

## The Development and Validation of a Natural Circulation Analysis Code for Marine Reactors

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### Abstract

A one-dimensional two-fluid model in a noninertial frame of reference is set up by implementing the dynamics of mass point in the noninertial frame into the two-fluid model in an inertial frame. The body force term in the momentum equation is treated as a time dependent function and the inertial force induced by ship three-dimensional motions is added into this term. Staggered mesh, finite volume method, semi-implicit first order upwind scheme and SOR method are used to discretize and solve two-phase mass, momentum and energy conservation equations. A code named PNCMC for studying natural circulation of marine reactor under motion condition is developed based on the difference equations. Single-phase natural circulation experiments under rolling condition performed in INET and two-phase natural circulation experiments under rolling condition are used to validate PNCMC. The validation of results indicates PNCMC is able to investigate the single-phase and two-phase natural circulation under rolling motion.

**Keywords:** Natural circulation, Marine reactor, Motion condition, Thermal hydraulics

### 1. Introduction

Innovative small (less than 300MWe) and medium (between 300MWe and 700MWe) sized reactors is highly valued in both developing and developed countries. The Intensified effort on SMRs designs and research generally aims to provide increased benefits in the area of safety and security, as well as power generation flexibility. Apart from designed as land-based nuclear power plants, small and medium sized LWRs can be mounted on a barge as FNPPs (floating nuclear power plants) or as propulsion power. For example, the SMART 330MW integral pressurized water reactor was designed for seawater desalination, small-scale power generation and ship propulsion. KLT-40s is an upgrade version of ice-breaker ship reactor KLT-40, which is designed for FNPP. So, operating in the ocean becomes an important development trend of small and medium sized LWRs.

Most notably, natural circulation is widely used in the design of small and medium sized LWRs, such as ABV-6M, VK-300, NuScale and IMR. Almost all the small and medium sized LWRs introduce passive residual heat removal systems, and the essence of these systems is also natural

circulation.

However, during operation in the ocean, the sea wave together with wind induce the barge rolling, pitching and heaving, and these motions affect the reactor thermal hydraulics, especially natural circulation behavior. Hence considering the nuclear safety, a ton of research activities aiming at investigating natural circulation under motion condition have been performed over the years. Japan Atomic Energy Research Institute (JAERI) conducted experiments using the first Japanese experimental ship Mutsu in various sea conditions to investigate the behavior of the whole marine reactor system from May 1991 to February 1992 [1]. They measured the parameters of wave height, wind force, ship motion (acceleration) and reactor thermal hydraulics, the results indicated that the ship motions hardly affect the operation parameters of primary loop (forced circulation loop), but prominently affect the steam generator water level, evaporator steam void fraction and recirculation flow rate. In order to study the effects of heaving motion on two-phase flow dynamics of marine reactor, Ishida et al. mounted a small scale rig on a suspended platform oscillated in vertical direction by hydraulic device [2]. Murata et al. performed a series of single-phase natural circulation tests in a model reactor with rolling motion to investigate effects of the rolling motion on its thermal-hydraulic behavior [3]. Tan et al. also mounted a flow loop on a rolling platform and carried out a series of experiments to study single-phase natural circulation flow, heat transfer and two phase flow instability [4, 5]. Yan et al. also mounted a PRHRS apparatus on rolling equipment to investigate operational characteristics of passive residual heat removal system under rolling motion [6]. The experimental results indicate that loop flow rate oscillates with ship motions and two-phase natural circulation instability may overlap flow oscillation induced by ship motions.

Except experiments, a lot of work has been focused on theoretical analysis and code modifying to study reactor thermal hydraulics under ocean condition. Yan et al. modified the RELAP5 code by adding a module calculating the effect of rolling motion and introducing new flow and heat transfer models to investigate the thermal hydraulic behavior of nuclear power systems [7]. JAERI developed RETRAN-02/GRAV code, which is the modified version of RETRAN-02 code, to simulate the experimental voyages of Mutsu and analyze the effects of ship motions on natural circulation of deep sea research reactor DRX [8]. For thermal-hydraulic analysis of ship reactor under multi-dimensional motions, RETRAN-03/MOV code developed from RETRAN-03 in Seoul National University [9].

The effects of ship motion focus on two principles. Firstly, inclination decrease the height difference between the source (core) and sink (SG), and thus decrease the thermal driving head of natural circulation; Secondly, inertial forces imparted by the ship motion in addition to the gravity act upon the primary coolant, which together with flow resistance control the natural circulation flow rate. The direction and magnitude of inertial forces imparted by rolling depend on the position of rolling axis, rolling acceleration and reactor system arrangement in three dimensional spaces. So the natural circulation behaviors under ocean conditions are much more complicated

One dimensional two-fluid model in noninertial frame of reference is proposed in this study.

Staggered mesh, finite volume method and semi-implicit first order upwind scheme are used to discretize and solve mass, momentum and energy equations. An analysis program for reactor natural circulation under motion conditions, named PNCCMC, is developed based on the difference equations and verified by experimental data.

## 2. Two-fluid model in noninertial frame of reference

When studying the thermal hydraulic of marine reactor that has an accelerated motion relative to the ground, if take the ground as the frame of reference, the flow of fluid is a combination of the reactor motion relative to the ground and fluid motion relative to system boundary (pipe wall). However, if we build a noninertial frame which has the same motion as reactor, the flow of fluid in this frame is just the fluid motion relative to the system boundary, which is the main concern in most engineering. Hence studying the thermal hydraulic in non-inertial reference system has its own advantages. The boundary condition is changed from the moving condition in inertial frame of reference to stationary boundary condition in noninertial frame of reference, so the mathematical description of flow is simplified.

Thermal dynamic variables and net viscous stress are independent of the frame of reference and the continuity and energy equations are unchanged in a noninertial frame of reference. The primary effect of a noninertial frame is the addition of extra body force term to the momentum equations, and the extra body force term named inertial force arise from the motion of the noninertial frame.

Combining dynamics of mass point in noninertial frame with two-fluid model (Ishii et al, 1984) in inertial frame, one-dimensional two-fluid momentum equations in noninertial frame of reference is proposed as follows,

Gas momentum equation:

$$\alpha_g \rho_g \frac{\partial V_g}{\partial t} + \alpha_g \rho_g V_g \frac{\partial V_g}{\partial z} = -\alpha_g \frac{\partial P}{\partial z} + \Gamma_g (V_i - V_g) - p_w \alpha_{gw} \tau_{gw} + M_g^d + \langle \alpha_g \rho_g (g_z + f_{ine,z}) \rangle \quad (1)$$

Liquid momentum equation:

$$\alpha_f \rho_f \frac{\partial V_f}{\partial t} + \alpha_f \rho_f V_f \frac{\partial V_f}{\partial z} = -\alpha_f \frac{\partial P}{\partial z} + \Gamma_f (V_i - V_f) - p_w \alpha_{fw} \tau_{fw} - M_f^d + \langle \alpha_f \rho_f (g_z + f_{ine,z}) \rangle \quad (2)$$

In PNCCMC, the constitutive relations are in accordance with that used in RELAP5/MOD3.3, and the detail description of those can be found in RELAP5/MOD3 code manual volume IV [10].

## 3. Inertial forces arise from ship motion

The inertial forces of a fluid particle are obtained in this general expression,

$$m \cdot \mathbf{f}_{ine} = m \left( \mathbf{a}_0 + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{V} \right) \quad (3)$$

The four terms in bracket of eq. (3) may each be significant. The first term  $\mathbf{a}_0$  accounts for the acceleration of noninertial frame relative to stationary frame. It provides the apparent force that pushes occupants back into their seats or makes them tighten their grip on a handrail when a vehicle accelerates. The second term  $\dot{\boldsymbol{\omega}} \times \mathbf{r}$  is the acceleration caused by angular acceleration of the noninertial frame, so it is of little importance for geophysical flows or for flows in machinery that rotate at a constant rate about a fixed axis. However, it does play a role when rotation speed or the direction of rotation varies with time. The third term  $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$  is the centripetal acceleration, depends strongly on the rotation rate and the distance of the fluid particle from the axis of rotation. The final term  $2\boldsymbol{\omega} \times \mathbf{V}$  is the Coriolis acceleration, depends on the fluid particle's velocity, not on its position. It has important consequences for navigation during air and sea travel.

As shown in Fig.1, noninertial frame OXYZ with coordinate unit vectors  $\mathbf{i} \ \mathbf{j} \ \mathbf{k}$  has a rolling motion around axis  $\mathbf{S} = (S_x, S_y, S_z)$  and acceleration  $\mathbf{a}_0 = (a_{0,x}, a_{0,y}, a_{0,z})$  relative to stationary frame. An important convention of OXYZ is that the direction of  $\mathbf{k}$  is straight up and contrary to gravity, which is beneficial to calculate  $g_z$ .  $\Sigma$  is a arbitrary cross section of channel and points in  $\Sigma$  belong to the domain  $\{x_0 \pm \Delta x, y_0 \pm \Delta y, z_0 \pm \Delta z\}$ . The unit vector  $\mathbf{n} = (\cos \theta, \cos \beta, \cos \gamma)$  represents for positive main stream of channel.

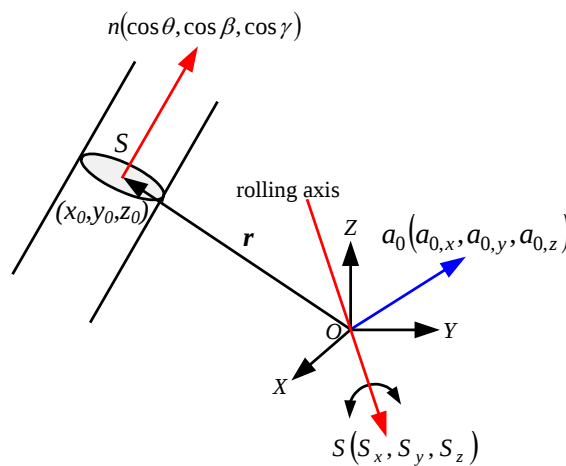


Fig.1. Geometry showing flow channel and noninertial frame

The rolling of ship is similar to sine wave and is expressed as

$$\varphi = \varphi_m \sin(\varpi_d t), \quad (4a)$$

$$\omega = \varphi_m \varpi_d \cos(\varpi_d t), \quad (4b)$$

$$\dot{\omega} = -\varphi_m \varpi_d^2 \sin(\varpi_d t), \quad (4c)$$

Where  $\varphi$  is rolling angle,  $\varphi_m$  is the rolling amplitude,  $\varpi_d$  is the rolling frequency.

According to eq. (4),  $f_{ine}$  is obtained as

$$\begin{aligned} f_{ine} = & \left[ a_{0,x} - \omega^2 (S_x S_y y - S_y^2 x - S_z^2 x + S_x S_z z) - \dot{\omega} S_y z + \dot{\omega} S_z y - 2\omega S_y V_z + 2\omega S_z V_y \right] \mathbf{i} \\ & + \left[ a_{0,y} - \omega^2 (S_y S_z z - S_z^2 y - S_x^2 y + S_x S_y x) - \dot{\omega} S_z x + \dot{\omega} S_x z - 2\omega S_z V_x + 2\omega S_x V_z \right] \mathbf{j} . \quad (5) \\ & + \left[ a_{0,z} - \omega^2 (S_x S_z x - S_x^2 z - S_y^2 z + S_y S_z y) - \dot{\omega} S_x y + \dot{\omega} S_y x - 2\omega S_x V_y + 2\omega S_y V_x \right] \mathbf{k} \end{aligned}$$

The flow field in a channel is actually three-dimensional, but the velocity of main stream is much greater than that of cross section, which is suitable for most flow channel. And the Coriolis acceleration is perpendicular to velocity, so the component of Coriolis acceleration along main stream is ignored in PNCMC. Then  $f_{ine,n}$  is simplified as

$$\begin{aligned} f_{ine,n} = & \left[ a_{0,x} - \omega^2 (S_x S_y y - S_y^2 x - S_z^2 x + S_x S_z z) - \dot{\omega} S_y z + \dot{\omega} S_z y \right] \cos \theta \\ & + \left[ a_{0,y} - \omega^2 (S_y S_z z - S_z^2 y - S_x^2 y + S_x S_y x) - \dot{\omega} S_z x + \dot{\omega} S_x z \right] \cos \beta . \quad (6) \\ & + \left[ a_{0,z} - \omega^2 (S_x S_z x - S_x^2 z - S_y^2 z + S_y S_z y) - \dot{\omega} S_x y + \dot{\omega} S_y x \right] \cos \gamma \end{aligned}$$

For reactor systems, especially for experimental facilities, we can assume that

$$\frac{\Delta x}{x_0} \ll 1, \quad \frac{\Delta y}{y_0} \ll 1, \quad \frac{\Delta z}{z_0} \ll 1. \quad (7)$$

So  $f_{ine,n}$  is approximately uniform in cross section  $\Sigma$ .

Then  $\langle f_{ine,n} \rangle$  is written as

$$\begin{aligned} \langle f_{ine,n} \rangle = & \left[ a_{0,x} - \omega^2 (S_x S_y y_0 - S_y^2 x_0 - S_z^2 x_0 + S_x S_z z_0) - \dot{\omega} S_y z_0 + \dot{\omega} S_z y_0 \right] \cos \theta \\ & + \left[ a_{0,y} - \omega^2 (S_y S_z z_0 - S_z^2 y_0 - S_x^2 y_0 + S_x S_y x_0) - \dot{\omega} S_z x_0 + \dot{\omega} S_x z_0 \right] \cos \beta . \quad (8) \\ & + \left[ a_{0,z} - \omega^2 (S_x S_z x_0 - S_x^2 z_0 - S_y^2 z_0 + S_y S_z y_0) - \dot{\omega} S_x y_0 + \dot{\omega} S_y x_0 \right] \cos \gamma \end{aligned}$$

$\langle f_{ine,n} \rangle$  is changed into  $\langle f_{ine,z} \rangle$  in eqs. (1) and (2).

During rolling,  $g_z$  is changed with the angle between channel and vertical direction. Rolling is in some sense a rotation with varying speed around a fixed axis, the new unit vector of main stream  $\mathbf{n}' = (\cos \theta', \cos \beta', \cos \gamma')$  at the rolling angle  $\varphi$  can be obtained through the steps as follows:

$$\begin{aligned} \hat{\mathbf{A}} = & \begin{bmatrix} S_x S_x & S_x S_y & S_x S_z \\ S_y S_x & S_y S_y & S_y S_z \\ S_z S_x & S_z S_y & S_z S_z \end{bmatrix}, \mathbf{A}^* = \begin{bmatrix} 0 & -S_z & S_y \\ S_z & 0 & -S_x \\ -S_y & S_x & 0 \end{bmatrix}, \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \mathbf{M} = & \hat{\mathbf{A}} + \cos \varphi \cdot (\mathbf{I} - \hat{\mathbf{A}}) + \sin \varphi \cdot \mathbf{A}^* \quad (15) \end{aligned}$$

$$\mathbf{n}' = \mathbf{n} \cdot \mathbf{M}^T \quad (16)$$

$$g_z = g \cdot \cos \gamma' \quad (17)$$

Where  $\mathbf{M}^T$  is the transposition of  $\mathbf{M}$ ,  $g$  is gravitational constant.

#### 4. Numerical solution

PNCMC uses a staggered-mesh in which the velocities  $V_g$ ,  $V_f$  are defined at the mass and energy cell interfaces and the pressure  $p$ , gas volume fraction  $\alpha_g$ , temperature  $T$ , internal energy  $U$ , and density  $\rho$  are defined at the mass, energy cell centers. The difference equations for mass and energy cell are obtained by integrating the mass and energy equations with respect to the spatial

variable  $z$ , from the junction at  $z_j$  to  $z_{j+1}$ . The momentum equations are integrated with respect to the spatial variable from cell center  $z_k$  to adjoining cell center  $z_L$ .

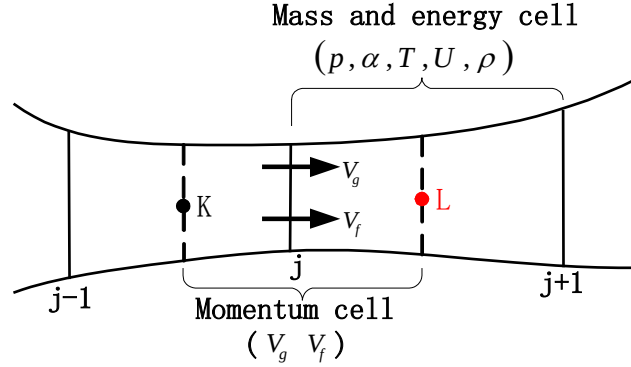


Fig.2. Schematic of staggered-mesh

Completion of the problem definition requires a choice of two independent variables from the four thermodynamic variables: pressure, temperature, density, and specific internal energy. Density is not a good choice because a small error in a solution for density can translate to a significant error in pressure. If pressure is chosen as an independent variable, a small error in the solution for pressure results in an even smaller fractional error in density and temperature. Constitutive relations for interphase and wall-to-fluid heat transfer depend on differences of temperatures that must be evaluated implicitly with respect to these temperature differences in the numerical equations. So  $p$ ,  $T_f$ ,  $T_g$ ,  $\alpha_g$ ,  $V_f$ ,  $V_g$  are chosen as independent variables in PNCMC. The method of numerical solution is referred to RELAP5/MOD3.3.

## 5. Code validation

Code validation should cover a broad range of conditions and phenomena. Because only seldom natural circulation experiments under motion conditions were performed in previous studies, there is a little experimental data available in public for validation. In this study, two-phase natural circulation experiments under rolling condition performed by Tan et al. are used for the validation of PNCMC [4, 5].

Tan et al. experimentally studied the two-phase flow instability of natural circulation under rolling motion condition. The test facility is shown in Fig.3 and composed of a simple natural circulation loop and a rolling plate, the detailed operating and geometry parameters can be found in article [5]. According to the configuration of the test facility, the natural circulation loop was modeled with 56 volumes shown in Fig.4. Because the detailed local pressure drop coefficients were not available, this validation focused on the repeat of phenomenon, not on numerical accuracy.

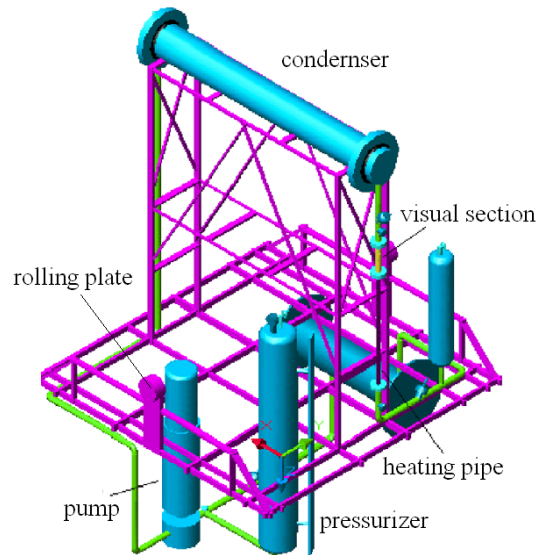


Fig. 3. Schematic of test facility (Fig2 in Tan et al., 2009b)

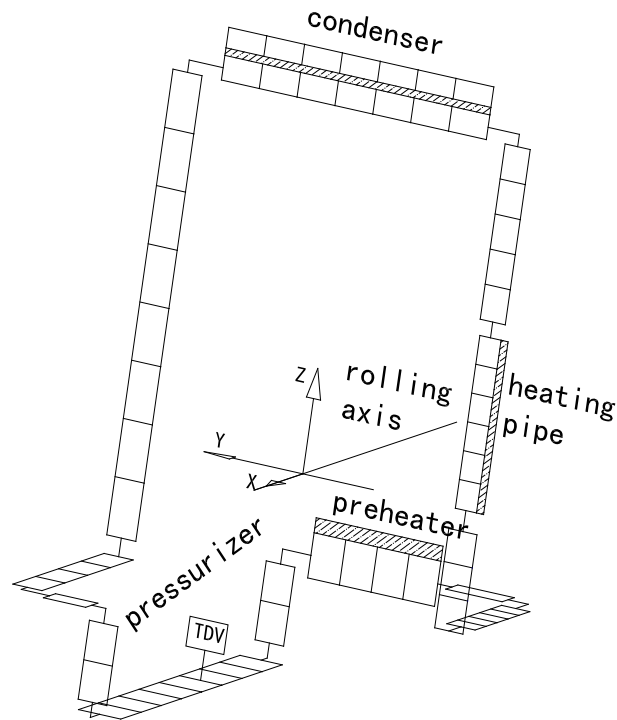
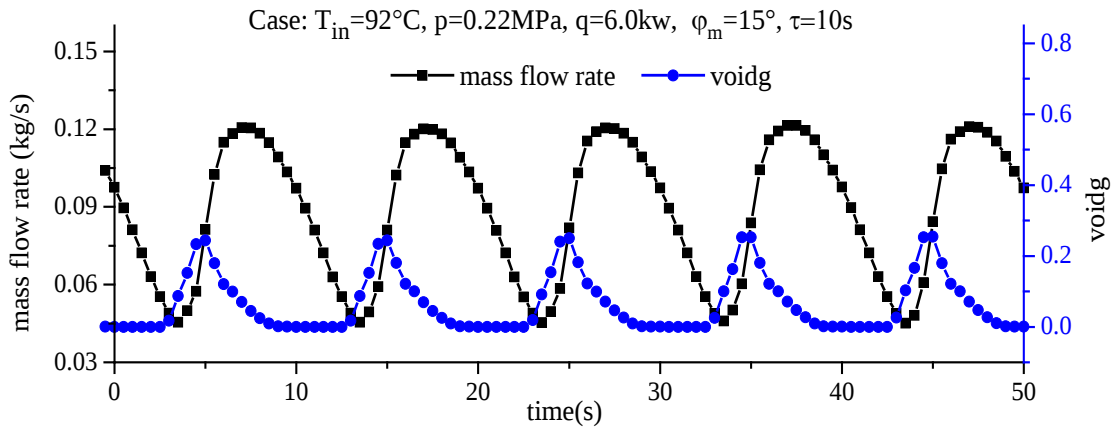
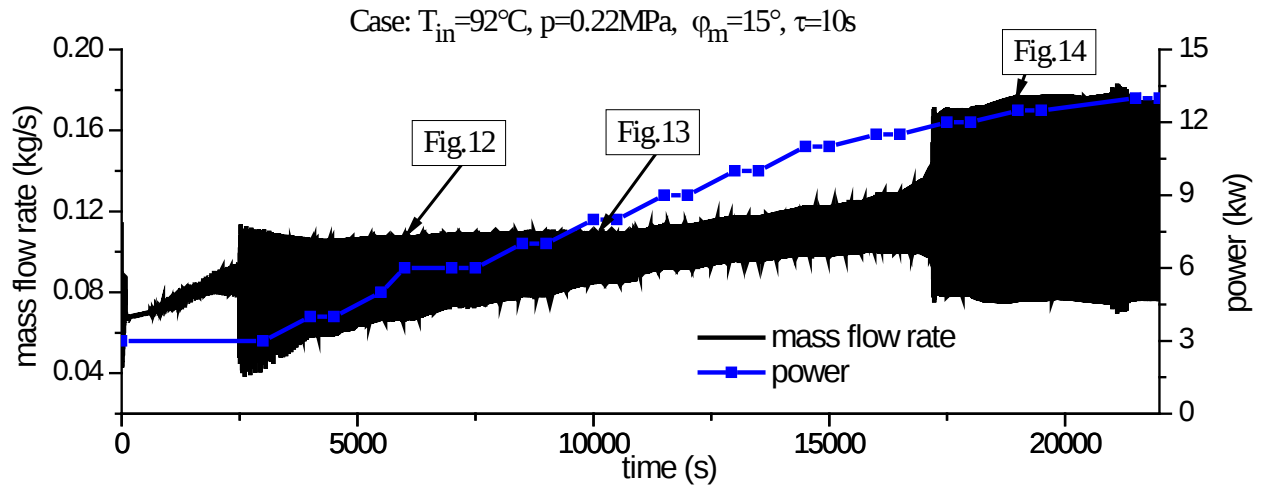
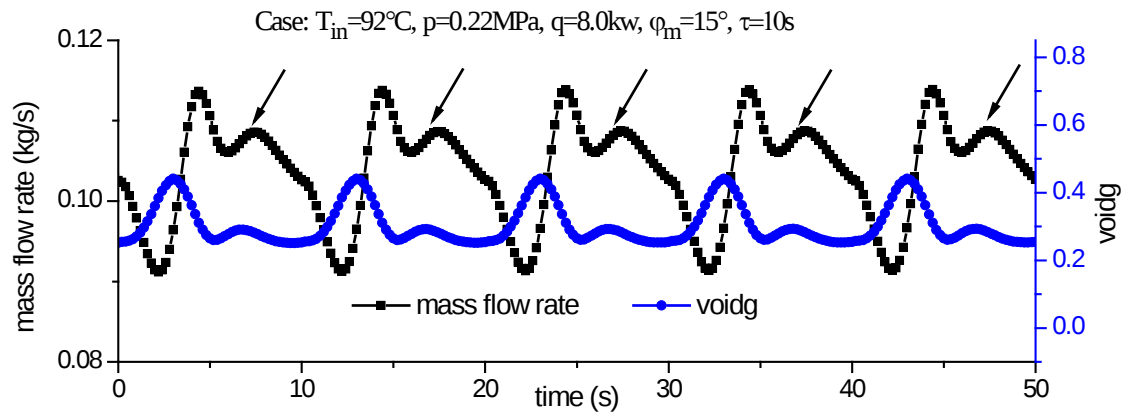


Fig. 4. The Nodalization of Tan's test facility for PNCMC

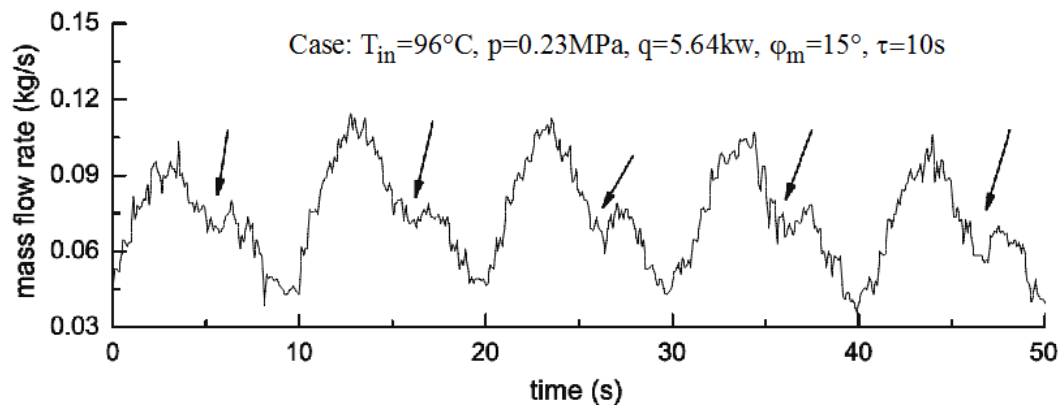
Fig.5 shows the mass flow rate variation under rolling motion with increasing the heating power from 3kW to 13kW. The rolling motion began at 2500s, and then the mass flow rate oscillated periodically. As the heating power increased, the oscillation amplitude decreased slowly and then enlarged suddenly when the power reached 12.0kw. Fig.6 indicate that vapor bubbles generated around the trough point of the flow oscillation, and single-phase flow and two-phase flow alternatively appeared in heating pipe, which was also observed in Tan's experiments. This oscillation was similar to density wave oscillation (DWO), but it was not self-excited, the flow would become a steady single phase flow if stop the rolling motion. Increased heating power

continuously, the variation of mass flow rate was no longer approximate sine wave. In one rolling period, the mass flow rate increased faster than decreased, and a short-lived increase occurred in the process of decrease marked by arrow in Fig.7. This irregular complex flow oscillation was a transition process, and the oscillation kept evolving until an overlapping of flow oscillation caused by rolling motion and DWO occurred in Fig.8. The prediction of PNCCMC was consistent with experiment result. In one rolling period, there were three short periodic oscillations from 3.1s to 3.6s, which was because the period of DWO was about 3s. As shown in Fig.5, the overlapping effect was obvious very much and enhanced the oscillation, the system became more unstable. To sum up, PNCCMC has the ability to simulate two phase flow under rolling motion.



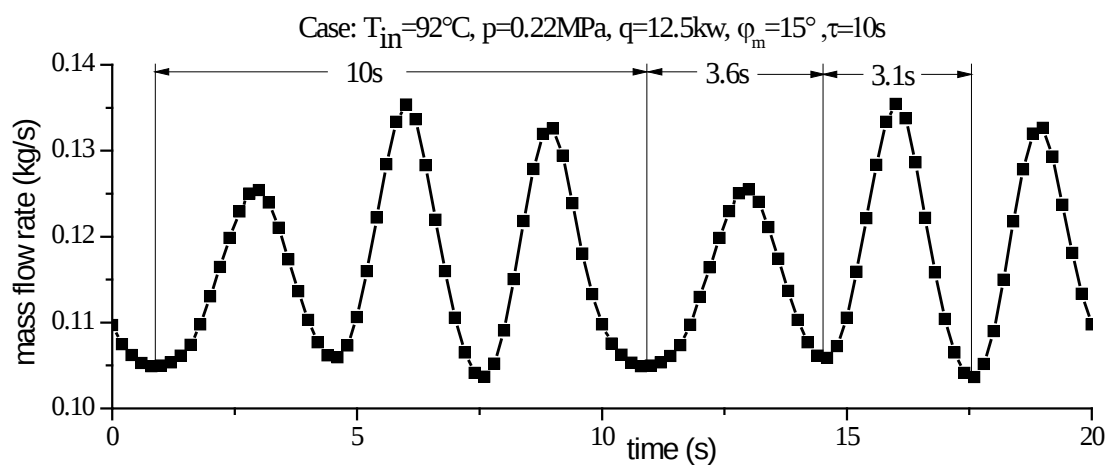


(a) Prediction of PNCMC

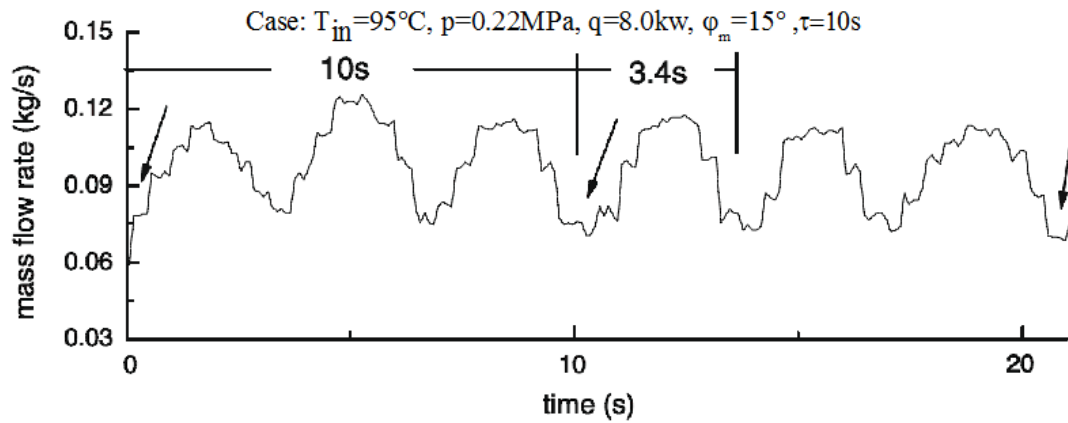


(b) Results of Tan's experiment (Fig.10 in article Tan et al. 2009b)

Fig. 7. Irregular complex flow oscillation



(a) Prediction of PNCMC



(b) Results of Tan's experiment (Fig.12 in article Tan et al. 2009b)

Fig. 8. Overlapping of flow oscillation caused by rolling motion and density wave oscillation

## 6. Conclusions

In the present work, one-dimensional two-fluid model in noninertial frame of reference is proposed. The body force term in the momentum equation is modified as time dependent function and inertial force induced by ship three-dimensional motion, such as rolling, heaving and inclination are added into this term. The constitutive relations include models for defining flow regimes and flow-regime-related models for total interfacial shear force, wall friction, wall heat transfer, interphase heat transfer are in accordance with that used in RELAP5/MOD3.3. Staggered mesh, finite volume method, semi-implicit first order upwind scheme and SOR method are used to discretize and solve two-phase Mass, momentum and energy equations. PNCMC code is developed based on these difference equations and compiled using C++ language.

Single-phase natural circulation experiments under rolling conditions performed in INET and two-phase natural circulation experiments under rolling condition performed by Tan et al. are used for the validation of PNCMC. The calculation results are in good agreement with single phase experiments. And PNCMC can figure out the phenomenon that was found in two-phase experiments. On all accounts, PNCMC is capable to investigate the natural circulation characteristic of marine reactor under motion condition.

## 7. References

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## Appendix A: Nomenclature

|                    |                                                                                   |
|--------------------|-----------------------------------------------------------------------------------|
| $A$                | area of cross section ( $\text{m}^2$ )                                            |
| $a_0$              | acceleration of noninertial frame relative to stationary frame ( $\text{m/s}^2$ ) |
| $FI$               | interphase drag coefficients (liquid, vapor) ( $\text{s}^{-1}$ )                  |
| $\mathbf{f}_{ine}$ | extra acceleration ( $\text{m/s}^2$ )                                             |
| $g$                | gravity acceleration ( $-9.8 \text{ m/s}^2$ )                                     |
| $H_{Loss}$         | pressure drop (Pa)                                                                |
| $h$                | enthalpy (J/kg)                                                                   |
| $M_g^d$            | total interfacial force (N)                                                       |
| $m$                | mass (kg)                                                                         |
| $P$                | pressure (Pa)                                                                     |
| $p_w$              | channel perimeter (m)                                                             |
| $q$                | heating power (w)                                                                 |
| $q'''$             | power source ( $\text{w/m}^3$ )                                                   |
| $\mathbf{r}$       | radial vector of fluid particle in noninertial frame (m)                          |
| $T$                | temperature (K)                                                                   |
| $t$                | time (s)                                                                          |
| $V$                | velocity(m/s)                                                                     |

|                |                                                                |
|----------------|----------------------------------------------------------------|
| $x, y, z$      | spatial coordinate (m)                                         |
| Greek letters  |                                                                |
| $\alpha$       | gas volume fraction                                            |
| $\rho$         | density, (kg/m <sup>3</sup> )                                  |
| $\Gamma_{wg}$  | volumetric mass exchange rate near wall (kg/m <sup>3</sup> •s) |
| $\varphi$      | rolling angle (rad)                                            |
| $\omega$       | angle velocity of noninertial frame (rad/s)                    |
| $\tau$         | shear stresses (N), rolling period (s)                         |
| $\dot{\omega}$ | angle acceleration of noninertial frame (rad/s <sup>2</sup> )  |
| $\varpi_d$     | rolling frequency of noninertial frame (Hz)                    |
| Subscripts     |                                                                |
| $f$            | liquid phase                                                   |
| $g$            | gas phase                                                      |
| $k$            | k= $f$ or k= $g$                                               |
| $w$            | pipe wall                                                      |
| $z$            | main stream                                                    |