APPLICATION OF SIMILARITY PRINCIPLES FOR HEAT TRANSFER AND FLUID-DYNAMICS TO DIFFERENT FLUIDS AT SUPERCRITICAL PRESSURE

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Abstract

The paper reports on the results of a comparison between different fluids at supercritical pressures, basing on similarity principles recently proposed to establish a rationale for fluid-to-fluid comparison. The dimensionless groups proposed in past work for the analysis of flow stability in heated channels with fluids at supercritical pressure were recently extended to general heat transfer conditions, in order to scale also thermo-physical property groups having relevance for heat transfer. The capabilities of this new rationale are investigated by the use of a CFD code considering different fluids and drawing conclusions about its suitability.

1. Introduction

Dynamic similarity is a powerful tool often adopted in physics to extend the information obtained by experiments or by analytical means beyond the strict limitations of the particular boundary conditions for which it was derived. Thermal-hydraulics, in particular, makes a widespread use of dimensionless groups in engineering correlations, defining the ranges in which they are believed to be reasonably applicable to avoid undue extrapolations.

The two classical methodologies that are commonly adopted to establish the dimensionless groups relevant for each particular physical phenomenon make use of the π Buckingham theorem or of the technique of reverting governing equations into dimensionless form [1]. The latter technique is obviously preferable whenever suitable governing equations are available and are believed to include all the necessary phenomena. This route is taken in this work to scale heat transfer conditions at supercritical pressure in a fluid-to-fluid comparison perspective.

In past work, the dimensionless groups proposed by Ambrosini and Sharabi [2-3] for the analysis of flow stability in heated channels with fluids at supercritical pressure were shown to be effective in scaling the dynamic behaviour predicted by 1D models at imposed heat flux conditions. A key feature of these dimensionless groups is, in fact, their capability to establish a dimensionless equation of state between density and specific enthalpy that is nearly independent of fluid pressure and, at a lesser extent, of the specific considered fluid. Since the study of the dynamics of heated channels by 1D models requires only a single equation of state defining the relationship between density and specific enthalpy, a reasonable generality of stability predictions in dimensionless form followed. Another interesting feature of the proposed dimensionless groups is the key role played by the pseudo-critical point in the adopted definitions, that replaces saturation conditions in similar dimensionless numbers adopted for fluids at subcritical pressure.

Recently, an attempt was made to extend the above similarity principles in order to scale also thermo-physical property groups having relevance for heat transfer [4]. The obtained theory, that is going to be described in detail in a future paper [5], represents a more complete tool for establishing similarity between the behaviour of fluids at supercritical pressures, thus providing indications on both dynamic and heat transfer phenomena. The peculiar trends shown by thermo-physical properties at the

pseudo-critical temperature is one of the key features at the basis of the selected dimensionless definitions.

The capabilities of this new rationale are investigated in this paper with the aid of CFD models, considering different fluids and drawing preliminary conclusions about its suitability.

2. Basis for the similarity theory for one-dimensional flow

The Appendix of Ref. 3 reports a detailed account of the developments leading to the set of dimensionless numbers proposed in that paper for stability analysis of heated channels with fluids at supercritical pressure. In summary, the one-dimensional balance equations in terms of cross section averaged variables for a variable density fluid in channels with uniform cross sections are made dimensionless by suitable definitions.

The 1D balance equations are written in a classical form also applied in the case of boiling channel analyses:

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho w}{\partial z}=0 \tag{1}$$

$$\frac{\partial \rho w}{\partial t}+\frac{\partial \rho w^2}{\partial z}+\frac{\partial p}{\partial z}=-\rho g-\left[\frac{f}{D}+2K_{in}\delta_d(z)+2K_{out}\delta_d(z-L)\right]\frac{\rho w^2}{2} \tag{2}$$

$$\frac{\partial \rho h}{\partial t}+\frac{\partial \rho h w}{\partial z}=q_0''\frac{\Pi_h}{A}f_q(z) \tag{3}$$

By adopting the definitions

$$\rho^*=\frac{\rho}{\rho_{pc}} \qquad h^*=\frac{\beta_{pc}}{C_{p,pc}}\left(h-h_{pc}\right) \qquad w^*=\frac{w}{w_0} \qquad t^*=\frac{t\,w_0}{L} \tag{4}$$

$$z^*=\frac{z}{L} \qquad p^*=\frac{p}{\rho_{pc}\,w_0^2} \qquad G^*=\rho^*\,w^* \qquad Fr=\frac{w_0^2}{g\,L} \tag{5}$$

$$\Lambda=\frac{f\,L}{2\,D_h} \qquad N'_{TPC}=\frac{q_w''\,\Pi_h\,L}{\rho_{pc}\,w_0\,A}\frac{\beta_{pc}}{C_{p,pc}} \qquad N_{SPC}=\left(h_{pc}-h_{in}\right)\frac{\beta_{pc}}{C_{p,pc}} \tag{6}$$

$$N_{TPC}=\frac{\dot{Q}}{W}\frac{\beta_{pc}}{C_{p,pc}}=N'_{TPC}\,/\,\rho_{in}^* \tag{7}$$

the balance equations are rewritten in dimensionless form

$$\frac{\partial \rho^*}{\partial t^*}+\frac{\partial G^*}{\partial z^*}=0 \tag{8}$$

$$\frac{\partial G^*}{\partial t^*}+\frac{\partial}{\partial z^*}\left(\frac{G^{*2}}{\rho^*}\right)+\frac{\partial p^*}{\partial z^*}=-\frac{\rho^*}{Fr}-\left[\Lambda+K_{in}\delta^*(z^*)+K_{out}\delta^*(z^*-1)\right]\frac{G^{*2}}{\rho^*} \tag{9}$$

$$\frac{\partial \rho^* h^*}{\partial t^*}+\frac{\partial G^* h^*}{\partial z^*}=N'_{TPC}\,f_q^*(z^*) \tag{10}$$

The velocity scale in the above equations, w_0, is defined as the actual inlet velocity in considered working conditions, w_{in}.

This formalism is clearly similar to the one adopted in the past in works performed at the Rensselaer Polytechnic Institute (see, e.g., [6-7]) in relation to boiling channels and also adopted by one of the authors of this paper in developing a numerical methodology for transient and linear analysis of boiling channel instabilities (see, e.g., [8]). Actually, as in other proposals appeared in literature [9-10], dimensionless numbers for stability of heated channels with fluids at supercritical pressure can be differently derived with reference to previous experience in the case of boiling channels.

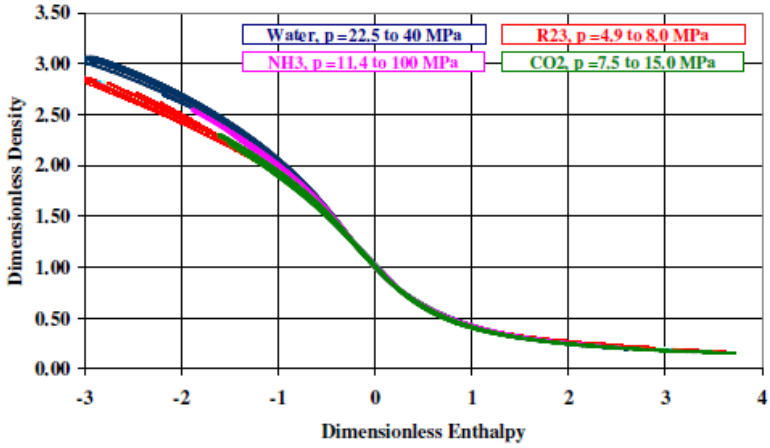
The proposed dimensionless numbers have some interesting properties, summarised as follows:

- a nearly unique trend of dimensionless density is observed as a function of dimensionless enthalpy for a given fluid at different pressures and for different fluids (Figure 1a); this is the basic feature allowing to establish the similarity: in fact, whenever a single dimensionless equation of state, $\rho^* \left(h^* \right)$, is considered acceptable, the dimensionless equations (8), (9) and (10) establish a rather universal similarity principle; for the time being, this equation of state is introduced as an approximate fit of the dimensionless trend observed for water at pressures in the range from 22.5 to 40 MPa, as obtained from the NIST property package [11];
- a second useful characteristic is that, in similarity with the case of boiling channels (see, e.g., [6-7]), each point in the N_{SPC}-N_{TPC} plane completely identifies the distribution of dimensionless enthalpy along the channel in steady-state conditions; in particular, loci at constant exit dimensionless enthalpy (the inlet value is defined by the opposite of N_{SPC}) can be identified in the plane as parallel straight lines, as well as loci at constant location along the channel of the pseudo-critical point are found to be straight lines through the origin (Figure 1b); as it will be shown, this feature represents an important point of strength of the set of dimensionless number being investigated.

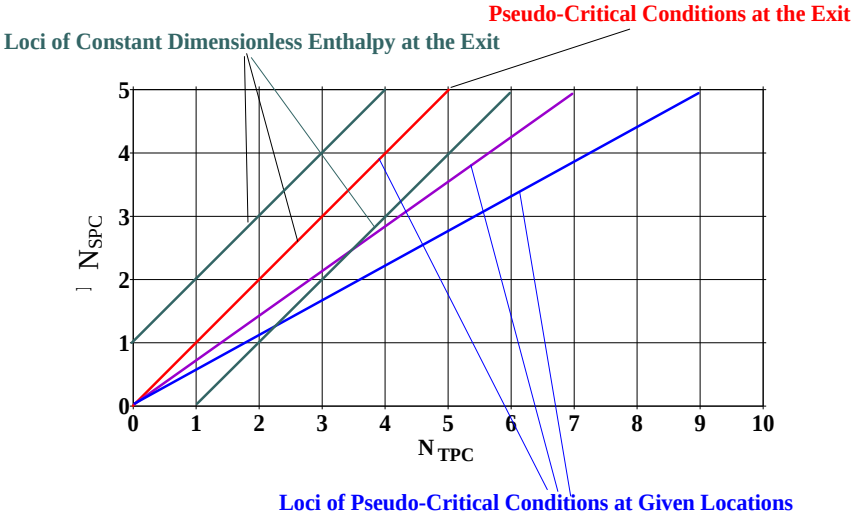
On this basis, it can be understood that the channel static and dynamic behaviour can be represented by the dimensionless equations (8), (9) and (10) with a relatively large degree of universality, no matter the value of supercritical pressure and the considered fluid, in the limits of what the adopted 1D approach can really model.

The usefulness of this theory was repeatedly shown in different past applications. In particular, in a paper presented at this Symposium [12], the results of a fluid-to-fluid comparison related to the stability of a simple heated channel clearly showed a rather large degree of independence on pressure and fluid of the stability thresholds identified for water, ammonia, carbon dioxide and refrigerant R23.

In this simple theory, anyway, an important feature was lacking, which represented the subject of repeated attempts aimed at its extension to boundary conditions other than imposed heat flux heating. In fact, whenever heat transfer is obtained by imposed wall (or secondary fluid) temperature, the thermo-physical properties of the fluid are necessarily called into play even in 1D equations. The problem faced in the trials to extend the theory to these conditions, motivated by the need to consider natural circulation experiments in which cooling is provided by a secondary flow, is that thermo-physical properties do not show any evident universal behaviour irrespective of pressure and fluid as shown by density. In addition, when considering heat transfer behaviour at supercritical pressure, it is often advisable to revert to CFD applications, which make use of a 3D approach, since the relevant phenomena can be hardly grasped by a 1D approach. These considerations led to search for a more general approach that was recently presented at an IAEA Meeting [4] and is being further elaborated with the aid of CFD models.



a) dimensionless equation of state



b) relevant loci in the N_{SPC}-N_{TPC} plane

Figure 1. Some useful characteristics of the adopted dimensionless definitions

3. Basis for the extension of the similarity theory to three-dimensional flows

3.1 Dimensionless numbers and local and instantaneous equations

In order to extend the similarity theory to include the effect of thermo-physical properties, the balance equations are rewritten in 3D local and instantaneous formulation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{w} \right) = 0 \tag{11}$$

$$\frac{\partial \rho \vec{w}}{\partial t} + \nabla \cdot \left(\rho \vec{w} \vec{w} \right) = \nabla \cdot \left\{ \mu \left[\nabla \vec{w} + \nabla \vec{w}^T - \frac{2}{3} \left(\nabla \cdot \vec{w} \right) \vec{\vec{I}} \right] \right\} - \nabla p + \rho \vec{g} \tag{12}$$

$$\frac{\partial \rho h}{\partial t} + \nabla \cdot \left(\rho h \vec{w} \right) = \nabla \cdot \left(k \nabla T \right) \tag{13}$$

Though the same assumption on the neglect of flow work is made in the energy equation as adopted in the 1D formulation, the present form is now suitable to include the local variation of fluid properties that is at the basis of relevant phenomena as heat transfer enhancement and deterioration [13-15].

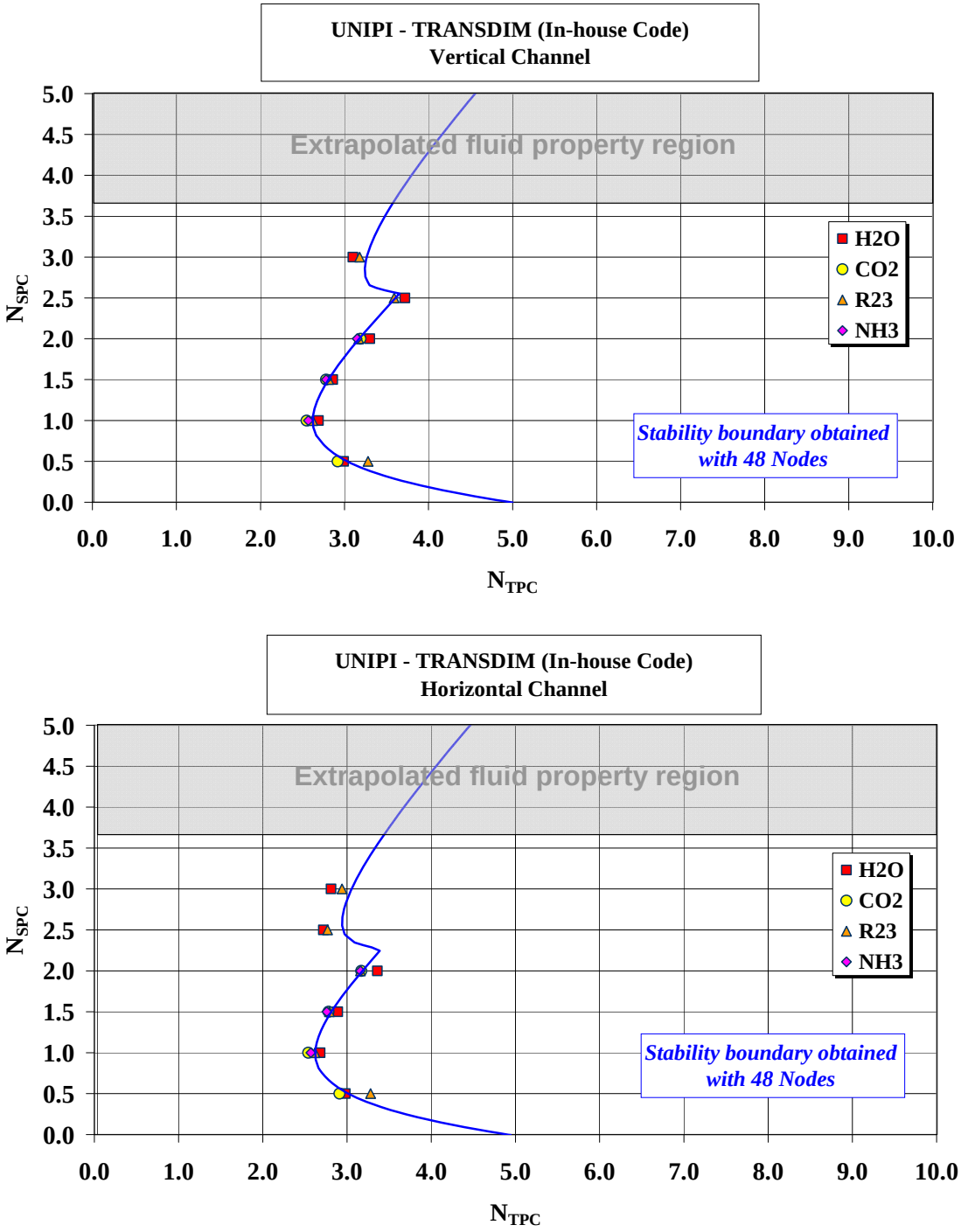


Figure 2. Reference data proposed for the second part of the benchmark exercise related to different fluids and vertical or horizontal channel [12]

Though the equations are written in 3D form, it is anyway borne in mind that channel flow conditions are here addressed, so that the boundary conditions involve imposed inlet enthalpy and wall heat flux, in addition to the obvious no-slip conditions at the wall and to an assigned velocity at the inlet:

$$-\left(k\frac{\partial T}{\partial r}\right)_{r=R} = q''_w \qquad h_{channel-inlet} = h_{in} \qquad w(r=R)=0 \quad w_{channel-inlet} = w_0 = w_{in} \tag{14}$$

Introducing the dimensionless definitions already adopted for the 1D balances, extending the dimensionless definition of the *z* spatial coordinate also to the radial coordinates

$$r^* = \frac{r}{L} \tag{15}$$

the dimensionless form of the above equations is found to be:

$$\frac{\partial \rho^*}{\partial t^*} + \nabla^* \cdot \left(\rho^* \vec{w}^* \right) = 0 \tag{16}$$

$$\frac{\partial \rho^* \vec{w}^*}{\partial t^*} + \nabla^* \cdot \left(\rho^* \vec{w}^* \vec{w}^* \right) = \nabla^* \cdot \left\{ \frac{1}{Re_{L,loc,pc}} \left[\nabla^* \vec{w}^* + \nabla^* \vec{w}^{*T} - \frac{2}{3} \left(\nabla^* \cdot \vec{w}^* \right) \vec{\vec{I}} \right] \right\} - \nabla^* p^* + \frac{\rho^*}{Fr} \vec{k} \tag{17}$$

$$\frac{\partial \rho^* h^*}{\partial t^*} + \nabla^* \cdot \left(\rho^* h^* \vec{w}^* \right) = \nabla^* \cdot \left(\frac{1}{Pe_{L,loc,pc}} \nabla^* h^* \right) \tag{18}$$

where the following definitions for the local Reynolds and Peclet number based on pseudo-critical density were adopted

$$Re_{L,loc,pc} = \frac{\rho_{pc} w_{in} L}{\mu} \qquad Pe_{L,loc,pc} = \frac{\rho_{pc} C_p w_{in} L}{k} \tag{19}$$

It is interesting to verify that the ratio of these two dimensionless numbers is expectedly the local Prandtl number

$$Pr_{loc} = \frac{Pe_{L,loc,pc}}{Re_{L,loc,pc}} = \frac{C_p \mu}{k} \tag{20}$$

The dimensionless boundary conditions have the form

$$-\frac{4}{Pe_{D,w}} \left(\frac{\partial h^*}{\partial r^*} \right)_{r^* = R^*} = N_{TPC} \qquad h^*_{channel-inlet} = -N_{SPC} \qquad w^* \left(r^* = R^* \right) = 0 \qquad w^*_{channel-inlet} = 1 \tag{21}$$

where

$$Pe_{D,w} = \frac{G C_{p,w} D}{k_w} = \frac{GD}{\mu_w} \frac{C_{p,w} \mu_w}{k_w} = Re_{D,w} Pr_w \qquad Re_{D,w} = \frac{GD}{\mu_w} \qquad Pr_w = \frac{C_{p,w} \mu_w}{k_w} \tag{22}$$

As an alternative to the above thermal boundary conditions, imposed wall temperature (i.e., wall fluid enthalpy at a constant operating pressure) can be assumed:

$$T \left(r = R \right) = T_w \qquad \Rightarrow \qquad h^* \left(r^* = R^* \right) = h^*_w \tag{23}$$

Nevertheless, the boundary condition of imposed heat flux will be the more interesting one in the present discussion.

In this regard, it is useful to rewrite the Peclet number at the wall in the form

$$Pe_{D,w} = \frac{\rho_{pc} w_{pc} C_{p,w} D}{k_w} \tag{24}$$

where the pseudo-critical density is made to appear. Furthermore, assuming steady-state conditions and uniform channel cross section, it is

$$G = \rho w = \rho_{in} w_{in} = \rho_{pc} w_{pc} \tag{25}$$

Then, considering the dimensionless equation of state $\rho^* = \rho^* \left(h^* \right)$, it is:

$$w_{pc} = \frac{\rho_{in}}{\rho_{pc}} w_{in} \quad \Rightarrow w_{pc} = \rho^*_{in} w_{in} \tag{26}$$

resulting in

$$Pe_{D,w} = \rho_{in}^{*} \frac{\rho_{pc} C_{p,w} w_{in} D}{k_w} \tag{27}$$

3.2 Considerations on the obtained dimensionless numbers

Let us now revise the above discussion considering some consequences of the resulting developments. The dimensionless numbers appearing in the equations, which establish the similarity principles, can be now summarised.

- N_{TPC} and N_{SPC}, defined as

$$N_{TPC} = \frac{q_w'' \Pi_h L}{G A} \frac{\beta_{pc}}{C_{p,pc}} = \frac{q_w'' 4L}{G D} \frac{\beta_{pc}}{C_{p,pc}} = \frac{\dot{Q}}{W} \frac{\beta_{pc}}{C_{p,pc}} \qquad N_{SPC} = \left(h_{pc} - h_{in}\right) \frac{\beta_{pc}}{C_{p,pc}} \tag{28}$$

  represent the dimensionless power-to-flow ratio and the inlet enthalpy conditions; the latter is measured as a function of its dimensionless distance to the pseudo-critical point. Since for uniform heating and steady flow it is

$$h^{*}\left(x^{*}\right) = -N_{SPC} + N_{TPC} x^{*} \tag{29}$$

  these numbers completely define the steady-state distribution of "bulk" dimensionless enthalpy along the channel and, through the dimensionless equation of state, of "bulk" density. Therefore, *specifying this couple of values uniquely defines the dimensionless bulk conditions along the channel axis in steady-state.*

- The local values of the Reynolds and the Peclet numbers can be rewritten as

$$Re_{L,loc,pc} = \left(\frac{\rho_{pc}}{\mu}\right)\left(w_{in} L\right) \qquad Pe_{L,loc,pc} = \left(\frac{\rho_{pc} C_p}{k}\right)\left(w_{in} L\right) \tag{30}$$

  highlighting the role the two diffusivities

$$\nu_{loc,pc} = \frac{\mu}{\rho_{pc}} \qquad \alpha_{loc,pc} = \frac{k}{\rho_{pc} C_p} \tag{31}$$

  In order to obtain similar conditions in a fluid-to-fluid perspective, it would be necessary to assure the equality of the above defined Reynolds and Peclet numbers; this, in turn, would imply the equality of their ratio, i.e. of the Prandtl number

$$Pr_{loc} = \frac{C_p \mu}{k} \tag{32}$$

  This requirement represents a major problem for establishing a real similarity since, as it will be shown later on, it is quite difficult (or impossible) to find fluids having the same value and *the same trend* of the Prandtl number over the complete range of addressed dimensionless enthalpy: an imperfect similarity necessarily follows. However, since the value of the Prandtl number at the pseudo-critical point strongly depends on pressure, it is not impossible to find pressures at which two fluids have at least the same maximum value of the Prandtl number. This choice was suggested by Rhode et al. [16] in choosing R23 at a pressure of 5.7 MPa to represent water and 25 MPa; in fact, both fluids at the two respective pressures show a maximum of the Prandtl number close to pseudo-critical conditions having a numerical value of about 8. The same choice is adopted in this work extending these considerations also to other fluids.

- An attempt to compensate for the possible differences between fluid diffusivities, establishing an approximate equality between the Peclet number, $Pe_{L,loc,pc}$, could be made by scaling the group

$w_{in}L$, having the dimensions of a diffusivity as well. However, in order to comply at the same time with the criteria of equality the Reynolds and the Peclet number, *it would require the same Prandtl number as a function of dimensionless enthalpy, which, as already mentioned, is not easily satisfied.*

- Note that a possible scaling of the group $w_{in}L$ could be made preserving the value of the Froude number, also depending on inlet velocity: $Fr = w_{in}^2 / gL$. This requirement suggests a possible strategy for scaling the velocity and the length, to be made clearer in future applications.
- In the case of imposed heat flux, preserving similarity between heat transfer phenomena would require to achieve a same value of the Peclet number at the wall:

$$Pe_{D,w} = \left(\rho_{in}^*\right)\left(\frac{\rho_{pc}C_{p,w}}{k_w}\right)\left(w_{in}D\right) \tag{33}$$

Since the equality in the dimensionless inlet density is an obvious requirement for similarity, in view of establishing the same dimensionless bulk fluid conditions at the inlet, and assuming to scale uniformly the geometry in the axial an radial direction (i.e., using the same scale for length and diameter), again the role of the trend of the diffusivity $\alpha_{loc,pc}$ as a function of dimensionless enthalpy (at the wall, in this case) appears of overwhelming importance.

In summary, the role of the trend of the diffusivities $\nu_{loc,pc}$ and $\alpha_{loc,pc}$ as a function of dimensionless enthalpy seems one of the most important aspects to be considered to understand how to establish a proper similarity in a fluid-to-fluid perspective for momentum and heat transfer phenomena. This aspect is given attention in the next section.

4. Trends of diffusivities as a function of dimensionless enthalpy

Following the above considerations and in agreement with the choice suggested in [16], three different operating pressures were selected for carbon dioxide, ammonia and refrigerant R23 in order to have the a similar maximum value of the Prandtl number at the pseudo-critical point as observed for water at 25 MPa (roughly equal to 8). Figure 3 reports the trends of the Prandtl number and of the diffusivities $\nu_{loc,pc}$ and $\alpha_{loc,pc}$ as a function of dimensionless enthalpy for the four fluids at the considered pressures. The following considerations can be made:

- though the maximum Prandtl number is nearly the same at the pseudo-critical point (note that this equality is only approximate for the selected pressures), there are evident deviations in the trends of this parameter in the liquid-like region (Figure 3a);
- in this respect, it can be noted that ammonia has a Prandtl number trend closer to the one of water than carbon-dioxide and R23; this is in agreement with the known greater similarity of ammonia to water, resulting from the values of the compressibility and the acentric factors [17], and is coherent also with the trends of density shown for the four fluids in Figure 1a;
- the trends of $\alpha_{loc,pc}$ and $\nu_{loc,pc}$ reported in Figure 1b and Figure 1c confirm the greater similarity of ammonia with water; a closer analysis of the data shows that, though obtaining a completely similar trend of the Reynolds and of the Peclet numbers is out of reach, scaling the groups $w_{in}D$ and $w_{in}L$ by factors equal to 2 or 2.5 for CO_2 and R23 would make a bit closer the trends of these two dimensionless numbers, possibly improving similarity.

5. Preliminary discussion of the theory with the aid of a CFD code

The above considerations suggest that some degree of similarity between operating conditions with different fluids could be reached by imposing in a given geometry the same boundary conditions in terms of N_{SPC}, N_{TPC} and Fr. Among the fluids considered herein, ammonia should provide results closer to those of water in terms of dimensionless fluid temperatures in bulk and at the wall.

Though a decisive confirmation of this rationale could be given only by experimental observation, it is hardly possible that experimental data in corresponding conditions exist in sufficient quantity to draw meaningful conclusions at present time. As a consequence, the only viable route to get a preliminary validation of the above considerations is the use of computational techniques that proved to show a reasonable representation of observed heat transfer phenomena. As shown in a paper presented at this Symposium [18], sufficient work has been performed in the past to support the conclusion that k-ε turbulence models have capabilities to predict the onset of deterioration in different operating conditions, though their response is not completely accurate, especially when the pseudo-critical temperature is exceeded in the bulk fluid or at the wall.

In this respect it is necessary to clarify that:

- though the equations reported in section 3.1 do not include any turbulent diffusivity, it can be assumed that turbulence effects are somehow implied in the treatment, considering that the same equations could be used in a direct numerical simulation analysis of heat and momentum transfer phenomena, that is probably impossible to perform with the presently available hardware;
- possibly, the weakest point in this attempt of proving the above rationale is that *we will implicitly assume that the closure laws included in the adopted RANS k-ε approach are coherent with the presented treatment*; this is obviously not sure, depending on details of the model and of its empirical constants that are not easy to discern at present time.

However, with the due caution dictated from the above warnings, results obtained by the STAR-CCM+ code [19] with the built-in standard low-Re k-ε model [20] are presented hereafter for a real experiment performed by Watts [21] with water at 25 MPa and for corresponding "similar" conditions with carbon-dioxide, ammonia and R23. The operating pressures are those at which the maximum of the Prandtl number is nearly the same for the four fluids (see Figure 3). In the calculations, the same geometry and the same values of N_{TPC}, N_{SPC} and Fr were imposed, evaluating heat fluxes, flow rates and inlet velocities in compliance with them.

Figure 4 presents the results obtained in the calculations by the CFD code for wall temperature in comparison with experimental data. The inlet temperature is 200 °C in one case, labelled "higher N_{SPC}", and 250 °C, in the case labelled "lower NSPC". As it can be noted, the model is able to predict the experimental wall temperature trend with an accuracy that can be considered quite exceptional for these applications. In both cases, heat transfer deterioration occurs, though at different extents, owing to the effects of laminarisation.

Figure 5 and Figure 6 present the results obtained by the calculations for the different fluids in dimensionless form. In particular, the bulk and wall dimensionless wall temperatures are presented, together with the distribution of the Prandtl number at the wall. The case at higher N_{SPC} was calculated only for ammonia and R23, in addition to water, because CO_2 does not allow so large values of the pseudo-subcooling number (namely 2.17) without freezing. The case with lower N_{SPC} (namely 1.79) was instead affordable also with CO_2.

The analysis of the figures shows that:

- deteriorated heat transfer is always predicted, in similarity to what happens for water; however, for fluids other than water, a more severe deterioration is predicted;

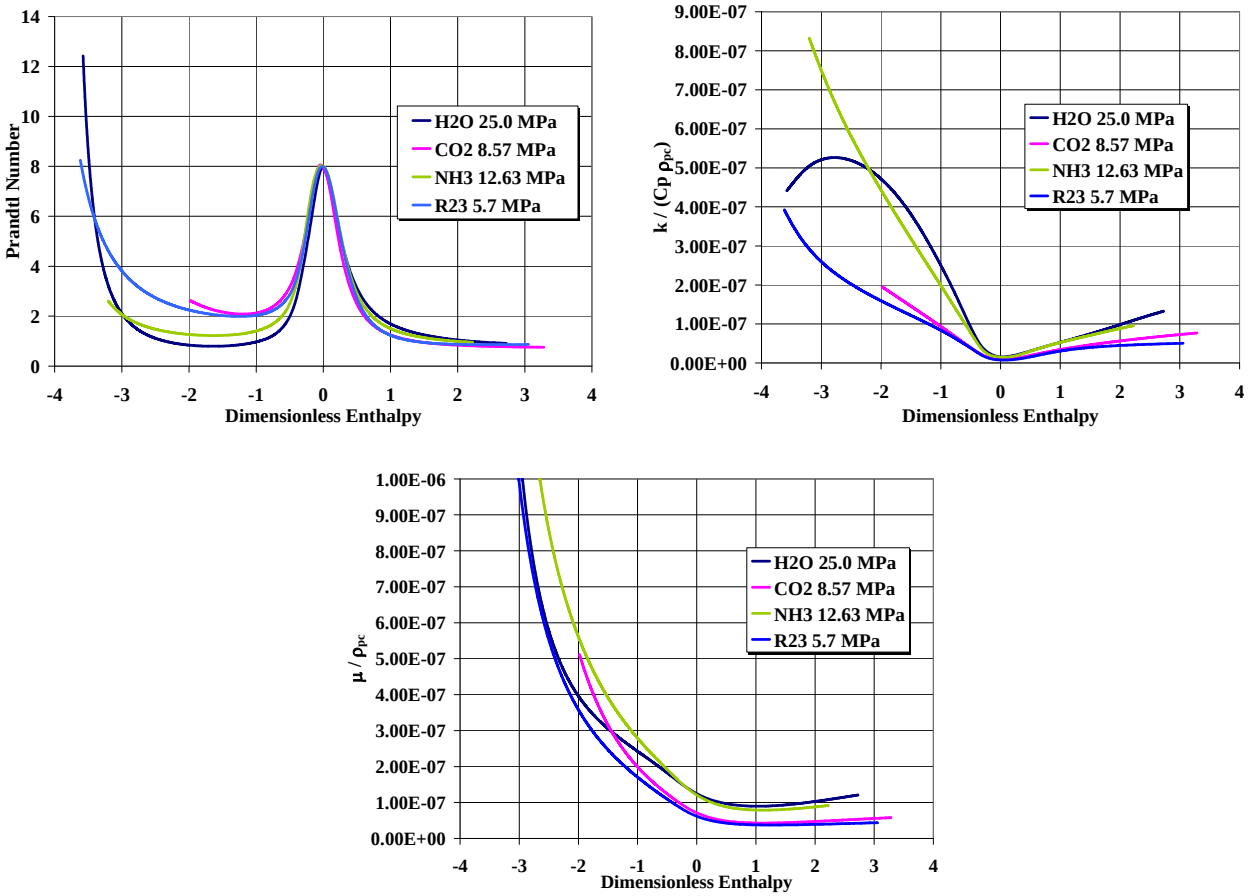
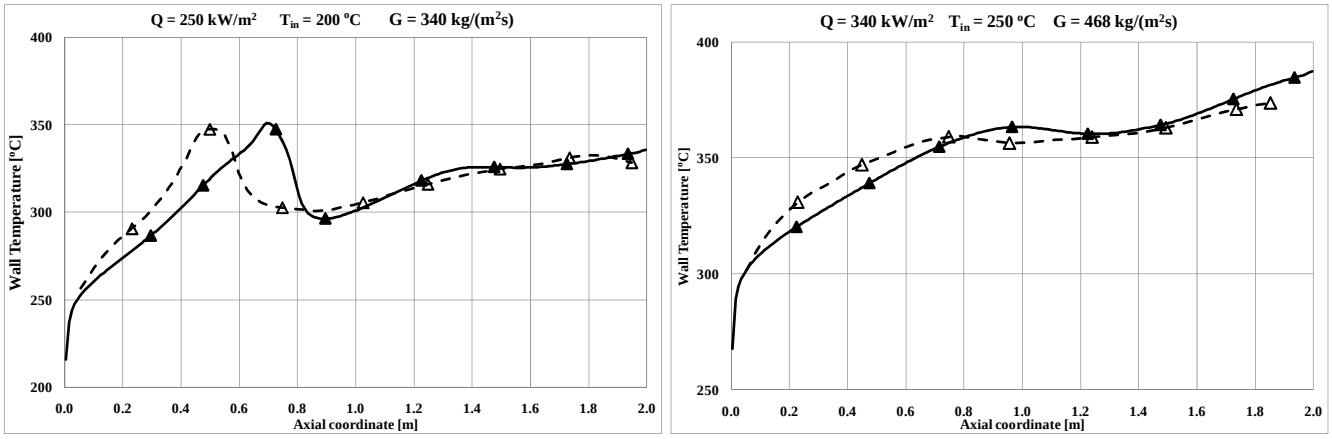


Figure 3. Prandtl number and reference diffusivities as a function dimensionless enthalpy for water, carbon dioxide, ammonia and R23 at different pressures



a) higher N_{SPC} b) lower N_{SPC}

Figure 4. Calculated (continuous line) and predicted wall temperatures (dashed line) for the cases selected to be predicted with different fluids

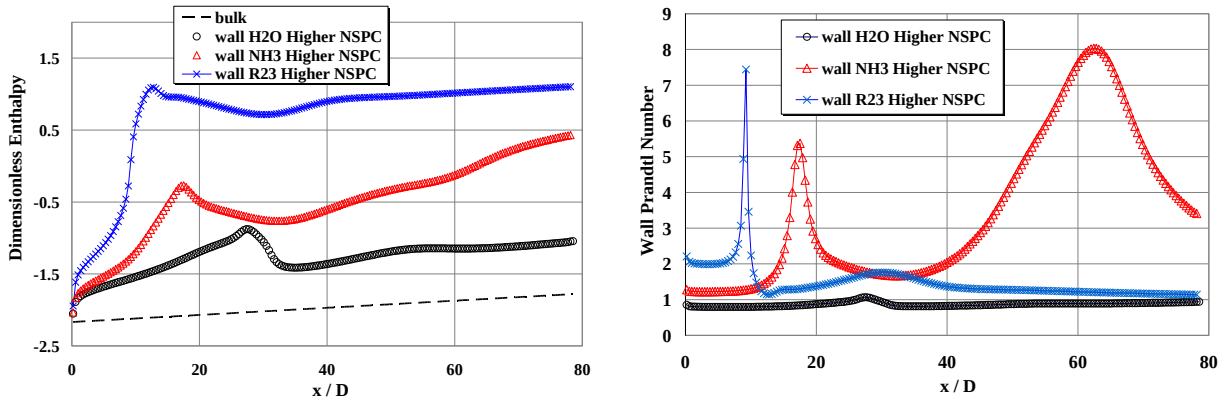


Figure 5. Bulk and wall dimensionless enthalpy and wall Prandtl number for the case with higher N_{SPC}

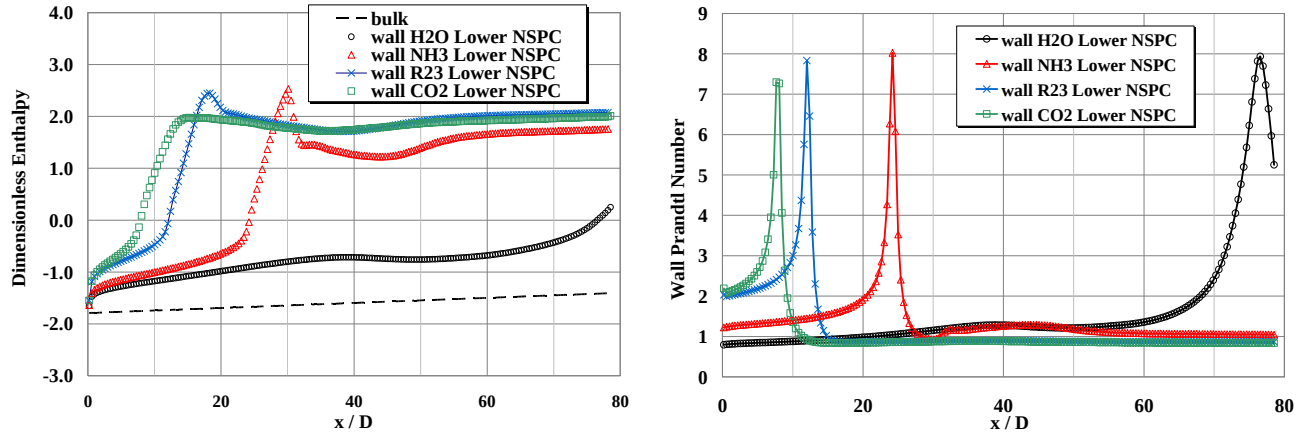


Figure 6. Bulk and wall dimensionless enthalpy and wall Prandtl number for the case with lower N_{SPC}

- a possible reason for this effect is related to the value of the wall Prandtl number, that is higher than for water in the liquid-like region for the other fluids;
- coherently with what expected, though the behaviour predicted for water shows a milder deterioration, the results obtained for ammonia are much closer to ones for water than those obtained for the other fluids; in particular, the occurrence of deterioration is delayed along the channel in closer similarity with the case of water;
- the highly deteriorated heat transfer observed for ammonia, carbon dioxide and R23 with respect to water must be considered remembering that k-ε models tend to predict very large wall temperatures whenever deterioration is obtained by exceeding the pseudo-critical threshold; this effect probably plays a role in worsening the comparison of fluid dimensionless enthalpy at the wall.

6. Conclusion

The similarity criteria proposed in this paper for heat transfer with fluids at supercritical pressures were obtained as an extension of those applied with reasonable success to 1D stability problems at imposed heat flux. Including the effect of locally variable thermo-physical properties required a 3D formulation of balance equations, in which specific forms of the heat and momentum molecular diffusivities appeared. It was noted that the trends of these diffusivities as a function of dimensionless enthalpy is not unique, as it is for dimensionless fluid density, though similarities appear in qualitative terms.

This first attempt to validate the similarity criteria by the use of a CFD code confirmed the role of the fluid Prandtl number in the liquid-like region in determining the severity of deterioration. In particular, it was seen that, while water has a Prandtl number lower than unity in that region, ammonia,

carbon dioxide and R23 are characterised by larger values, systematically leading to a progressive anticipation of deterioration. The difficulty of the problem is clearly shown in Figure 3, revealing that, even assuming a same value for the maximum of the Prandtl number for all the fluids, it is not possible to achieve a same trend of this parameter as a function of dimensionless enthalpy. It must be remarked, in fact, that with respect to heat transfer with fluids having nearly constant properties, in the case of fluids at supercritical pressures an important aspect is the way in which fluid properties change as a consequence of heating or cooling.

In summary, the first results of this similarity analysis allow to clarify the main difficulties involved in the problem. Additional results are being produced, aiming to propose a viable strategy for establishing a correlation among operating conditions obtained with different supercritical fluids.

7. References

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NOMENCLATURE

Latin Letters

A	cross section area [m^2]
C_p	specific heat at constant pressure [J/(kgK)]
D	diameter [m]
f	friction factor
$f_q(z)$	normalized distribution of heat flux
Fr	Froude number
g	gravity [m/s^2]
G	mass flux [kg/(m^{2}s)]
h	fluid specific enthalpy [J/kg]
K_{in}	localized pressure drop coefficient at the channel inlet
K_{out}	localized pressure drop coefficient at the channel outlet
L	channel length [m]
N_{SPC}	sub-pseudocritical number
N'_{TPC}, N_{TPC}	apparent and true trans-pseudocritical numbers
p	pressure [Pa]
q'	linear power [W/m]
q''	heat flux [W/m^2]
$\dot{Q}$	power [W]
r	radial coordinate [m]
R	radius [m]
s	specific entropy [J(kgK)]
t	time [s]
v	specific volume [m^3/kg]
W	mass flow rate [kg/s]
w	velocity [m/s]
z	axial coordinate along the channel [m]

Greek Letters

α	thermal diffusivity [m^2/s]
β	isobaric thermal expansion coefficient [K^{-1}]
δ_d	dimensional Dirac delta function [m^{-1}]
δ^*	dimensionless Dirac delta function
Λ	friction dimensionless group
μ	dynamic viscosity [Pa s]
ν	momentum diffusivity [m^2/s]
Π_h	heated perimeter [m]
ρ	density [kg/m^3]
Ω_p	expansion frequency [s^{-1}]

Subscripts

in	inlet
loc	local
out	outlet
p	constant pressure
pc	pseudocritical
0	reference value

Superscripts

*	dimensionless values