A MODEL OF DEVELOPING MIXED CONVECTION HEAT TRANSFER IN VERTICAL TUBES TO FLUIDS AT SUPERCRITICAL PRESSURE

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Abstract

The physical and transport properties of fluids at pressures just above the critical value change very rapidly with temperature over a particular range where such fluids make the transition from being liquid-like to gas-like. Consequently, when heat transfer takes place within them, strong spatial non-uniformity of density can be encountered. Problems can then arise as a result of the influence of buoyancy on mean flow, turbulence and heat transfer. Partial laminarisation of the flow accompanied by severe deterioration of heat transfer and localised overheating sometimes occur. The empirical equations currently available for calculating heat transfer to fluids at supercritical pressure are not able to account for such effects. Thus, with the aim of achieving an improved understanding of the physics of such flows and also constructing a sound, theoretically-based, empirical framework able to support reliable calculational procedures, the author has extended an existing semi-empirical model of fully developed mixed convection in vertical tubes to account for, non-uniformity of fluid properties, inertia and axial development of the effect of buoyancy on heat transfer. Firstly, the approach used to incorporate non-uniformity of fluid properties into the model is presented. Next the method adopted for bridging the discontinuous heat transfer behaviour exhibited by the model is described. Finally, two possible approaches designed to capture the observed axial development of buoyancy-influence on heat transfer are presented. Computational work is in progress using the extended model to try to reproduce the heat transfer behaviour found in experiments with water at supercritical pressure. Preliminary results have indicated that the extended model is valid for downward flow and that it is capable of reproducing the impairment of heat transfer found with upward flow. The work is ongoing and an interim report on it is presented here.

1. Introduction

In some respects the effects of buoyancy on turbulent heat transfer in vertical tubes are contrary to what might be intuitively expected, even in the case of conventional fluids such as water or air at normal pressure where fluid property non-uniformity is not usually very strong (see, for example, Jackson et. al., [1] and Jackson, [2]). For upward flow in a heated tube, impairment of heat transfer develops with onset of buoyancy influence. It occurs due to a reduction in turbulence production and impaired diffusion of heat. This occurs in spite of the fact that the upward motion of buoyant near-wall fluid is aided by buoyancy and advection of heat is improved. The impairment of heat transfer builds up with increase of buoyancy influence until a stage is reached where heat transfer coefficients fall to about one half of those expected for the same flow rate in the absence of any buoyancy influence. Then, with further increase in buoyancy the heat transfer process recovers until it eventually becomes enhanced in relation to that for forced convection.

With downward flow in a heated tube, a systematic improvement in heat transfer effectiveness occurs with increase of buoyancy influence, even through the buoyant near-wall fluid is opposed by buoyancy and advection of heat is worsened.

Thus buoyancy-influenced heat transfer in tubes exhibits complicated trends even in the absence of strong non-uniformity of fluid properties. This will be illustrated later in Section 4, using experimental data for air. Such trends are also found with fluids at supercritical pressure, where the non-uniformity fluid properties can be very strong and lead to striking localized effects on heat transfer. The empirical equations currently available for calculating convective heat transfer are not able to account for such effects. Thus, there is a need to develop means whereby heat transfer behaviour with fluids at supercritical pressure can be reliably described.

In the following section of this paper, attention is focussed on accounting for the effects of fluid property non-uniformity for conditions of mixed convection. The approaches for dealing with inertia and axial development of buoyancy effect are then addressed separately in subsequent sections.

2. Model of fully developed variable property mixed convection heat transfer in a vertical tube with a specified imposed wall heat flux

A physically-based, semi-empirical model of fully developed turbulent, buoyancy-influenced heat transfer in a vertical heated tube is presented in which account is taken of non-uniformity of fluid properties. It is an extended and improved version of a model for conditions of constant properties which was developed much earlier by the author (see Jackson and Hall [3,4]).

The effect of buoyancy on the distribution of shear stress across the thermal layer in a vertical heated tube can be found by approximate analysis using the following simplified equation of motion in which the inertia terms are neglected.

$$0 = \pm\rho g(a-y) - (a-y)\frac{dp}{dx} + \frac{d[(a-y)\tau]}{\partial y} \tag{1}$$

The symbol x is used for the coordinate in the flow direction, y is the transverse coordinate (measured inwards into the fluid from the wall), p is pressure, ρ is density and τ is the local total shear stress (molecular plus turbulent) at a distance y from the wall. The symbol *a* is used for tube radius.

It is assumed that the density of the fluid varies with y across the buoyant thermal layer within a region of extent δ_T according to the equation $\rho = \rho_w + (\rho_b - \rho_w)y/\delta_T$, and is uniform at the bulk value ρ_b across the core region $(y > \delta_T)$. On integrating Equation 1, term by term across the entire flow and next across the wall layer only, then re-organising the two resulting equations and subtracting them to eliminate axial pressure gradient, the following expression for the reduction of shear stress across the buoyant wall-layer is obtained, after neglecting higher order terms.

$$\tau_w - \tau_{\delta_T} = \pm\delta_T(\rho_b - \rho_w)g/2 \tag{2}$$

The symbol τ_w refers to wall shear stress. The positive sign in Equation 2 applies for upward flow and the negative one for downward flow.

The thermal layer thickness δ_T can be related to turbulent buffer layer thickness δ by an approximate empirical relationship $\delta_T = \delta / \overline{Pr}^n$ which is based on observed heat transfer behaviour in turbulent-pipeflow. $\overline{Pr}$ is the integrated mean value of Prandtl number over the range of temperature from T_b to T_w. The index n is frequently assigned the value 0.4 for gaseous fluids and 1/3 for liquid-like fluids. Substituting for δ_T in Equation 2 using this relationship we obtain

$$\tau_w - \tau_{\delta_T} = \pm\delta(\rho_b - \rho_w) g \overline{Pr}^{-n} / 2 \tag{3}$$

The following equation expresses δ in terms of δ^+, a universal, turbulent wall layer thickness defined using integrated mean values of density and viscosity and near-wall shear stress τ_{δ_T}.

$$\delta = \frac{\delta^+ \overline{\mu}}{\tau_{\delta_T}^{1/2} \overline{\rho}^{1/2}} \tag{4}$$

Thus, on substituting for δ in Equation 3 we obtain

$$\frac{\tau_{\delta_T}}{\tau_w} = 1 \pm \frac{\delta^+ \overline{\mu}(\rho_b - \rho_w) g \overline{Pr}^{-n}}{2\tau_w^{3/2} \overline{\rho}^{1/2}} \left(\frac{\tau_{\delta_T}}{\tau_w}\right)^{-1/2} \tag{5}$$

Next, wall shear stress τ_w in Equation 5 is related to ρ_b, u_b and friction factor $f_{b_o} (= \tau_w / \frac{1}{2}\rho_b u_b^2)$. Then, expressing f_{b_o} in terms of Reynolds number Re_b $(=\rho_b u_b d/\mu_b)$ using an empirical equation of the form

$$f_{b_o} = K_1, Re_b^{-m_1} \tag{6}$$

the following equation is obtained

$$\frac{\tau_{\delta_T}}{\tau_w} = 1 \pm \sqrt{2}\delta^+ K_1^{-3/2} \frac{F_g}{Re_b^{m_3} Pr_b^n} \left(\frac{(\rho_b - \rho_w)}{\rho_b}\right) \left(\frac{\overline{\mu}}{\mu_b}\right) \left(\frac{\overline{\rho}}{\rho_b}\right)^{-\frac{1}{2}} \left(\frac{\overline{Pr}}{Pr_b}\right)^{-n} \left(\frac{\tau_{\delta_T}}{\tau_w}\right)^{-\frac{1}{2}} \tag{7}$$

In Equation 7, $F_g (= gd^3 / \nu_b^2)$ is a gravitational body force group. The index m_3 to which Re_b is raised is related to m_1 by the expression $3(1-m_1/2)$.

The ratio of, Nusselt number for mixed convection Nu_b to that for variable property forced convection Nu_{b_o}, can be related to τ_{δ_T} / τ_w using the idea that the effect of buoyancy in modifying the near-wall distribution of shear stress enables a buoyancy-influenced flow to thought of as one which is not affected by buoyancy but is flowing at some different mean velocity (reduced or increased, depending on whether the flow direction is upward or downward). Thus, on the basis that f_{b_o} is proportional to $Re_b^{-m_1}$ and Nu_{b_o} is proportional to $Re_b^{m_2}$, we can show that

$$\frac{\mathrm{Nu_b}}{\mathrm{Nu_{b_o}}} = \left(\frac{\tau_{\delta_T}}{\tau_w}\right)^{m_4} \tag{8}$$

in which $m_4 = m_2/(2-m_1)$.

When Equation 7 is used in conjunction with Equations 8 the effect of buoyancy on heat transfer in a heated vertical tube, as represented by the ratio $\mathrm{Nu_b/Nu_{b_o}}$, is given by

$$\frac{Nu_b}{Nu_{b_o}} = \left[\left|1 \mp \frac{C_B}{F_{TD}F_{VP_2}} \frac{Gr_b^*}{\mathrm{Re}_b^{m_3}\,\mathrm{Pr}_b^{2n}} \left(\frac{\overline{\mu}}{\mu_b}\right)\left(\frac{\overline{\rho}}{\rho_b}\right)^{-1/2}\left(\frac{\overline{\mathrm{Pr}}}{\mathrm{Pr}_b}\right)^{n}\left(\frac{\overline{\rho\beta}}{\rho_b\beta_b}\right)\left(\frac{Nu_b}{Nu_{b_o}}\right)^{m_5}\right|\right]^{m_6} \tag{9}$$

in which $m_3 = m_1 + m_2$, $C_B = \sqrt{2}\delta^+ K_1^{-3/2}, \mathrm{Gr_b^*} = g\beta_b q_w d^4/(k_b \nu_b^2)$ and $m_5 = 1/(2m_4) + 1$. The –ve sign applies for upward flow and the +ve one for downward flow. The modulus signs ensure that for upward flow a real solution is always obtained for $\mathrm{Nu_b/Nu_{b_o}}$.

The Nusselt number $\mathrm{Nu_{b_o}}$, for developing, variable property forced convection in a tube, can be determined using an empirical equation of the form

$$\mathrm{Nu_{b_o}} = K_2 F_{TD}\, \mathrm{Re_b^{m_2}}\, \mathrm{Pr_b^n}\, F_{VP_2} \tag{10}$$

in which F_{TD} is an empirical thermal development function which, according to Petukhov and Polyakov [5], can take the form

$$F_{TD} = 1 + 2.35\mathrm{Re_b^{-0.15}}\mathrm{Pr_b^{-0.4}}(x/d)^{-0.6}\exp[-0.39\mathrm{Re_b^{-0.1}}(x/d)] \tag{11}$$

Thus, F_{TD} can be determined at any location x/d long the tube knowing Re_b and Pr_b.

F_{VP_2} is a correction factor which accounts for the effect of non-uniformity of fluid properties on heat transfer under conditions of turbulent variable property forced convection. According to Krasnoschekov and Protopopov [6], this can take the form $(\rho_w/\rho_b)^{0.3}(\overline{c}_p/c_{p_b})^{0.4}$ in which $\overline{c}_p$ is an integrated mean value of specific heat over the temperature range from T_b to T_w.

This general form of the model of buoyancy-influenced heat transfer in a vertical tube with a specified imposed wall heat involves several empirical indices which need to be specified. Different combinations of the coefficient K_1 and the index m_1 have been used in the various empirical power law equations for friction factor which are in common use. However, they all give rather similar values of f_b. The same can be said of the values of K_2 and m_2 used in the various empirical relationships of Dittus-Boulter form for Nusselt number.

If in Equation 9 the indices m_1 and m_2 are assigned the values 0.25 and 0.8, respectively, and the coefficients K_1 and K_2, 0.079 and 0.023, the following particular form of the model is obtained

$$\frac{Nu_b}{Nu_{b_o}} = \left[\left|1 \mp \frac{C_B}{F_{TD}F_{VP_2}} \frac{Gr_b^*}{\mathrm{Re}_b^{3.425}\,\mathrm{Pr}^{0.8}} \left(\frac{\bar{\mu}}{\mu_b}\right)\left(\frac{\bar{\rho}}{\rho_b}\right)^{-\frac{1}{2}} \left(\frac{\overline{\mathrm{Pr}}}{\mathrm{Pr_b}}\right)^{-0.4} \left(\frac{\overline{\rho\beta}}{\rho_b\beta_b}\right)\left(\frac{Nu_b}{Nu_{b_o}}\right)^{-2.1}\right|\right]^{0.46} \tag{12}$$

The coefficient C_B has an estimated value of about 10^5 based on a universal turbulent buffer layer thickness δ^+ of 30. However, it should be borne in mind that in view of the many simplifying assumptions which have been made in order to arrive at this model the estimated value of C_B might well need to be adjusted to make modelling results fit observed behaviour.

Again, the –ve sign applies for upward, flow and the –ve one for downward, flow.

Notice that in Equation 12, the dimensionless groups $\mathrm{Gr}_b^*, \mathrm{Re}_b$ and Pr_b combine together in the manner $\mathrm{Gr}_b^*/(\mathrm{Re}_b^{3.425}\,\mathrm{Pr}_b^{0.8})$ to form a buoyancy parameter Bo_b^* which in conjunction with the various property ratio terms shown characterises the strength of buoyancy influence under conditions of variable property mixed convection.

Equation 12 can be written in the following condensed form

$$\frac{\mathrm{Nu_b}}{\mathrm{Nu_{b_o}}} = \left[\left|1 \mp \mathrm{C_B Bo_b^*} \frac{\mathrm{F_{VP_1}F_{VP_3}F_{VP_4}}}{\mathrm{F_{TD}F_{VP_2}}} \left(\frac{\mathrm{Nu_b}}{\mathrm{Nu_{b_o}}}\right)^{-2.1}\right|\right]^{0.46} \tag{13}$$

in which

$$\mathrm{F_{VP_1}} = \left(\frac{\bar{\mu}}{\mu_b}\right)\left(\frac{\bar{\rho}}{\rho_b}\right)^{-1/2}, \quad \mathrm{F_{VP_3}} = \left(\frac{\overline{\mathrm{Pr}}}{\mathrm{Pr_b}}\right)^{-0.4} \text{ and } \mathrm{F_{VP_4}} = \left(\frac{\overline{\rho\beta}}{\rho_b\beta_b}\right)$$

The effect of buoyancy on heat transfer given by Equation 13 is shown on Figure 1.

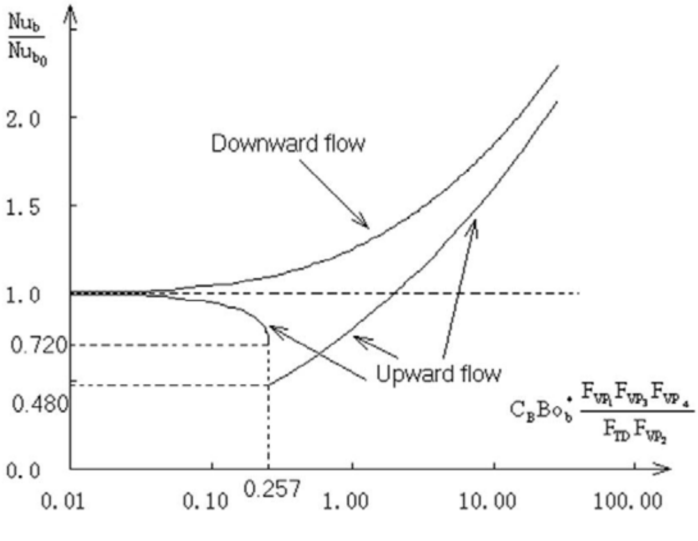


Figure 1 Predicted effect of buoyancy on heat transfer

As can be seen, if the parameter $\mathrm{C_B Bo_b^* F_{VP_1}F_{VP_3}F_{VP_4}/(F_{TD}F_{VP_2})}$ is sufficiently small $\mathrm{Nu_b/Nu_{b_o}}$ tends to unity and the effect of buoyancy on heat transfer is negligible. Noting that $F_{VP_1}F_{VP_4}$ and F_{VP_2} are of order one, a criterion for the effect of buoyancy on heat transfer to be less than about 2% downstream of the thermal entry region is that $C_B Bo_b^* F_{VP_3}$ must be less than 0.04. Thus, in terms of the parameter $Bo_b^* (= Gr_b^* / Re_b^{3.425} Pr_b^{0.8})$, a criterion for buoyancy to have a negligible effect on heat transfer is

$$Bo_b^* < 4 \times 10^{-7} \left(\overline{Pr}/Pr_b\right)^{0.4} \tag{14}$$

Furthermore, as can again be seen from Figure 1, and also from Equation 13, if for upward flow the parameter $C_B Bo_b^* F_{VP_1} F_{VP_3} F_{VP_4} / F_{VP_2}$ reaches a value of about 0.4 severe impairment of heat transfer will be encountered, with Nu_b / Nu_{b_o} being reduced to about 0.5. Thus, noting again that $F_{VP_1} F_{VP_2} F_{VP_3}$ and F_{VP_4} are each of order unity, a criterion in terms of the buoyancy parameter $Bo_b^* (= Gr_b^* / Re_b^{3.425} Pr_b^{0.8})$ for 'partial laminarisation' of a buoyancy-aided flow is

$$Bo_b^* \approx 4x10^{-6} \tag{15}$$

The influence of buoyancy on convective heat transfer will be dominant when

$$Bo_b^* \approx 10^{-3} \tag{16}$$

Then the model yields an expression for Nu_b which is independent of flow rate and takes the form

$$Nu_b = CGr_b^{*0.23} Pr_b^{0.23} F_{VP} \tag{17}$$

in which F_{VP} is a combined property ratio term. Note the similarity between this equation and those which are used to describe turbulent free convection from vertical heated surfaces.

For conditions of mixed convection under conditions of negligible non-uniformity of fluid properties Equation 13 reduces to

$$\frac{Nu_b}{Nu_{b_o}} = \left[\left| 1 \mp C_B Bo_b^* \left(\frac{Nu_b}{Nu_{b_o}} \right)^{-2.1} \right| \right]^{0.46} \tag{18}$$

which is the result obtained earlier by the present author for fully developed, constant property mixed convection using a similar but simpler analysis (see Reference 2).

In conclusion, we next consider how the model might be used to produce results which can be compared with experimental data. To do this we need to be able to calculate the distribution of wall temperature T_w for variable property, buoyancy influenced flow and heat transfer using Equation 13.

Firstly, the pressure and temperature of the fluid entering the tube must be specified and also the mass flow and wall heat flux q_w. This information enables the axial distribution of bulk enthalpy h_b of the fluid, and hence its bulk temperature T_b, to be determined by using the steady flow energy equation (conservation of energy). Then, values of Re_b, Pr_b, Gr_b^* and Bo_b^* can be found at any axial location x/d.

Next, the axial distribution of Nusselt number Nu_{b_o} for variable property forced convection with negligible influence of buoyancy should be determined using Equation 10. In that equation, the thermal entry development factor F_{TD} can be determined for any axial location x/d, knowing Re_b and Pr_b. The determination of the variable property correction factor for forced convection F_{VP_2} involves knowing T_w as well as T_b and thus an iterative computational procedure needs to be employed using Equation 10 to determine T_w and hence Nu_{b_o} and F_{VP_2}.

Finally, the unknown variable Nu_b in the model equation for variable property buoyancy-influenced flow and heat transfer, (Equation 13), should be expressed in terms of the unknown variable T_w by replacing it by $q_w d/(k_b(T_w - T_b))$. T_w is the unknown variable in the expression. Thus, Bo_b^*, F_{TD}, Nu_{b_o} and F_{VP_2} are parameters which have been determined at this stage. The property ratio terms F_{VP_1}, F_{VP_2} and F_{VP_4} are all functions of the unknown variable T_w and also involve T_b, which is known. Thus, with these changes Equation 13 is a non-linear algebraic equation from which local values of the unknown variable T_w can be determined at any specified axial location x/d using standard iterative computational procedures. The process can conveniently be initiated at x=0, where T_w has a value equal to the fluid temperature at entry, and continues step by step along the tube using the converged value of T_w at each location as the initial value at the next one. Nu_b can be determined at each stage using the converged value of T_w in the expression $q_w d/(k_b(T_w - T_b))$.

3. Accounting for the inertia effects associated with velocity profile development under the action of buoyancy during laminarisation with upward flow

As seen in Section 2, the physically-based, semi-empirical model of variable property mixed convection heat transfer in a vertical heated tube led to an non-linear algebraic expression, Equation 13, relating Nusselt number ratio Nu_b/Nu_{b_o} to a buoyancy function consisting of a parameter $Bo_b^* (= Gr_b^*/(Re_b^{3.425} Pr_b^{0.8}))$, a thermal development factor F_{TD} and four property ratio terms. The variation of Nu_b/Nu_{b_o} with that function shown on Figure 1 provides a detailed picture of fully developed, variable property mixed convection in vertical tubes for upward and downward flow. It covers the entire mixed convection range from forced convection with negligible influence of buoyancy to buoyancy-dominated convection where the effectiveness of heat transfer becomes the same as that for free convection, being independent of the direction of the imposed rate of flow and, as a consequence of this is, independent of the direction of the imposed flow.

For downward flow, buoyancy in the near-wall region opposes the imposed motion with the result that in the region where turbulent eddies are mainly produced shear stress is caused to increase so that the turbulence in the flow is increased and turbulent diffusion of heat is enhanced. This effect builds up systematically as the conditions are varied so as to strengthen the influence of buoyancy. Thus, Nu_b/Nu_{b_o} increases systematically.

For upward flow, buoyancy in the near-wall region aids the imposed motion, shear stress is caused to decreases in the region where turbulent eddies are mainly produced, the turbulence in the flow is reduced and turbulent diffusion of heat is impaired. The effect again builds up systematically with increase of buoyancy influence so that Nu_b/Nu_{b_o} falls, until a critical value of the buoyancy function is reached where according to the model there is a sudden change as the modulus signs in Equation 13 come into action. Then, according to the model, discontinuous transition occurs to an operating point on the lower curve on Figure 1 at some increased value of the buoyancy function. A step decrease of Nu_b/Nu_{b_o} occurs associated with which would be a sudden change in the radial distributions of shear stress, with this becoming negative in the core flow, so that the velocity profile then inverted in that region. However, we need to remember that in the derivation of the model a simplified equation of motion was used in which the inertia terms were omitted. In practice, such a discontinuous change will not happen because of the inertia of the fluid. Therefore, a method needs to be devised to bridge this transition.

To do this it is necessary to find a way of estimating the finite axial length scale over which this change of velocity profile might actually be achieved as a result of inertia effects. An approach is proposed whereby the reduction in Nu_b/Nu_{b_o} associated with that predicted by the model as a result of the discontinuous change is determined first. Then the axial distance over which a similar reduction in Nu_b/Nu_{b_o} has occurred just prior to the discontinuous change can be found and used as the axial distance over which the change of velocity profile occurs beyond the transition location. A simple exponential function decaying with axial distance downstream of that location will be used to calculate local values of Nu_b/Nu_{b_o} in a step by step calculation to bridge the transition over such a distance. The difference between that Nusselt number ratio and the one calculated using the model for the same value of buoyancy function decays with axial distance and the ratio eventually follows the model. Hence, wall temperatures will be calculated over the bridging region and beyond it with a smooth transition onto the lower curve of the model.

4. Extension of the model to include developing buoyancy-influenced heat transfer

The model of fully developed, variable property mixed convection in a heated vertical tube presented in Section 2 involved the assumption that the inertia terms in the equation of motion could be neglected. Thus, the applicability of that model is limited to locations sufficiently far downstream in the tube where a pseudo-developed flow condition is approached. In practice, it is found that, with downward flow, such a condition is readily achieved about 20 diameters downstream from the start of heating but with upward (buoyancy-aided) flow it takes about 50 diameters (see below).

Figure 2 shows some sample results for four values of buoyancy parameter Bo_b^* from a detailed study of mixed convection heat transfer to air in a very long, uniformly heated vertical tube with upward and downward flow, Li [7]. They are presented in the form of plots of local Nusselt number Nu_b versus dimensionless axial distance x/d from the start of heating. Also shown are distributions of Nusselt number Nu_{b_o} for developing, variable property forced convection with negligible influence of buoyancy (the chain-dashed curves). These were calculated using an empirical equation for such conditions established from experimental data obtained in the course of that study. As can be seen, the trends of impairment and enhancement of heat transfer due to buoyancy described earlier in the introductory section of this paper are clearly evident. With downward flow systematic enhancement of heat transfer occurs and a fully developed condition is readily achieved for x/d values greater than 20. With upward flow impairment of heat transfer with increase of buoyancy influence is followed recovery and then enhancement. A pseudo-developed mixed convection condition is eventually achieved but only for values of x/d greater than about 50.

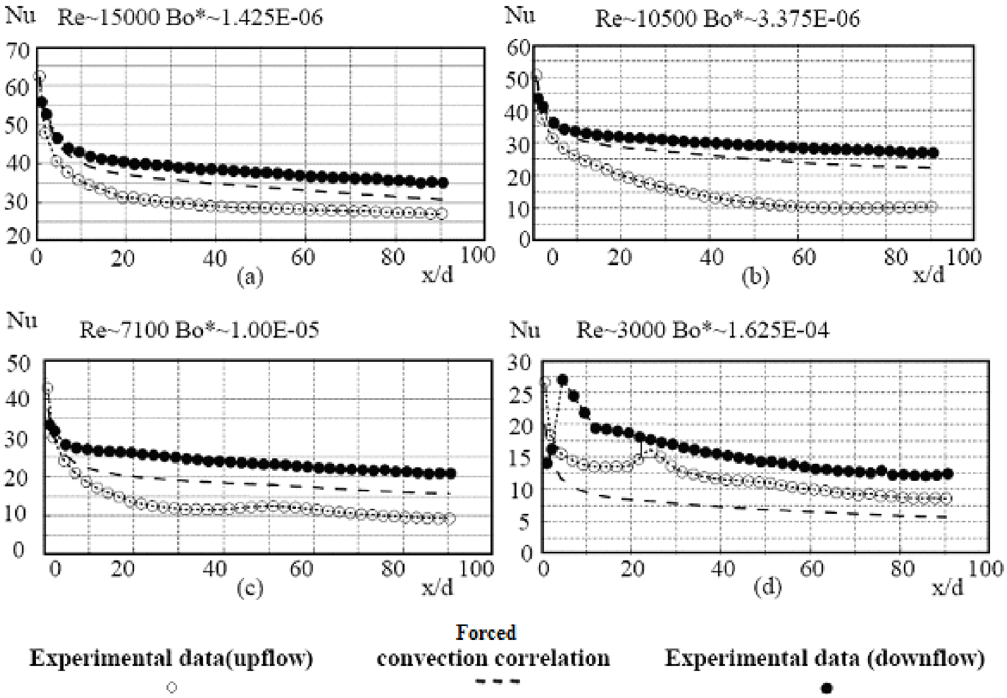


Figure 2 Impairment and enhancement of turbulent mixed convection heat transfer to air at atmospheric pressure in a heated vertical tube, Li, [7]

Figure 3 shows downward flow data for x/d greater than 20 plotted in the form Nu_b/Nu_{b_o} versus Bo_b^* along with the curve obtained for that flow configuration from the constant properties version of the model (Equation 18) with the coefficient C_B assigned the value $1.5x10^{-5}$. As can be seen, the data correlate extremely well and are closely fitted by the model equation. Thus the constant property, model of mixed convection is able to describe the experimental data for fully developed mixed convection with downward flow remarkably well.

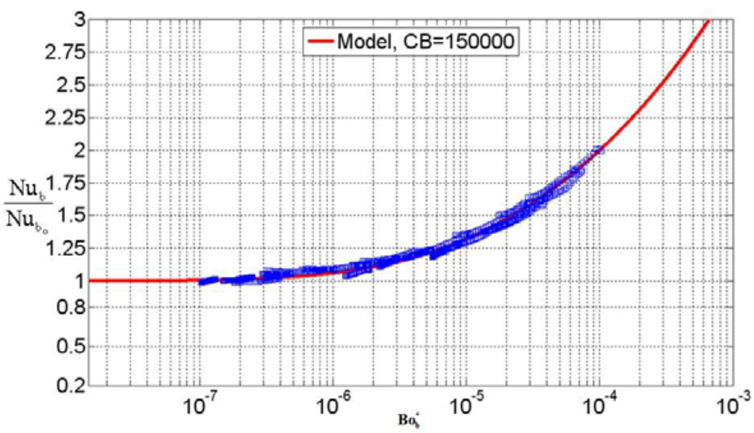
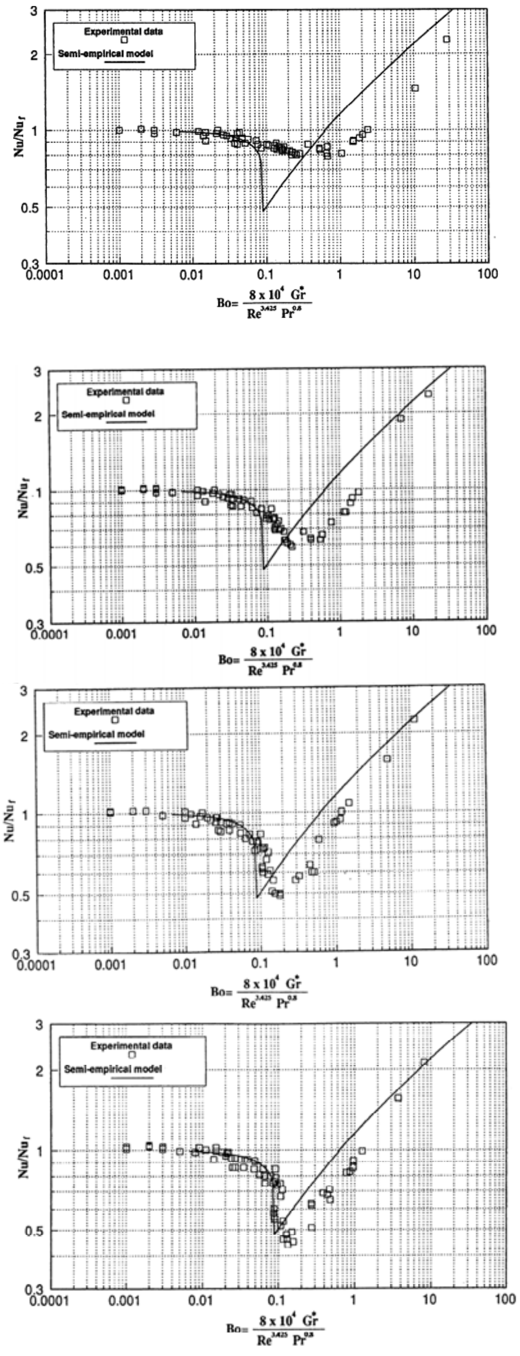


Figure 3 Correlation of experimental data and comparison with semi-empirical model

Figures 4(a), (b), (c) and (d) show experimental values for upward flow of Nu_b / Nu_{b_o} for particular values of x/d from each test plotted against Bo_b^*. As can be seen, the maximum impairment of heat transfer increases with increase of x/d. For x/d=11.9 it is only about 20% but for x/d=26.6 it is over 40% and for x/d=41.3 it is over 50%. It can also be seen from the curves shown along with the data, which were obtained using the fully developed constant property model of mixed convection (Equation 18) completely fail to reproduce observed behaviour for x/d=11.9, do somewhat better for x/d=26.6 and do quite well in terms of matching the maximum impairment of heat transfer and following the general trends of the experimental data for x/d=41.3 and 56.0. A pseudo-developed condition has then been approached.



(a)
x/d=11.9

(b)
x/d=26.6

(c)
x/d = 41.3

(d)
x/d=56.0

Figure 4 Developing buoyancy-aided mixed convection heat transfer to air, Li [7]

Clearly the model of fully developed mixed convection does quite well in correlating and fitting data from the experimental investigation of Li [7] for conditions where such a fully developed mixed convection condition is approached. However, there is a need to consider how it might be modified to extend its coverage to include developing mixed convection.

5. An extended version of the model to include developing mixed convection

Two different approaches have been devised with a view to extending the model to cover developing mixed convection. The first one applies a rising scaling factor F(x/d)=1-exp(-A(x/d)), to the index 0.46 in Equation 13 and associated with this change the index 2.1 is modified to 1.1/F(x/d)+1. Thus, the extended version of Equation 13 becomes

$$\left(\frac{Nu_b}{Nu_{b_o}}\right) = \left[\left|1 \mp C_B Bo_b^* F_5 \left(\frac{Nu_b}{Nu_{b_o}}\right)^{-(1/0.92F(X)+1)}\right|\right]^{0.46F(X)} \tag{19}$$

in which $F_4 = F_{VP_1} F_{VP_3} F_{VP_4} / F_{VP_2}$ and X is dimensionless axial distance x/d.

The parameter A which controls the distance over which the scaling factor rises and asymptotes to unity has initially been assigned a value 0.1 but this might need to be modified to really optimise the performance of the model. Figure 5 shows the manner in which the scaling function F(X) increases with X.

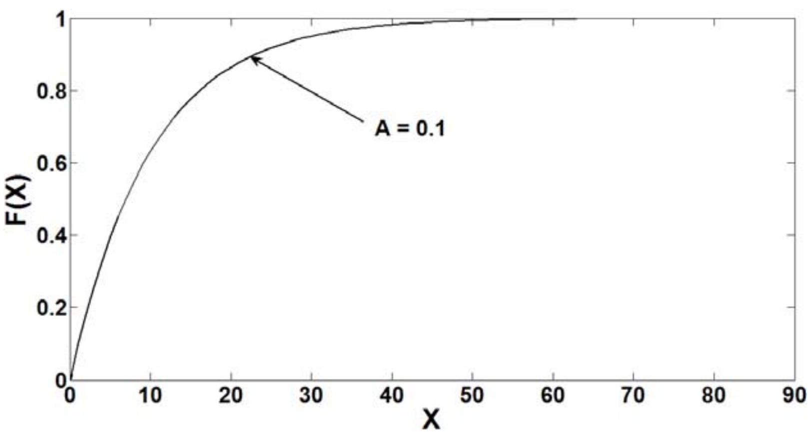
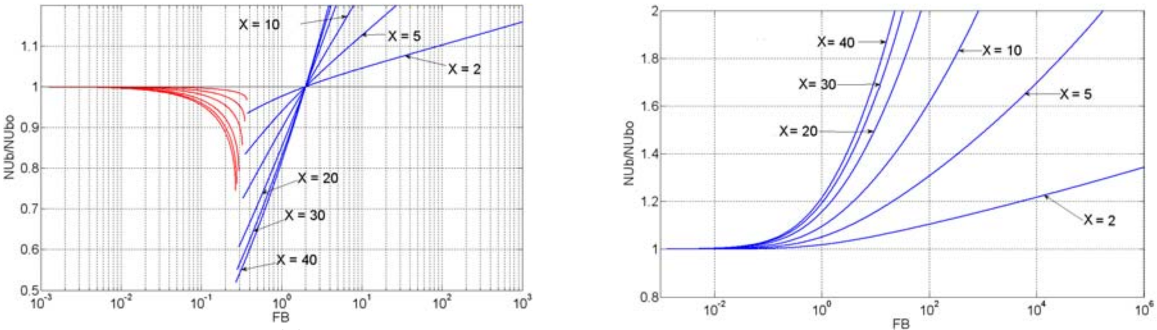


Figure 5 Variation of the scaling function with axial location

Equation 19 gave the family of curves of Nusselt number ratio Nu_b / Nu_{b_o} versus $FB(= C_B Bo_b^* F_4)$ for a range of values of X shown on Figures 6(a) and 6(b), respectively for upward and downward flow. For X=10 to X=40 they broadly reproduce the trends seen in the data of Li [7] shown on Figures 3 and 4.



(a) (b)

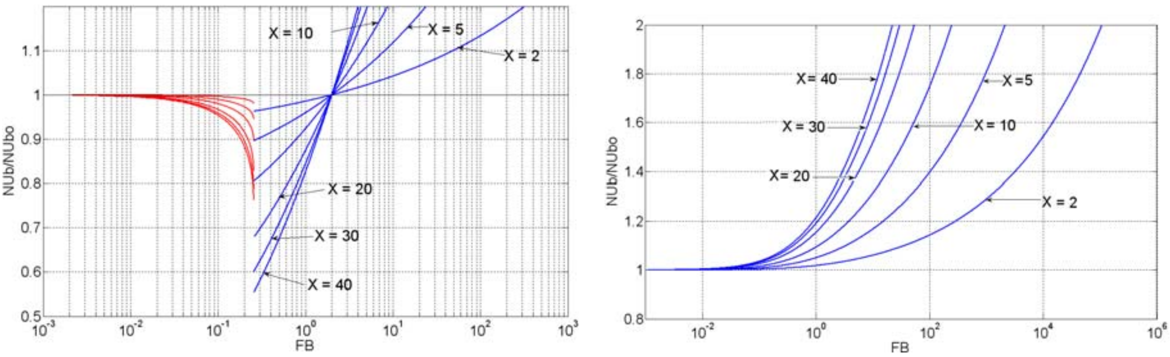
Figure 6 $\mathrm{Nu_b / Nu_{b_o}}$ versus $\mathrm{C_B Bo_b^* F_4}$ using the first approach (a) upward and (b) downward flow

The second approach simply uses the same factor F(X) to scale the normalised relative Nusselt number $\mathrm{(Nu_b - Nu_{b_o})/Nu_{b_o}}$ given by the fully developed model. Thus

$$\left(\frac{Nu_b}{Nu_{b_o}}\right) = \left[1 + F(X)\left\{\left(\frac{Nu_b}{Nu_{b_o}}\right)_{fd} - 1\right\}\right] \tag{20}$$

in which $\mathrm{(Nu_b / Nu_{b_o})_{fd}}$ is given by Equation 13.

The resulting family of curves of $\mathrm{Nu_b / Nu_{b_o}}$ versus $\mathrm{FB(= C_B Bo_b^* F_4)}$ for a range of values of X for upward and downward flow is shown on Figures 7(a) and 7(b), respectively.



(a) (b)

Figure 7 Behaviour predicted by the second approach for (a) upward and (b) downward flow

Again, for X=10 to X=40 they broadly reproduce the trends of the variation of $\mathrm{Nu_b / Nu_{b_o}}$ versus $\mathrm{C_B Bo_b^* F_4}$ seen in the experimental work of Li [7] shown on Figures 3 and 4.

6. Mixed convection heat transfer to water at supercritical pressure

The extended model presented here is being evaluated using some experiments on mixed convection heat transfer to water at a supercritical pressure of 250 bar in a vertical tube of inside diameter 25.4mm from a detailed study of mixed convection heat transfer with upward and downward flow carried out Watts, [9]. The tube was uniformly heated over a length of 2m by passing electricity along it from a variable power AC power supply system. Measurements of the temperature of the surface of the test section were made at 50 locations using chromel-alumel thermocouples with the two wires of each pair resistance-welded to the outside of the tube. The measured values were corrected to account for the temperature drop across the tube wall material. Mineral-wool thermal insulation of thickness 60mm was used to minimise heat losses from the outside of the test section.

Figure 8 shows some typical results which exhibit very different behaviour for upward and downward flow due to the influence of buoyancy. Heat transfer for upward flow is generally impaired in relation to that for downward flow. The development of localised peaks on the wall temperature distribution for upward flow is clearly evident whereas for downward flow no significant non-uniformity is seen.

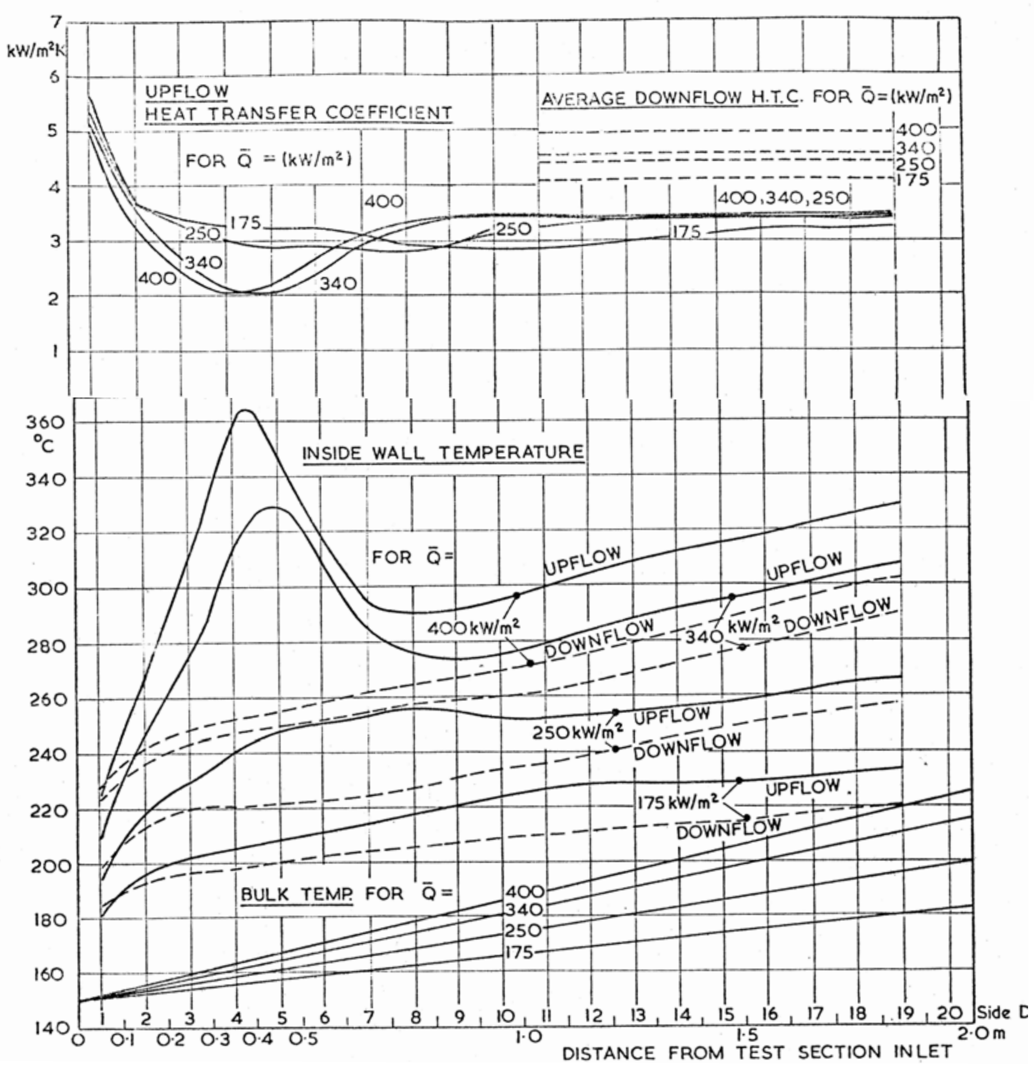


Figure 8 Developing mixed convection heat transfer to water at 250 bar for upward and downward flow; tube bore 22.5mm and mass velocity 380 kg/m^{2}s, Watts [9]

Concluding remarks

The computational work aimed at validating the model of developing mixed convection heat transfer described here is still in progress. It involves determining the values of the buoyancy coefficient C_B and the decay constants which optimise the agreement between modelling results and experimentally observed behaviour.

Nomenclature

a	Radius of tube (=d/x), m
A	Decay constant in Equation 5
Ac^*	Acceleration parameter $(=Q^*/(\mathrm{Re}_b^{1.625}\,\mathrm{Pr}_b))$
c_p	Specific heat at constant pressure, kJ/kgK
Bo^*	Buoyancy parameter $Gr_b^*/(\mathrm{Re}_b^{3.425}\,\mathrm{Pr}_b^{0.8})$
C_A	Acceleration coefficient
C_B	Buoyancy coefficient
d	Tube diameter, m
g	Acceleration due to gravity, m/s^2
Gr_b^*	Grashof number $(=g\beta_b q_w d^4/(k_b \nu_b^2))$
f	Friction factor
F_g	Gravitational group $(=gd^3/\nu_b^2)$
FB	Buoyancy function $(=C_B Bo_b^* F_4)$
F_{TD}	Thermal development function (defined below Equation 2)
F_{VP_1}	Variable property function $((\overline{\mu}/\mu_b)(\overline{\rho}/\rho_b)^{-1/2})$
F_{VP_2}	Variable property function $(=(\rho_w/\rho_b)^{0.3}(\overline{c}_p/c_{p_b})^{0.4})$
F_{VP_3}	Variable property function $(=(\overline{\mathrm{Pr}}/\mathrm{Pr}_b)^{-0.4}$
F_{VP_4}	Variable property function $(=(\overline{\rho\beta}/(\rho_b\beta_b))$
F_4	Combined function of $(=F_{VP_1}F_{VP_3}F_{VP_4}/F_{VP_2})$
k	Thermal conductivity, kW/mK
K_1	Coefficient in Equation 6
K_2	Coefficient in Equation 10
m_1	Index in Equation 6
m_2	Index in Equation 10
m_3	Index in Equation 9 ($=m_1+m_2$)
m_4	Index in Equation 8 ($=m_2/(2-m_1)$)
m_5	Index in Equation 9 ($=1/((2m_4)+1)$)
Nu_b	Nusselt number $(=q_w d/(k_b(T_w-T_b)))$
Nu_{b_o}	Nusselt number for variable property forced convection $(=q_w d/(k(T_{w_o}-T_b)))$
P	Pressure, MPa
Pr_b	Prandtl number $(=\mu_b c_{p_b}/k_b)$
q_w	Wall heat flux, kW/m^2
Q_b^*	Thermal loading parameter $(=q_w d\beta_b/k_b)$
Re_b	Reynolds number $(=\rho_b u_b d/\mu_b)$
T_b	Local bulk temperature, ^oC

T_w Local wall temperature, ^oC
u_b Local bulk velocity
X Dimensionless axial coordinate (=x/d)
y Transverse coordinate (measured inwards from wall), m

Greek symbols
β Thermal expansion coefficient $(= -(1/\rho)(\partial\rho/\partial T)_p)$
μ Viscosity, kg/ms
ν Kinematic viscosity, m^2/s
ρ Density, kg/m^3
δ Turbulent buffer layer thickness, m
δ_T Thermal layer thickness, m
τ Shear stress, N/m^2
τ_w Wall shear stress, N/m^2

Subscripts
b Denotes properties evaluated at the bulk temperature
fd Denotes fully developed condition
in Denotes properties evaluated at the inlet fluid temperature
pc Denotes properties evaluated at the pseudocritical temperature
w Denotes properties evaluated at the wall temperature

Superscripts
\- Bar on top of a property symbol denotes that it is integrated over the range between the wall and the bulk temperature
\+ Denotes thickness of buffer layer in universal wall coordinate form, $\delta^+ = \rho_b \tau_w^{1/2} \delta / \mu_b$

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