41E-07	1.42E-07	3.13E-07	1.72E-07	5.48E-07	1.68E-07
$\Sigma_{s,2,2}$	2.92E-08	2.04E-07	1.28E-07	5.19E-08	3.05E-08	2.02E-07	2.22E-07	1.68E-07	1.40E-07
Δσ(R) |%	1.04E-01	6.84E-02	3.43E-02	8.10E-02	1.09E-01	6.90E-02	8.39E-02	7.41E-02	3.74E-02

It can be observed that the uncertainty results obtained by the statistical sampling method agree well with those by DNP method, which can prove that the development of the statistical sampling method in UNICORN is correct. And the difference exits between the uncertainties by DNP and the statistical sampling method may be caused by that the DNP method has the first-order approximation or that the statistical sampling method has the statistical error inevitably. It can be observed that the standard deviations of few-group cross sections due to the uncertainties of multigroup microcosmic cross sections can be over 1.0% for $\Sigma_{s,1,2}$ and almost 1.0% for the others important parameters. These uncertainties are un-ignorable and significant for neutronics calculations.

4. Conclusions

In this paper, a new sensitivity and uncertainty analysis code for neutronics calculations, named UNICORN, has been developed. For sensitivity analysis, the DNP method has been chosen, and the statistical sampling method has been applied for uncertainty analysis. By comparison with TSUNAMI-1D, the multigroup cross-section perturbation model can be proved correct. Based on the verified cross-section perturbation model, uncertainty analysis for pin-cell few-group cross sections has been performed by UNICORN. And the uncertainty results shows that the uncertainties of few-group cross sections are un-ignorable and significant, thus uncertainty analysis is essential and significant for neutronics calculations for much more confident and reliable results.

As the statistical sampling method for uncertainty analysis has the advantages that there exists no limit to the input parameters and responses, the further researches plan of this paper will be focused on propagating the nuclear-data uncertainties to the responses of burnup calculations and the core calculations in sequence.

Reference

[1] K. Ivanov, M. Avramova, et al. Benchmarks for Uncertainty Analysis in Modelling (UAM) for the Design, Operation and Safety Analysis of LWRs. OECD Nuclear Energy Agency, 2013.
[2] M. Pusa. Incorporating Sensitivity and Uncertainty Analysis to a Lattice Physics Code with Application to CASMO-4. Annals of Nuclear Energy, 2012, 40: 153-162.
[3] W. Wieselquist, A. Vasiliev, et al. Nuclear Data Uncertainty Propagation in a Lattice Physics Code Using Stochastic Sampling. PHYSOR 2012, Knoxville, Tennessee, USA.
[4] Artem Yankov, Markus Klein, et al. Comparison of XSUSA and “Two-step” Approaches for Full-core Uncertainty Quantification. PHYSOR 2012, Knoxville, Tennessee, USA.
[5] C. R. Weisbin, J. H. Marable, et al. Application of FORSS Sensitivity and Uncertainty Methodology to Fast Reactor Benchmark Analysis. Neutron Physics Division, 1976.
[6] B. T. Rearden. TSUNAMI-1D: Control Module for One-dimension Cross-section Sensitivity and Uncertainty Analysis for Criticality. Nuclear Science and Technology Division, 2009, Vol. I, Sect. C8.
[7] B. T. Rearden, M. A. Jessee. TSUNAMI Utility Modules. Nuclear Science and Technology Division, 2009, Vol. III, Sect. M18.
[8] B. L. Broadhead, B. T. Rearden, et al. Sensitivity- and Uncertainty-Based Criticality Safety Validation Techniques. Nuclear Science and Engineering, 2004, 146: 340-366.
[9] G. Marleau, A. Hébert, et al. A User Guide for DRAGON Version 4. Technical Report, 2014.
[10] R. E. Macfarlane, A. C. Kahler, et al. The NJOY Nuclear Data Processing System, Version 2012. Los Alamos National Security, 2012.
[11] F. Leszczynski, D. López Aldama, et al. WIMS-D Library Update: Final Report of a Coordinated Research Project. International Atomic Energy Agency, 2007.
[12] J. C. Helton, J. D. Johnson, et al. Survey of Sampling-based Methods for Uncertainty and Sensitivity Analysis. Reliability Engineering and System Safety, 2006, 91: 1175-1209.
[13] G. E. B. Archer, A. Saltelli, et al. Sensitivity Measures, Anova-like Techniques and the Use of Bootstrap. Journal of Statistical Computation and Simulation. 1997, Vol. 58: 99-120.
[14] Hongchun Wu, Weiyan Yang, Yulong Qin, et al. Benchmark Problems and Results for Verifying Resonance Calculation Methodologies. Physicor2012, Tennessee, USA, 2012.