ASSESSMENT OF NON-DIMENSIONAL PARAMETERS FOR STATIC INSTABILITY IN SUPERCRITICAL DOWN-FLOW CHANNELS

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Abstract

An analytical study of static instability of supercritical down-flow is presented herein. The study was conducted as part of an ongoing study to help identify relevant non-dimensional parameters governing supercritical flow instability in parallel channels. By and large, static instability was uncovered to be the dominant mode in down-flow, but oscillatory instability was found possible. Results are presented for H_2O and CO_2. It was found that often the static instability boundary occurred when the channel outlet temperature was close to the pseudo-critical temperature, sometimes even less than the pseudo-critical temperature. The study also indicates what parameters need to be altered, and in which direction, to improve system stability. Three different pressures for water were examined and non-dimensional parameters were assessed and discussed.

1. Introduction

Supercritical light water is proposed as the primary coolant in new Generation IV nuclear reactors where outlet temperature is more than 400 ^{o}C. A supercritical fluid is one where the fluid pressure and temperature are above the critical thermodynamic temperature and pressure. Higher thermal efficiency of power plant is one of the main advantages of using supercritical fluid. Flow stability of a system operating in the supercritical pressure and temperature range has become an active research topic amongst the researchers. Flow instabilities are undesirable in any reactor design as it can lead to fuel overheating, maybe even burnout. Some researchers believe that supercritical flow instability would be similar to two-phase flow instability.

Since supercritical flow instability is a relatively new topic, the literature is limited. Pioro and Duffey [1] performed an extensive literature review of supercritical flow, including hydraulic resistance and heat transfer of water and carbon dioxide. The first in-depth analytical study of the various supercritical flow instability modes was reported by Zuber [2]. He showed that supercritical flow, like two-phase flow, has flow oscillations and instabilities. The first report on static instability in supercritical flow was reported by Ambrosini and Sharabi [3], followed by other relevant studies of supercritical flow instability by Chatoorgoon [4], Shah [5] and Chatoorgoon [6], who reported that static instability would occur frequently in supercritical parallel channel down-flow. An experimental setup at the University of Wisconsin at Madison and Argonne National Laboratory (ANL) was described by Jain, et al. [7]. Ambrosini and Sharabi [8-10] studied vertical up-flow in heated channels at supercritical pressures and developed non-dimensional parameters for the instability. Similarities between two-phase flow and supercritical flow instability were also reported by Ambrosini [8]. Ortega Gomez, et al. [11] studied the thermal-hydraulic stability of uniformly heated channel at supercritical water pressure and proposed non-dimensional parameters of their own.

Static stability of supercritical flow in vertical down-flow single channel is studied herein. This study is conducted to evaluate the non-dimensional parameters that involved in the instability of down-flow single channel. Horizontal, up-flow analysis and also dynamic instability are not considered in this paper, these topics will be studied in the future studies.

2. Problem definition

The geometry consists of a single vertical pipe channel of length = 4.2672 m, ID = 8.36 mm, ε = 2.5×10^{-5}m which is the same as the geometry used by Ambrosini and Sharabi [3].

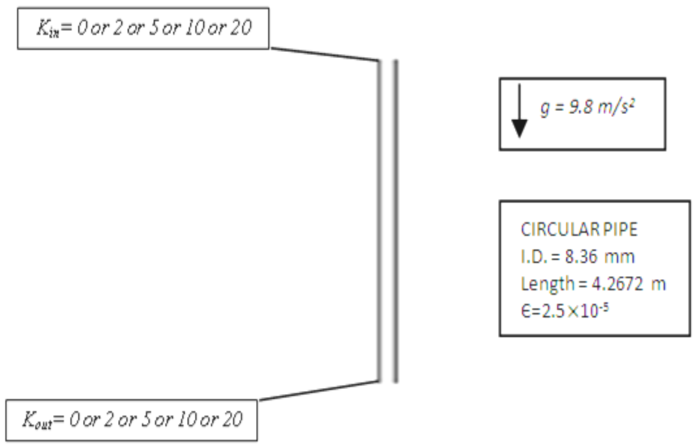

Fig. 1: Schematic of Geometry.

The fluids used were H_2O and CO_2. Different inlet and outlet K-factors were tested for a flow rate of 0.05 kg/s. For water, 25, 30 and 40MPa were used. One pressure of 8MPa pressure was used for CO_2.

3. Analysis

Static instability in two phase flow with a constant pressure drop was assessed by Rohatgi and Duffey [12]. They assumed at the instability boundary $\partial\Delta p/\partial\Delta G = 0$ and derived a quadratic form for the instability limit for homogenous equilibrium flow in parallel channels. In earlier work, Chatoorgoon [6] reported finding that the flow rate at the static instability boundary in supercritical horizontal flow was very close to the flow rate corresponding to

$$\frac{\partial\Delta p_{ch}}{\partial G} = 0 \tag{1}$$

Where $\partial\Delta p_{ch}$ was the channel frictional pressure drop. For horizontal flow, the steady momentum equation will show that Eq. (1) is mathematically equivalent to:

$$\frac{\partial\Delta(p+\rho u^2)}{\partial G} = 0 \tag{2}$$

While for horizontal flow, Eq. (1) and Eq. (2) are mathematically equivalent, for vertical flow Eq. (2) is preferred as it includes gravity effects. Hence, Eq. (2) is employed from here on as the condition for static instability in supercritical flow, and the flow rate that satisfies Eq. (2) would be a very good approximation of the flow rate at the static instability boundary.

Writing $\Delta p = p_e - p_i$, where p_e is the exit channel pressure and p_i is the inlet channel pressure (which is a constant).

$$\frac{\partial \Delta p}{\partial G} = \frac{\partial p_e}{\partial G} \tag{3}$$

Writing $\Delta \rho u^2 = G^2 \Delta \nu = G^2(\nu_e - \nu_i)$, where ν_e is the channel outlet specific volume and ν_i is the channel inlet specific volume.

$$G \equiv \rho u \tag{4}$$

Therefore,

$$\frac{\partial \Delta \rho u^2}{\partial G} \equiv 2G\left(\nu_e - \nu_i\right) + G^2 \frac{\partial \nu_e}{\partial G} \tag{5}$$

From the equation of state,

$$\frac{\partial \nu_e}{\partial G} = \frac{\partial \nu_e}{\partial h_e} \frac{\partial h_e}{\partial G}$$

And

$$\frac{\partial h_e}{\partial G} = -\frac{Q}{G^2 A}$$

Hence

$$\frac{\partial \nu_e}{\partial G} = -\frac{\partial \nu_e}{\partial h_e} \frac{Q}{G^2 A} = \frac{1}{\rho_e^2} \frac{\partial \rho_e}{\partial h_e} \frac{Q}{G^2 A} \tag{6}$$

Combining Eqs (2) – (6) gives:

$$\hat{Q} = 2\xi \tag{7}$$

Where

$$\hat{Q} = \frac{Q\theta_e}{GA} \tag{8}$$

$$\theta_e = -\frac{1}{\rho_e} \frac{\partial \rho_e}{\partial h_e} \tag{9}$$

And

$$\xi = \left(1 - \frac{\rho_e}{\rho_i}\right) + \frac{\rho_e}{2}\left(\frac{1}{G} \frac{\partial p_e}{\partial G}\right) \tag{10}$$

The ξ parameter is dimensionless, and it is a function only of the system steady-state frictional pressure drop and state parameters at the condition where Eq. (2) holds. It depends on the channel inlet and outlet densities, the mass flux and the rate of change of exit pressure with mass flux at the condition where Eq. (2) holds.

$\hat{Q}/\xi = 2$ is the theoretical result for static instability. ξ must be evaluated at the flow rate (for a given Power) where Eq. (2) holds – this is important!. For a given power, a steady-state flow-rate sweep is performed to determine the flow rate at the minimum Δp_{ch}.

On the other hand, $\hat{Q}$ is determined from the power at the instability boundary, as obtained from an experiment, or from an instability program (linear or nonlinear). The results for $\hat{Q}$ presented here were obtained using a linear stability program [13]. The steady-state parameters required by the linear code were produced by SPORTS front end.

4. Results

Results are presented for the thirty-three (33) cases shown in Table 1, which also gives the inlet and outlet temperatures, system pressure, K-factors, the powers Q_s and Q_p. Q_s is the instability power obtained from an instability analysis and Q_p is the power where $\Delta(p + \rho u^2)$ reaches the minimum for the specified flow-rate (in this case 0.05 kg/s). A linear instability computer code [13] was used to obtain the Q_s values presented in Table 1. The values of ξ and $\hat{Q}$, given in Table 1, are the non-dimensional parameters introduced by Chatoorgoon [14] for static instability in supercritical flow. The error % given in the last column of Table 1 is the difference between $\hat{Q}$ and the dashed line for the same value of ξ.

Eq. (7) is plotted in Fig. 2 as a dashed line. $\hat{Q}$ is a measure of the applied power, while ξ is related to channel pressure drop, shown in eq. (10). The 29 down-flow cases plus 2 horizontal flow cases, plus 2 up-flow numerical results are plotted versus theoretical line in Fig. 2. It is interesting that all the instability data fall near or on the theoretical line. The largest deviation from the theoretical line is 5.4%, while most of them are within 3%. This deviation from an engineering point of view is considered 'good'. Thus, the theoretical line presents a good engineering approximation of the static instability, which is obtained solely from a steady-state analysis of the geometry and does not require a formal stability analysis (using either a time-dependent solution code or a classical linear stability code). Its accuracy is ~95% or better.

Fig. 3 plots the channel outlet temperature at the static instability boundary for both H_2O and CO_2. It is noteworthy that for most of the cases, the channel outlet temperature is close to the pseudo-critical temperature. It deviates from it when the inlet K-factor is large and the outlet K-factor is small.

The effect of increasing inlet and outlet K-factors on the system stability is shown in Fig. 4 for water and in Fig. 5 for CO_2 for different inlet temperatures. As expected, when the channel outlet K-factor increases, for constant inlet temperature and inlet K-factor, the system becomes more unstable.

Figs. 6 and 7 show the effect of inlet temperature on the stability for H_2O and CO_2, respectively. It is shown that as the inlet temperature increases, while keeping the inlet and outlet K-factors constant, for low K-factors (e.g. K_{in}=2 and K_{out}=2), the system becomes more unstable. Surprisingly, for high K-factors (e.g. K_{in}=10 and K_{out}=20), increasing the inlet temperature causes the system to become more unstable until it reaches the most unstable inlet temperature. Increasing the inlet temperature further causes the system to become more stable. This behaviour was the same for H_2O and CO_2 as Figs. 5 and 6 show, but the reasons are unclear.

Fig. 8 shows the instability data plotted in Chatoorgoon's non-dimensional parameters [14] ratio for H_2O and CO_2. For an ideal set of parameters all the data would fall on the y-ordinate = 1.0. In this Fig the ratio of $(\hat{Q}/\xi)$ for H_2O and CO_2 for the same K-factors are plotted versus Case No. The maximum deviation is ~9%. Note: The cases that yielded an oscillatory instability rather than a static one were automatically filtered out as a value for ξ could not be obtained for the steady-state flow-rate sweep, since there was no condition that satisfied Eq. 1. Down-flow is the most unstable situation when compared to horizontal and up-flow orientation, and is more prone to static instability.

5. Conclusions

The non-dimensional parameters of Chatoorgoon for static instability in supercritical down-flow channels have been investigated for different K-factors, different inlet temperatures, different pressures and two fluids. Over 30 cases are presented and the following conclusions are derived:

- Vertical down-flow is the most unstable orientation compared to horizontal or vertical up-flow, determined from this and previous published work.
- Down-flow is much more prone to static instability, but oscillatory instability can occur.
- Increasing the inlet K-factors stabilizes the system while increasing outlet K-factors destabilizes the system. This is similar to two-phase flow.
- Increasing the inlet temperature did not always destabilize the system, nor did it always stabilize the system. The outcome seemed to depend on the K-factors. When the inlet and outlet K-factors are low, increasing the inlet temperature was found to destabilize the system. When the K-factors were high, increasing the inlet temperature was found to stabilize the system. It is not known if this would happen with two-phase flow.
- Chatoorgoon's non-dimensional parameters correlated well the numerically produced instability data for the same K-factors between the fluids, the maximum deviation was ~9% (Fig. 8). The method naturally filtered out those cases that were not a static instability, as it is impossible to define the ξ parameter when a static instability is nonexistent.
- In addition, a theoretical relationship was produced that yielded a maximum error less that 6% (Fig. 2). Furthermore, the method correlates well static instability in horizontal flow and vertical up-flow, as Fig. 2 shows. Hence, the method is independent of flow orientation.
- In most of the down-flow cases presented (Fig. 3), the channel outlet temperature at the instability boundary was very close to the pseudo-critical temperature. The exception seems to be when the inlet K-factor dominates the outlet K-factor.

Case#	*Fluid*	*Pres*	T_{in}	K_{in}	K_{out}	T_{out}	Q_s	$\widehat{Q}$	Q_p	ξ	*Error%*
1	H_2O	25	154.1	2	2	382.17	65	1.979	66.65	1.043	5.40
2	H_2O	25	154.1	2	20	381.99	65.5	2.024	66.4	1.058	4.54
3	H_2O	25	154.1	5	20	382.1	66.6	2.114	67.6	1.082	2.36
4	H_2O	25	154.1	10	20	383.17	68.9	2.274	69.7	1.139	0.17
5	H_2O	25	154.1	20	20	384.34	72.51	2.453	74.5	1.237	0.85
6	H_2O	30	154.1	10	20	401.38	74.33	2.172	76.1	1.111	2.30
7	H_2O	30	154.1	5	20	399.01	71.75	2.058	73	1.033	0.38
8	H_2O	30	154.1	2	20	397.51	70.5	1.995	71.2	0.984	1.35
9	H_2O	30	154.1	20	20	406.75	80	2.347	83	1.179	0.46
10	H_2O	30	195.62	10	20	408.38	72.91	2.12	75.9	1.09	2.83
11	H_2O	30	195.62	5	20	404.02	68.64	2.024	70.75	1.039	2.66
12	H_2O	30	195.62	2	20	402.11	67	1.97	68.2	0.991	0.60
13	H_2O	40	100	20	20	442.81	96	2.32	99	1.163	0.25
14	H_2O	40	100	10	20	430.5	89.65	2.147	91.3	1.069	0.41
15	H_2O	40	100	5	20	425.33	86.23	2.006	87.8	1.011	0.79
16	H_2O	40	100	2	20	422.35	84	1.923	85.8	0.987	2.65
17	CO_2	8	-43.8	2	2	32.87	9.78	1.333	9.82	0.671	0.67
18	CO_2	8	-43.8	20	2	41.37	14.18	2.244	15.1	1.126	0.35
19	CO_2	8	-43.8	20	5	38.43	13.44	2.2	14.35	1.12	1.81
20	CO_2	8	-43.8	20	10	36.85	13.08	2.167	13.75	1.11	2.44
21	CO_2	8	-43.8	20	20	35.86	12.71	2.129	13.2	1.1	3.33
22	CO_2	8	-43.8	2	20	34.22	10.79	1.752	10.9	0.868	0.91
23	CO_2	8	-43.8	5	20	34.42	11.05	1.834	11.2	0.918	0.10
24	CO_2	8	-43.8	10	20	34.73	11.52	1.949	11.77	0.983	0.87
25	CO_2	8	-30	10	20	36	11.49	1.917	11.95	0.989	3.18
26	CO_2	8	-30	5	20	35.07	10.79	1.829	11	0.936	2.35
27	CO_2	8	-30	2	20	34.78	10.3	1.745	10.5	0.885	1.43
28	CO_2	8	-30	2	2	33.94	9.09	1.408	9.17	0.723	2.69
29	CO_2	8	-30	0	0	32.43	8.3	1.098	8.27	0.541	1.45
30	H_2O-hor	25	50	10	20	382.92	88.5	2.824	90.2	1.447	2.47
31	H_2O-hor	25	50	20	20	384.05	91.05	3.026	93.5	1.553	2.64
32	H_2O-ver	25	50	10	20	385.27	95.01	3.216	98.5	1.612	0.24
33	H_2O-ver	25	50	20	20	387.32	98.31	3.258	104.3	1.63	0.06

Table l: H_2O and CO_2 results for static instability in down-flow.

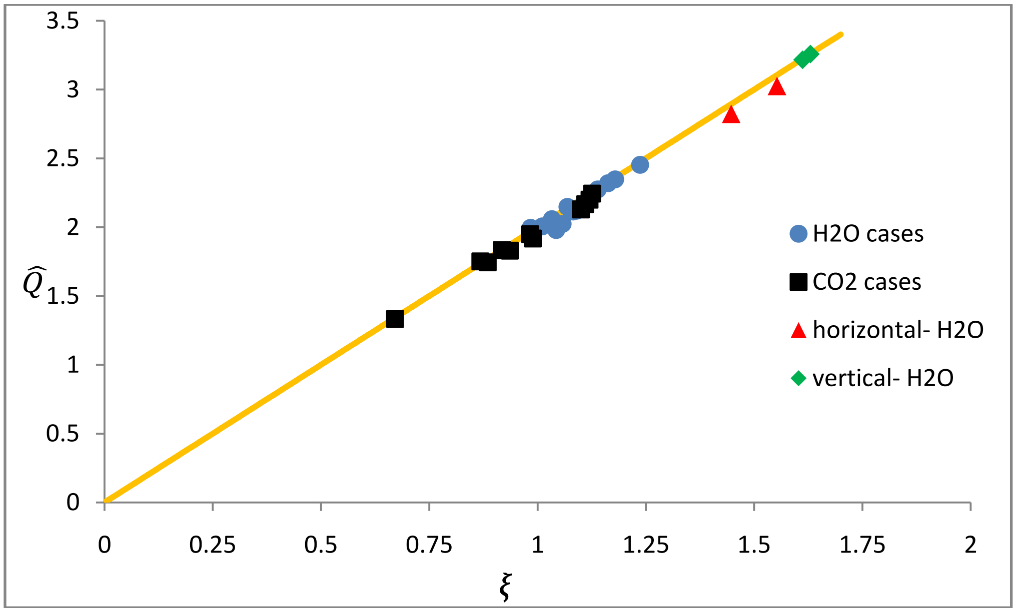

Fig. 2: Plotting the 33 cases in Chatoorgoon's [14] instability parameters.

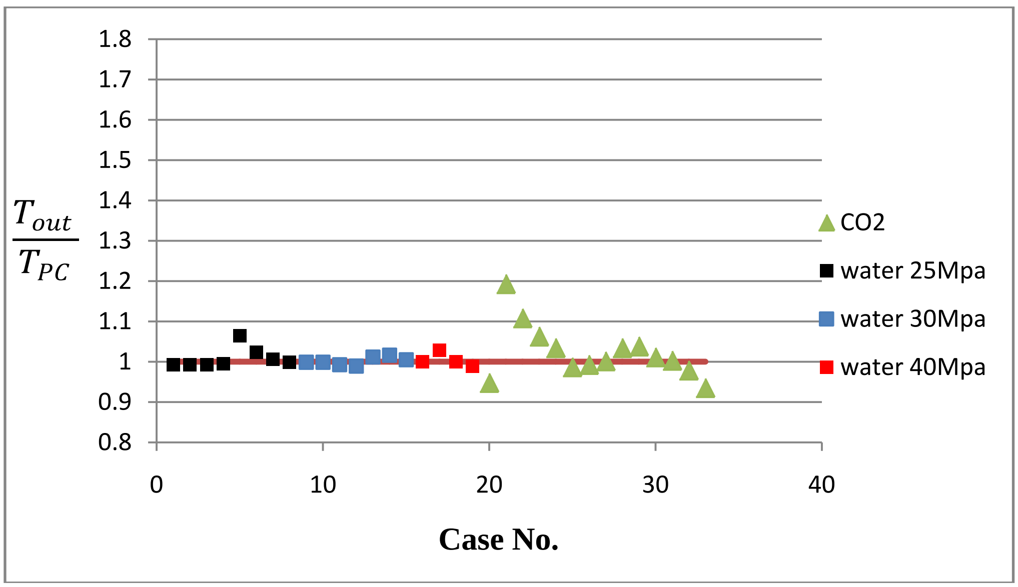

Fig. 3: Channel outlet temperature ratio for H_2O and CO_2 at instability boundary.

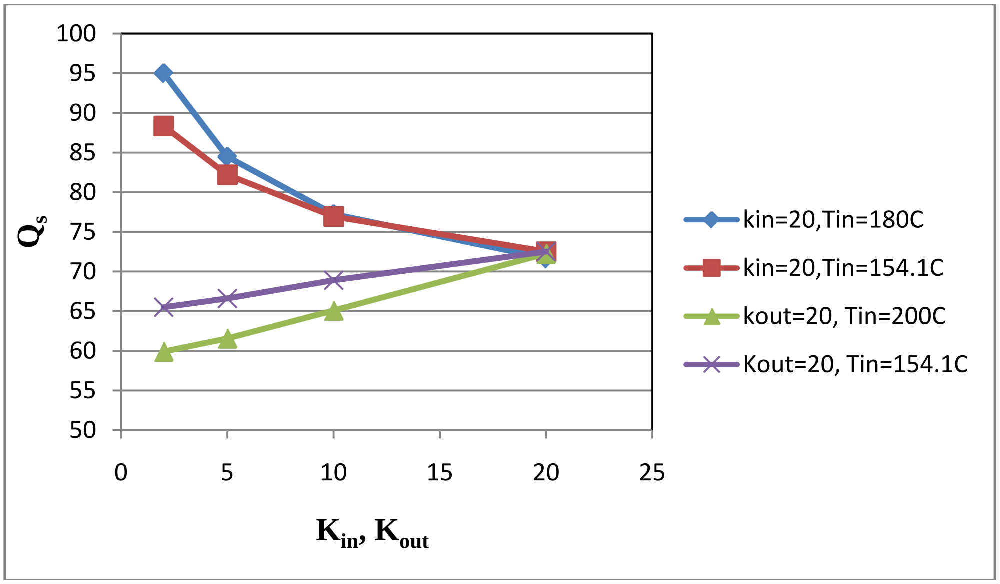

Fig. 4: Effect of inlet and outlet K-factors on static instability for H_2O.

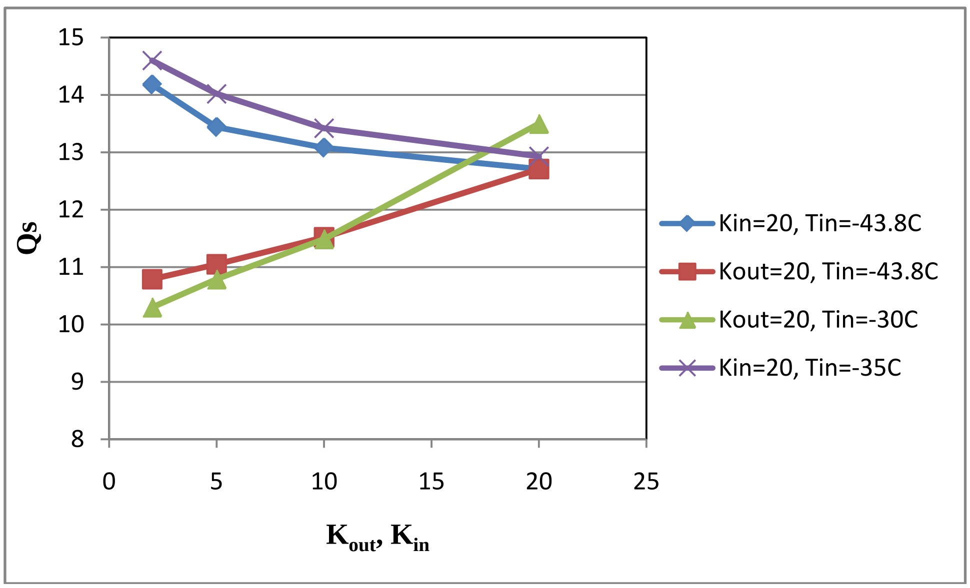

Fig. 5: Effect of inlet and outlet K-factors on static instability for CO_2.

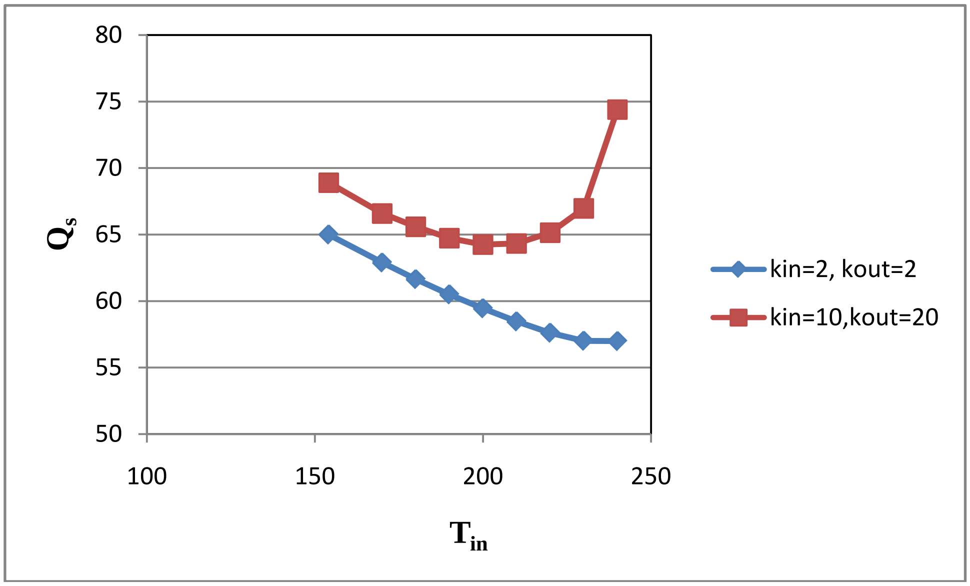

Fig. 6: Effect of inlet temperature on static instability for H_2O.

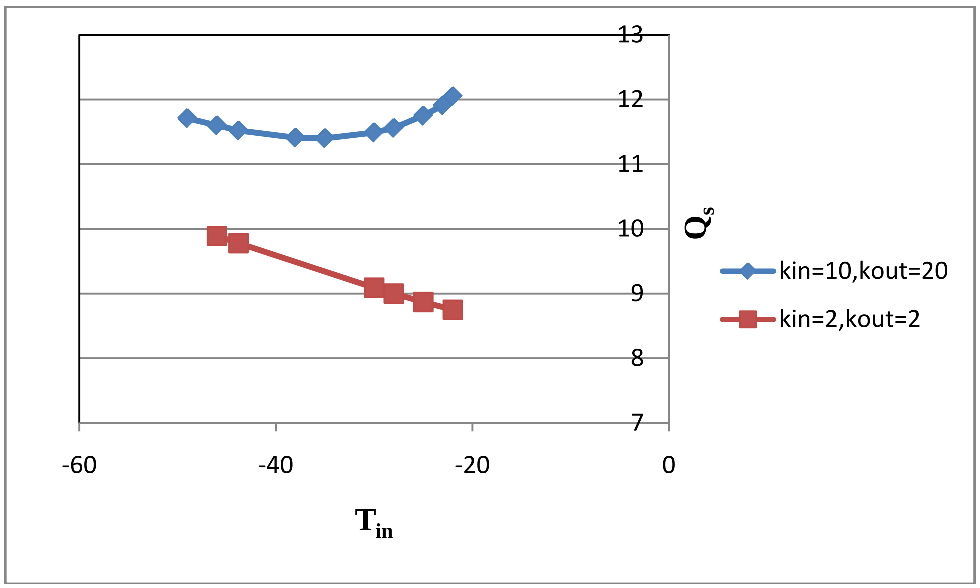

Fig. 7: Effect of inlet temperature on static stability for CO_2.

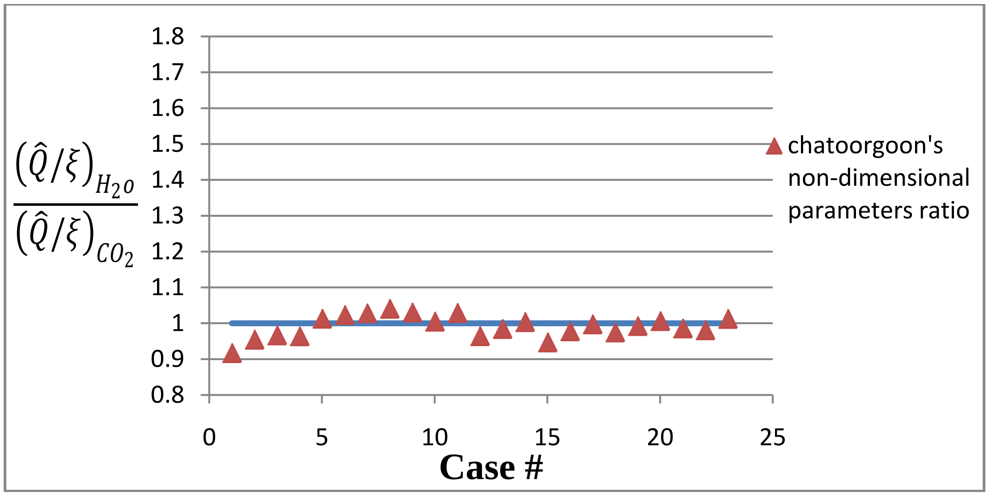

Fig. 8: Chatoorgoon's non-dimensional parameters ratio for H_2O and CO_2, same K factors, equal mass flow rate.

6. References

[1] Pioro, I.L., Duffey, R.B., 2003, "Literature Survey of Heat Transfer and Hydraulic Resistance of Water, Carbon Dioxide, Helium and Other fluids at Supercritical and Near-Critical Pressures", AECL-12127, FFC-FCT-409.

[2] Zuber, N., 1966. An Analysis of Thermally induced Flow Oscillations in the Near-Critical and Super-Critical Thermodynamic Region, Rept NASA-CR-80609, Research and Development Center, general Electric Company, Schenectady, New York, USA, May 25.

[3] Ambrosini, W., Sharabi, M., 2006. "Proceedings of Int. Conf. of Nuclear Engineering", ICONE 14-89862, July 17-20, Miami, Florida.

[4] Chatoorgoon, V., 2006. "Supercritical Flow Stability in Two Parallel Channels", Proceedings of Int. Conf. of Nuclear Engineering, ICONE 14-89692, July 17-20, Miami, Florida.

[5] Shah, M., 2007. "Linear Predictions of Static Instability in two parallel channels", M.Sc. thesis, Dept of Mechanical Engineering, University of Manitoba.

[6] Chatoorgoon, V, 2008. "Static Instability in Supercritical Parallel Channel Systems", ICONE16-48068.

[7] Jain, R., Litch, J., Anderson, M., Corradini, M., Lomperski, S., Chao, D. H., 2003, "Studies of Natural Circulation Heat Transfer and Flow Stability of a Supercritical Fluid", 10th International Topical Meeting on Nuclear Reactor Thermal Hydraulics (NURETH-10), Seoul, Korea, October 5-9

[8] Ambrosini, W., 2007. On the analogies in the dynamic behaviour of heated channels with boiling and supercritical fluids, Nucl. Eng. and Design, Vol. 237, No: 11, pp. 1164-1174, 2007.

[9] Ambrosini, W., Sharabi, M.B., 2008. Dimensionless parameters in stability analysis of heated channels with fluids at supercritical pressures, Nucl. Eng. & Design, 238, p 1917-1929.

[10] Ambrosini, W., Sharabi, M.B., 2007. Assessment of Stability Maps for Heated Channels with Supercritical Fluids versus the Prediction of a System Code, 3rd International Symposium on SCWR – Design and Technology, Shanghai, China, March

[11] Ortega Gomez, T., Class, A., Lahey, Jr., R.T., Schulenburg, T., 2006. "Stability Analysis of a Uniformly Heated Channel with Supercritical Water", Proceedings of Int. Conf. of Nuclear Engineering, ICONE 14-89733, July 17-20, Miami, Florida.

[12] Rohtagi, U. S., Duffey, R. B., 1998, "Stability, DNB, and CHF in Natural Circulation Two-Phase", Int. Comm. Heat Mass Transfer, Vol. 25, No. 2, pp. 161-174, 1998.

[13] Chatoorgoon, V., Upadhye, P., 2005, "Analytical Studies of Two-Phase Flow Stability in Natural-Convection Loops", 11th International Topical Meeting on Nuclear Reactor Thermal-Hydraulics (NURETH-11), Avignon, France, Oct 2-6, paper #165.

[14] Chatoorgoon, V. Yeylaghi, S., Leung, L., 2010, "Non-Dimensional Parameters for Oscillatory Instability in Supercritical Parallel Channel" ENC-2010, 30 May - 2 June 2010, Barcelona, Spain.

Nomenclature

A − flow area (m^2)

C_K − friction loss factor in (m^{-1})

g − gravitational constant

G − mass flux (kg/m^2/s).

h − enthalpy (kJ/kg).

K_{in} − channel inlet restriction loss coefficient.

K_{out} − channel outlet restriction loss coefficient.

$\dot{m}$ − mass flow rate (kg/s).

Q −applied power (kW).

Q_s − power at the instability boundary (kW).

Q_p − power where $\Delta(p + \rho u^2)$ vs. flow-rate is minimum (kW).

$\hat{Q}$ − normalized power, defined by Eq. (15).

ξ − normalized pressure drop, defined by Eq. (17).

ρ − flow density at channel (kg/m^3).

T_{in} − channel inlet temperature (^{o}C)

T_{out} − channel outlet temperature (^{o}C)

θ_e − value of θ (eq. (9)) at the channel exit (kg/kJ)

v − specific volume (m^3/kg)

Δp_{ch} − overall channel frictional pressure drop.