INTERPRETING AND CATEGORISING EXPERIMENTAL DATA ON FORCED AND MIXED CONVECTION HEAT TRANSFER TO SUPERCRITICAL PRESSURE FLUIDS USING PHYSICALLY-BASED MODELS

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Abstract

The need to improve the reliability of thermal design procedures for advanced pressurised water reactors has stimulated a renewed interest in heat transfer to fluids at supercritical pressure. Deterioration of heat transfer is sometimes encountered with such fluids, mainly as a result of the strong dependence of density on temperature. Two physically-based models of turbulent heat transfer in tubes are described here which can be used for categorising and interpreting experimental data on heat transfer to fluids at supercritical pressure. The first is a model of variable property mixed convection in vertical heated tubes. This predicts reduced or increased turbulence and, therefore, impaired or enhanced heat transfer, respectively, due to the influence of buoyancy on the flow. The second is a model of variable property forced convection with thermally-induced bulk flow acceleration due to high heat flux and low mass flux. This predicts reduced turbulence due to acceleration of the flow, and consequently, impaired heat transfer. Used together for conditions where influences of buoyancy and acceleration are both significant, the models enable the resulting complex heat transfer behaviour to be interpreted and better understood. Some recent experimental data on heat transfer to supercritical pressure carbon-dioxide in heated tubes of small diameter in the mini and micro ranges have been categorised and interpreted in a preliminary manner with the aid of the models and the conclusions arrived at are presented here. The work is ongoing and a more detailed demonstration of the usefulness of the models will be completed shortly.

1. Introduction

The properties of fluids at pressures just above the critical value vary rapidly with temperature over a particular range where they change from being liquid-like to gas-like.[1,2] Striking impairment of heat transfer can be encountered with such fluids as a result of the strong dependence of density on temperature causing turbulence to be modified.[3,4,5] The need to improve the reliability of procedures for the thermal design of advanced pressurised water reactors which can operate at supercritical pressure has stimulated a renewed interest in heat transfer under such conditions.[6] This has stimulated one of present authors to try to extend two physically-based, semi-empirical models which he developed earlier[7,8] to include effects of non-uniformity of fluid properties, so that they can be used for interpreting and categorising experimental data on heat transfer to fluids at supercritical pressure.[9] The first of these is a model of mixed convection in vertical tubes which predicts reduced or increased turbulence and impaired or enhanced heat transfer depending on the conditions. The second is a model of forced convection heat transfer with thermally-induced bulk flow acceleration due to high heat flux and low mass flux which predicts reduced turbulence and impaired heat transfer. They each provide criteria, in terms of values of appropriates dimensionless parameters (see later), which enable the conditions to be specified under which the effects on heat transfer considered will be either negligibly small or very strong. Used together for conditions where influences of both acceleration and buoyancy are significant, the models should enable the resulting complex heat transfer behaviour to be understood better and interpreted.

2. Mixed convection heat transfer in vertical tubes with imposed wall heat flux

2.1 A physically-based model

Such a model is described in a recent review of mixed convection heat transfer in vertical passages[10]. Fluid with reduced density in the thermal layer in a heated pipe experiences a force due to buoyancy which either aids or opposes the imposed flow depending on whether this is in the upward or downward direction, respectively.

In the case of upward flow, buoyancy tends to increase the extent to which the shear stress falls with distance from the wall in relation to that with fully developed in a tube without buoyancy. Thus near to the wall, in the region where turbulence production mainly occurs, shear stress is reduced, fewer turbulent eddies are produced and the flow becomes less turbulent. In the case of downward flow the trends are all in the opposite sense.

Thus, with upward flow in a vertical heated tube, impairment of turbulent heat transfer occurs with onset of buoyancy influence. This effect builds up until a stage is reached where the coefficient of heat transfer falls to about half of that expected in the absence of buoyancy. With further increase buoyancy influence the effectiveness of heat transfer recovers and it eventually becomes enhanced. In contrast, for downward flow a systematic improvement occurs in the effectiveness of heat transfer with increase of buoyancy influence.

2.2 A semi-empirical model of buoyancy-influenced heat transfer

A semi-empirical model of buoyancy-influenced heat transfer in a vertical heated tube with imposed wall heat flux q_w leads to the following equation for Nusselt number ratio,

$$\frac{\mathrm{Nu_b}}{\mathrm{Nu_{b_o}}} = \left[\left|1 \mp \frac{C_B}{F_{TD}} \frac{\mathrm{Gr_b^*}}{\mathrm{Re_b^{3.425}\,Pr_b^{2n}}} \left(\frac{\bar{\mu}}{\mu_b}\right)\left(\frac{\bar{\rho}}{\rho_b}\right)^{-1/2}\left(\frac{\rho_w}{\rho_b}\right)^{-0.3}\left(\frac{\bar{c}_p}{c_{p_b}}\right)^{-0.4}\left(\frac{\overline{\mathrm{Pr}}}{\mathrm{Pr_b}}\right)^{-0.4}\left(\frac{\overline{\rho\beta}}{\rho_b\beta_b}\right)\left(\frac{\mathrm{Nu_b}}{\mathrm{Nu_{b_o}}}\right)^{-2.1}\right|\right]^{0.46} \tag{1}$$

The –ve sign applies for upward flow and the +ve sign for downward flow.

The ratio $\mathrm{Nu_b}/\mathrm{Nu_{b_o}}$ is a measure of the effect of buoyancy on heat transfer. $\mathrm{Nu_b}$ is Nusselt number for variable property, buoyancy-influenced flow and $\mathrm{Nu_{b_o}}$ is Nusselt number for variable property convective heat transfer in the absence of buoyancy.

The latter can be determined from the empirical equation

$$Nu_{b_o} = KF_{TD}\,\mathrm{Re}_b^{0.8}\,\mathrm{Pr}_b^{0.4}\,F_{VP_2} \tag{2}$$

in which F_{TD} is a thermal development function, which according to Petukhov and Polyakov[11] can take the form $F_{TD} = 1 + 2.35\,\mathrm{Re}_b^{-0.35}\,\mathrm{Pr}_b^{-0.4}(x/d)^{-0.6}\exp(-0.39\,\mathrm{Re}_b^{-0.1}(x/d))$, and F_{VP_2} is a correction factor for the transverse non-uniformity of fluid properties which according to Krasnoshchekov and Protopopov[12] can take the form $(\rho_w/\rho_b)^{0.3}(\bar{c}_p/c_{p_b})^{0.4}$.

The Grashof number $Gr_b^*(=g\beta_b q_w d^4/k_b\nu_b^2)$ is defined in terms the imposed wall heat flux.

The coefficient C_B has an estimated value of about 10^5 for an assumed dimensionless universal wall layer thickness δ^+ of 30. As can be seen from Equation 1, the effect of buoyancy is characterised by the product of a buoyancy parameter $Bo_b^*\,(=Gr_b^*/(Re_b^{3.425}Pr_b^{0.8}))$ and several property ratio terms.

This physically-based model should be regarded as a framework designed to support an empirical equation for mixed convection in vertical heated tubes in which the coefficient C_B is assigned a value which causes the equation to fit experimental data. The value 10^5 quoted earlier is merely an estimate which can be used as a starting point such an exercise.

2.3 Condensed version of the equation

If the property ratio functions in Equation 1 are assigned the following symbols

$$F_{VP_1}=\left(\frac{\bar{\mu}}{\mu_b}\right)\left(\frac{\bar{\rho}}{\rho_b}\right)^{-1/2},\quad F_{VP_2}=\left(\frac{\rho_w}{\rho_b}\right)^{0.3}\left(\frac{\bar{c}_p}{c_{p_b}}\right)^{0.4},F_{VP_3}=\left(\frac{\overline{Pr}}{Pr_b}\right)^{-0.4}\text{ and }F_{VP_4}=\frac{\overline{\rho\beta}}{\rho_b\beta_b}$$

Equation 1 can be written in the following condensed form

$$\frac{Nu_b}{Nu_{b_o}}=\left[\left|1\mp C_B Bo_b^*\frac{F_{VP_1}F_{VP_3}F_{VP_4}}{F_{TD}F_{VP_2}}\left(\frac{Nu_b}{Nu_{b_o}}\right)^{-2.1}\right|\right]^{0.46}\qquad(3)$$

The effect of buoyancy on heat transfer predicted by Equation 3 is shown below on Figure 1.

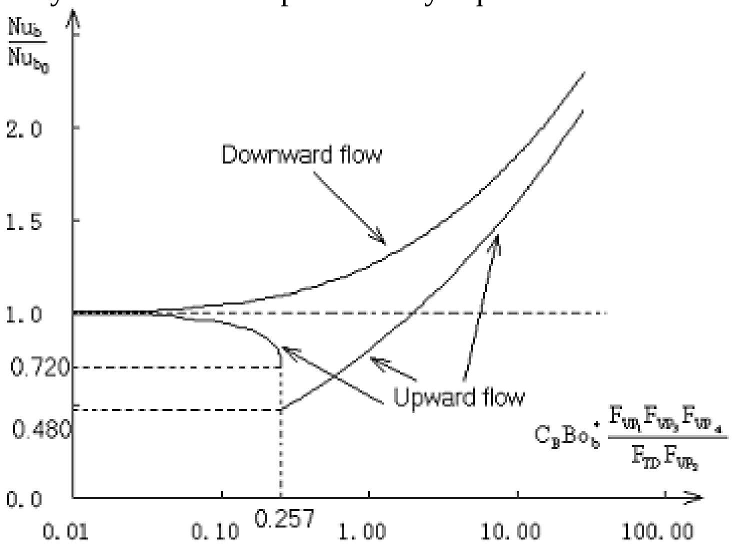

Figure 1 Effect of buoyancy on heat transfer in a vertical tube

2.4 Criterion for negligible effect of buoyancy on heat transfer

If the parameter $C_B Bo^*_b F_{VP_1} F_{VP_3} F_{VP_4} / F_{VP_2}$ is sufficiently small Nu_b / Nu_{b_o} tends to unity and the effect of buoyancy on heat transfer becomes negligible. Noting that $F_{VP_1} F_{VP_4}$ and F_{VP_2} are of order unity the model equation indicates that for the effect of buoyancy to be less than about 1%, $C_B Bo^*_b F_{VP_3}$ must less than 0.02 and thus a criterion in terms of $Bo^*_b (= Gr^*_b / Re_b^{3.425} Pr_b^{0.8})$ for the effect of buoyancy on heat transfer to be negligible is

$$Bo^*_b < 2 \times 10^{-7} \left(\overline{Pr}/Pr_b\right)^{0.4} \tag{4}$$

For fluids at supercritical pressure the term $(\overline{Pr}/Pr_b)^{0.4}$ can vary significantly and certainly needs to be included in the criterion under certain conditions.

2.5 Criterion for partial laminarisation of a buoyancy-aided flow in heated tube

As can be seen from Figure 1, if the parameter $C_B Bo^*_b F_{VP_1} F_{VP_3} F_{VP_4} / F_{VP_2}$ is increased to about 0.4 severe impairment of heat transfer is encountered for upward flow, with Nu_b / Nu_{b_o} being reduced to about 0.5 due to partial laminarisation. Thus, noting that $F_{VP_1} F_{VP_3}$ and F_{VP_4} are each of order unity, a criterion for partial laminarisation in terms of buoyancy parameter $Bo^*_b (= Gr^*_b / Re_b^{3.425} Pr_b^{0.8})$ is

$$Bo^*_b \approx 4x10^{-6} \tag{5}$$

2.6 Limiting behaviour with very strong buoyancy influence

For conditions of very strong buoyancy influence, $Bo_b^* \approx 10^{-3}$, the model predicts that Nusselt number becomes independent of flow rate. It yields an expression for Nu_b as a function of $Gr^*_b Pr_b$ which has a form similar to that found for turbulent free convection from a vertical surface $Nu_b = C Gr^*_b Pr_b^{0.23} F_{VP}$. The coefficient C is related to C_B and F_{VP} is a property function involving F_{VP_1}, F_{VP_2}, F_{VP_3} and F_{VP_4}.

3. Forced convection in a heated tube with thermally-induced flow acceleration

3.1 A physically-based model

The following physically-based model, which accounts for the effects of thermally-induced bulk flow acceleration on heat transfer for a fluid flowing in a heated tube with imposed wall heat flux, has been developed using ideas similar to those in the model of buoyancy-aided flow in a vertical tube presented in the preceding section.

As a result of energy being supplied to a fluid when it flows inside a heated tube its bulk enthalpy increases. Thus the bulk temperature rises and the density falls. Since the mass flow rate is the same at all axial locations, the fluid must accelerate.

An additional applied pressure difference (over and above that needed to overcome friction) is needed to produce this acceleration. However, within the boundary layer near the tube wall the velocity is lower than the bulk value and so the additional pressure gradient applied there is in excess of that needed to accelerate the fluid.

Thus, the gradient of shear stress in the wall region adjusts to balance the excess pressure gradient and consequently shear stress falls more quickly with distance from the wall than it otherwise would be, turbulence production is reduced and turbulent diffusion of heat is impaired.

3.2 Effect on heat transfer of thermally-induced bulk flow acceleration

A semi-empirical model of heat transfer with thermally-induced flow acceleration based on the proceeding ideas leads to the equation

$$\frac{Nu_b}{Nu_{b_o}} = \left[1 - C_A \frac{Q_b^*}{Re_b^{1.625} Pr_b}\left(\frac{\bar{\mu}}{\mu_b}\right)\left(\frac{\bar{\rho}}{\rho_b}\right)^{-\frac{1}{2}}\left(\frac{Nu_b}{Nu_{b_o}}\right)^{-1.1}\right]^{0.46} \tag{6}$$

Nu_b is Nusselt number for variable property forced convection with thermally-induced bulk flow acceleration and Nu_{b_o} is the value without such acceleration (which can be determined using Equation 2). The ratio Nu_b / Nu_{b_o} provides a measure the magnitude of the effect of thermally-induced flow acceleration on heat transfer. The dimensionless thermal loading group $Q_b^* (= \beta_b q_w d / k_b)$ combines with Re_b and Pr_b to form an acceleration parameter, $Q_b^* / (Re_b^{1.625} Pr_b)$ which, in conjunction with the property ratio term $(\bar{\mu}/\mu_b)/(\bar{\rho}/\rho_b)^{-1/2}$, characterises this effect.

Assigning the symbol Ac_b^* to the parameter, $Q_b / (Re_b^{1.625} Pr_b)$ and F_{VP_1} to the function, $(\bar{\mu}/\mu_b)/(\bar{\rho}/\rho_b)^{-1/2}$, Equation 6 can be re-written in the condensed form

$$\left(\frac{Nu_b}{Nu_{b_o}}\right) = \left[1 - C_A Ac_b^* F_{VP_1}\left(\frac{Nu_b}{Nu_{b_o}}\right)^{-1.1}\right]^{0.46} \tag{7}$$

The coefficient C_A has an estimated value of about 10^4 for an assumed universal turbulent buffer layer thickness δ^+ of 30. The remarks made at the end of Section 2.2 concerning the physically based model of mixed convection and the buoyancy coefficient C_B apply also in the case of this model and the coefficient C_A.

3.3 Behaviour predicted by the model

The variation of Nusselt number ratio, Nu_b / Nu_{b_o}, with $C_A F_{VP_1} Ac_b^*$ predicted by the model is as shown in Figure 2. The full line curve is a physical solution which indicates that heat transfer will be systematically impaired due thermally-induced bulk flow acceleration.

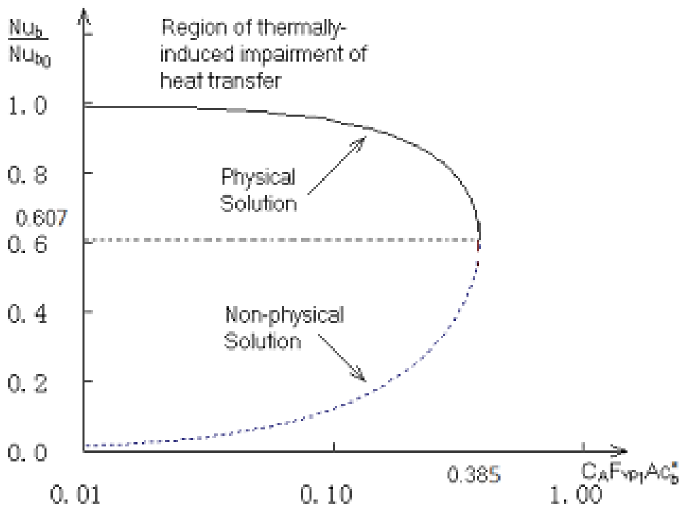

Figure 2 Effect on heat transfer of thermally-induced bulk flow acceleration

3.4 Criterion for the effect of flow acceleration on heat transfer to be negligible

Noting that the F_{VP_1} is of order unity, the criterion for impairment of heat transfer due to thermally-induced bulk flow acceleration to be less than 1% obtained from the equation is $C_A Ac_b^* < 0.02$, in which $Ac_b^* = Q_b^* / (Re_b^{1.625} Pr_b)$. Using the estimated value for the coefficient C_A of 10^4 this gives

$$Ac_b^* < 2x10^{-6} \tag{8}$$

3.5 Criterion for strong impairment of heat transfer due to flow acceleration

With increase of Ac_b^* and, therefore, increase of the parameter $C_A F_{VP_1} Ac_b^*$, the ratio Nu_b / Nu_{b_o} is predicted to decrease systemically due to bulk flow acceleration as a result of turbulence being reduced. Noting that Nu_b / Nu_{b_o} falls to about 0.6 when $C_A F_{VP_1} Ac_b^*$ reaches 0.385 the following criterion for strong impairment of heat transfer due to flow acceleration can be obtained from the equation noting that F_{VP_1} is of order unity

$$Ac_b^* \approx 2 \times 10^{-5} \tag{9}$$

This condition has been described as partial laminarisation.

3.6 Complete laminarisation of the flow

For values of Ac_b^* greater than $2x10^{-5}$ the excess pressure gradient acting on the wall layer is able to overcome the wall shear stress. Thus the core fluid does not experience any shear. The velocity there is uniform and the fluid is simply ‘taking ride’ with the wall layer. The conditions are then just right for the flow to become completely laminar.

3.7 The full picture provided by the model

Taking account of the preceding ideas, Nusselt number ratio Nu_b / Nu_{b_o} should vary with $C_A F_{VP_1} Ac_b^*$ in the manner shown below.

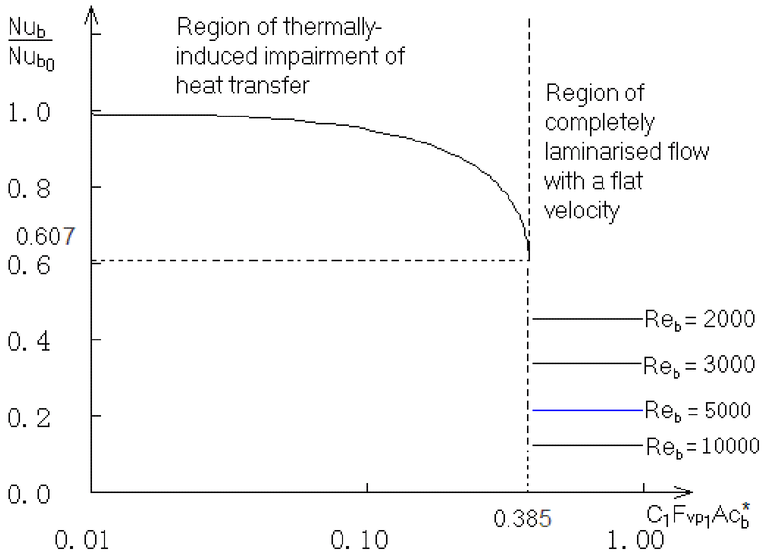

Figure 3 Effect of bulk flow acceleration leading to laminarisation

4. Combined influence of buoyancy and acceleration

The two models can be combined as follows to give an equation which accounts for the effects of both flow buoyancy and acceleration in heated tubes.

$$\frac{Nu_b}{Nu_o} = \frac{Nu_b}{Nu_{b_o}} = \left[1 - \left\{C_A Ac_b^* \pm C_B Bo_b^* \frac{F_{VP_1} F_{VP_2} F_{VP_4}}{F_{TD} F_{VP_2}} \left(\frac{Nu_b}{Nu_{b_o}}\right)\right\} \left(\frac{Nu_b}{Nu_{b_o}}\right)^{-1.1}\right]^{0.46} \qquad (10)$$

The positive sign in the $\pm$ applies for upward flow in a vertical tube and the negative one for downward flow. The ratio Ac_b^* / Bo_b^* is the Froude number u_b^2 / gd.

The models can be used to determine the distribution of wall temperature along a heated tube taking account of buoyancy and bulk flow acceleration. To do this, the mass flow rate and the pressure and temperature of the fluid entering the tube must be specified and also the heat input along the wall. Together, this information enables the distribution of bulk enthalpy h_b and bulk temperature T_b along the tube to be determined using the principle of conservation of energy. Hence values of Re_b, Pr_b, Gr_b^* and Q_b^* can then be found at all locations along the tube.

The Nusselt number Nu_{b_o} for developing, variable property forced convection in the absence of flow acceleration can be found from the empirical equation quoted earlier (Equation 2). The determination of F_{VP_2} in Equation 2 involves knowing T_w as well as T_b so an iterative approach is needed to determine Nu_{b_o} as a function of x/d.

The Nusselt number Nu_b for variable property forced convection with thermally-induced bulk flow acceleration and buoyancy can be expressed in terms of q_w, T_b and T_w. Thus, with T_b, Nu_{b_o}, Ac_b^* and Bo_b^* specified at all values of x/d, Equation 10 is a non-linear algebraic form which enables the distribution of the unknown variable T_w to be determined using standard iterative numerical procedures. The process can be initiated at x=0, where T_w=T_b, and continued step by step along the tube using the converged value of T_w after each step as the initial value at the next axial location.

5. Preliminary application of the models to interpret some recent experimental data

5.1 Introduction and test section details

Some recent experimental work carried out at Tsinghua University, Beijing with supercritical pressure carbon dioxide flowing upward and downward flow in four long, uniformly heated tubes has provided a valuable means of testing the validity of the semi-empirical models described here. In this section a preliminary attempt to use the models to interpret this data. The test sections were of relatively small bore (in the micro/mini range, 0.10mm, 0.27mm, 1.0mm and 2.0mm). The length/diameter ratios were large (400, 350, 150 and 145, respectively).

5.2 Experiments with a micro-tube of diameter 0.10mm

Some experimental results for Re_{in}3800 are shown on Figure 4.

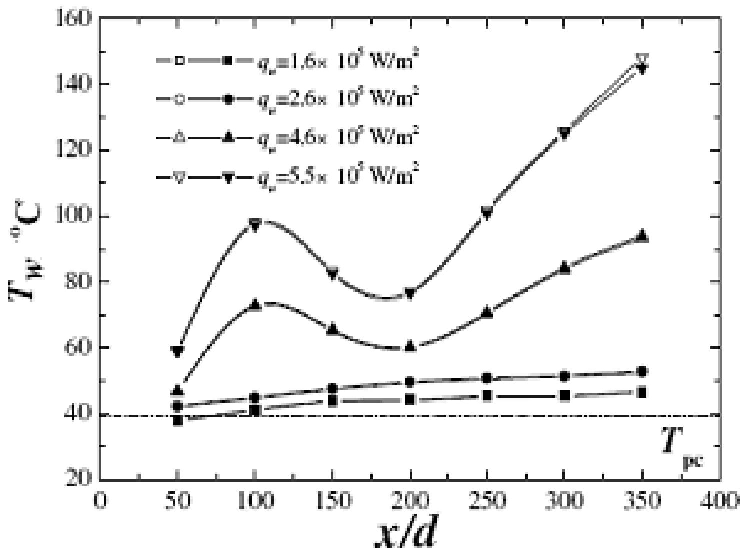

Figure 4 Local wall temperature distributions for upward and downward flow, P=8.80 Mpa, T_{in}=2.5^{o}C, d=0.10 mm, Re_{in}=3800

The values of Bo_b^* are all less than 3x10^{-9}, ie they are very much less than the criterion 10^{-7} for a 1% effect of buoyancy. Consequently, there should be no influence of buoyancy whatsoever and this is confirmed by the fact that there is no observable difference between the results for upward and downward flow. The acceleration parameter Ac_b for the highest heat flux was about 6x10^{-6} and, therefore, the model predicts that some effect of thermally-induced bulk flow acceleration would be expected. The non-monotonic characteristics on the wall temperature distribution for the two highest heat fluxes are indicative of an effect of thermally-induced bulk flow acceleration.

5.3 Experiments with a micro-tube of diameter 0.27mm

Experimental results for a Reynolds number of 2900 are shown below on Figure 5. At this value of Reynolds number of 2900 there is clearly some evidence of buoyancy influence for the two highest values of heat flux by virtue of the differences between the results for upward and downward flow results. For these two values of heat flux the values of Ac_b are in the range $6x10^{-6}$ to $8x10^{-6}$ which is indicative of some effect of thermally induced bulk flow acceleration. The non-monotonic behaviour of both the temperature distributions for the highest heat flux reinforces this idea. So, these results could be useful for the purpose of validating the model for combined influence of buoyancy and acceleration.

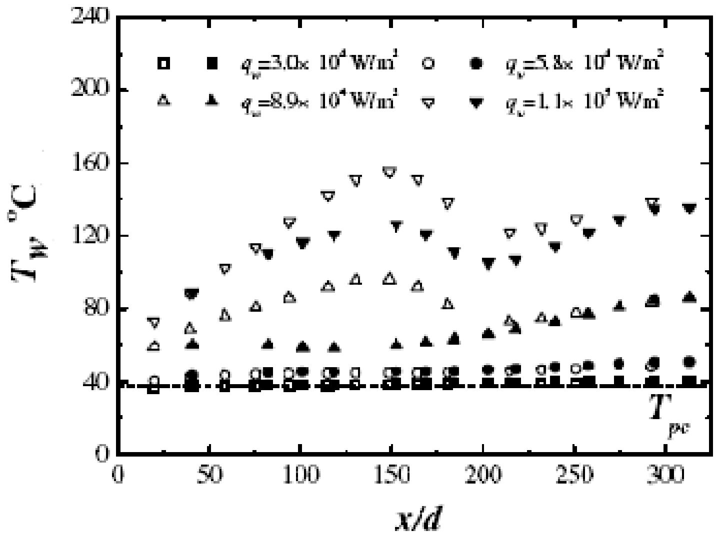

Figure 5 Local wall temperature distributions for upward and downward flow, P=8.60 MPa, T_{in}=30°C, d=0.27 mm, Re_{in}=2900

Results for a higher value of Reynolds number of Re_{in}=10600 are shown on Figure 6.

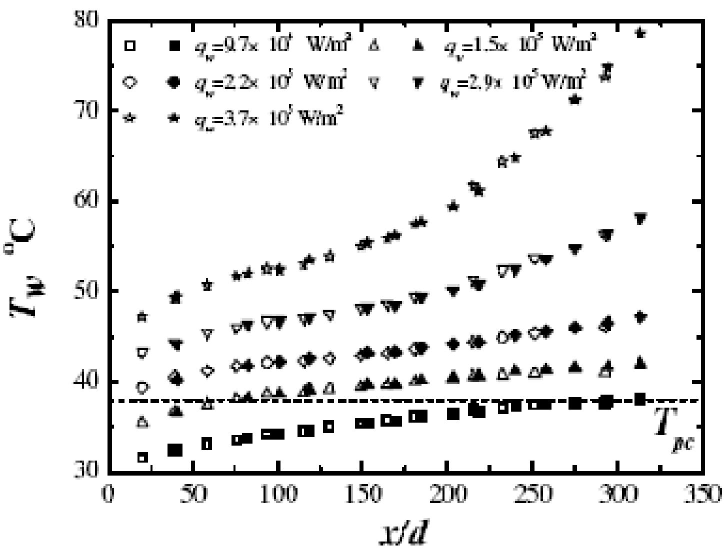

Figure 6 Local wall temperature distributions for upward and downward flow, P=860 MPa, T_{in}=25°C, d=0.27 mm, Re_{in}=10600

For this higher value of Reynolds number of 10600 there is no evidence of any significant influence of buoyancy because the results are very similar for upward and downward flow. Furthermore, there is no evidence of any non-monotonic behaviour and so it seems that no significant deterioration of heat transfer due thermally-induced bulk flow acceleration occurred.

5.4 Experiments with a micro-tube of diameter 1mm

Figure 7 shows experimental results for a Reynolds number, Re_{in}=5500. The experiments for the highest heat fluxes have Bo_b^* values of $2x10^{-6}$ and should, therefore, be influenced by buoyancy. The fact that they are is indicated by the clear difference between results for upward and downward flow. The results for the two lowest values of heat flux have values Bo_b^* have values of about $2x10^{-7}$ and should not be significantly influenced by buoyancy as is confirmed by the agreement between the results for upward and downward flow. The values of Ac_b are low but any influence of acceleration will add to the buoyancy influence for upward flow and the non-monotonic behaviour for that case gives a hint that this could be happening.

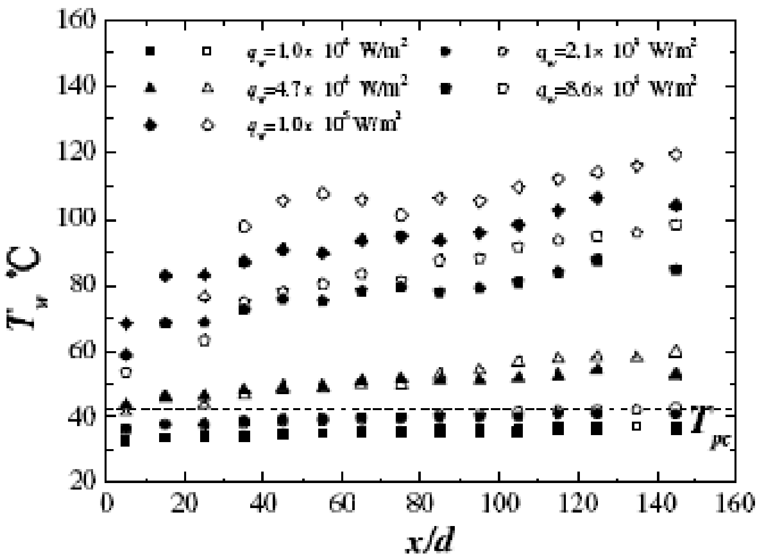

Figure 7 Local wall temperature distributions for upward and downward flow, P=9.50 MPa, T_{in}=30°C, d=1 mm, Re_{in}=5500

5.5 Experiments with a mini-tube of diameter 2mm

Figure 8 shows experimental results for at 2mm diameter mini-tube for Re_{in}=9000. The results for the four highest values of heat flux have Bo_b^* values of $1.9x10^{-6}$, $1.5x10^{-6}$, $1.2x10^{-6}$ and $1.1x10^{-6}$ and should be significantly influenced by buoyancy. That such influences are significant is clearly evident from the observed differences between the distributions for upward and downward flow and also by the forms of the localised peaks evident with upward flow. The values of Bo_b^* of $6x10^{-7}$ and $3x10^{-6}$ for the experiments with lower heat flux suggest only small influences of buoyancy and this is borne out by the similarity of the results for upward and downward flow. The values of the acceleration parameter Ac_b are small and indicate that the effects of thermally induced bulk flow acceleration should be negligible.

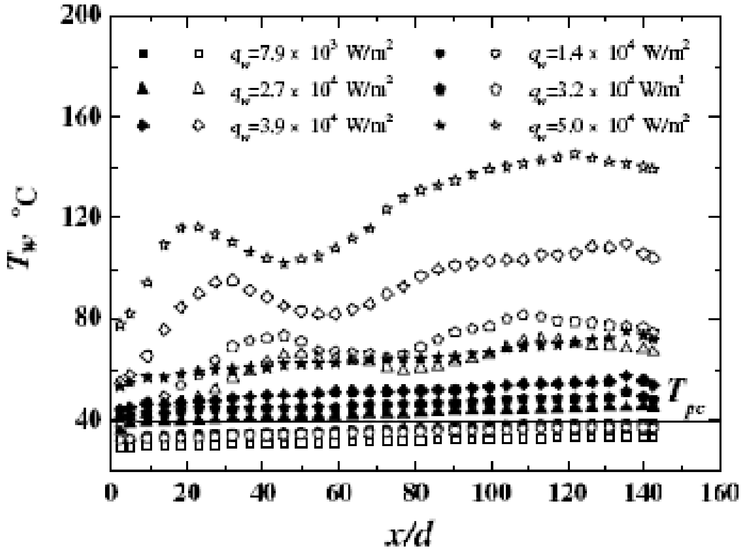

Figure 8 Local wall temperature distributions for upward and downward flow, P=8.80 MPa, T_{in}=25°C, d=2 mm, Re_{in}=9000

Conclusions

In this paper a description is given of two physically-based models of heat transfer, one concerned with mixed convection with non-uniform fluid properties in vertical heated tubes and the other with forced convection with high heat flux and low mass flux leading to thermally-induced bulk flow acceleration. A preliminary demonstration of how such models can be used for the purpose of categorising and interpreting experimental data on heat transfer to carbon dioxide at supercritical data pressure in mini/micro tubes then follows. The work is ongoing and a more detailed evaluation of the approach will be reported in due course.

Nomenclature

Ac^*	Acceleration parameter $(= Q^*/(\mathrm{Re}_b^{1.625}\,\mathrm{Pr}_b))$
c_p	Specific heat at constrant pressure, kJ/kgK
Bo^*	Buoyancy parameter $Gr_b^*/(\mathrm{Re}_b^{3.425}\,\mathrm{Pr}_b^{0.8})$
C_A	Acceleration coefficient
C_B	Buoyancy coefficient
d	Tube diameter, m
g	Acceleration due to gravity, m/s^2
Gr_b^*	Grashof number $(= g\beta_b q_w d^4/(k_b \nu_b^2))$
F_{TD}	Thermal development function (defined below Equation 2)
F_{VP_1}	Variable property function $((\bar{\mu}/\mu_b)(\bar{\rho}/\rho_b)^{-1/2})$
F_{VP_2}	Variable property function $(=(\rho_w/\rho_b)^{0.3}(\bar{c}_p/c_{p_b})^{0.4})$
F_{VP_3}	Variable property function $(=(\overline{\mathrm{Pr}}/\mathrm{Pr}_b)^{-0.4}$
F_{VP_4}	Variable property function $(=(\overline{\rho\beta}/(\rho_b\beta_b))$
k	Thermal conductivity, kW/mK
K	Coefficient in Equation 2

Nu_b	Nusselt number $(= q_w d / (k_b (T_w - T_b)))$
Nu_{b_o}	Nusselt number for variable property forced convection $(= q_w d / (k (T_{w_o} - T_b)))$
P	Pressure, MPa
Pr	Prandtl number $(= \mu c_p / k)$
q_w	Wall heat flux, kW/m^2
Q_b^*	Thermal loading parameter $(= q_w d \beta_b / k_b)$
Re_b	Reynolds number $(= \rho_b u_b d / \mu_b)$
T_b	Local bulk temperature, ^{o}C
T_w	Local wall temperature, ^{o}C
u_b	Local bulk velocity

Greek symbols

β	Thermal expansion coefficient $(= -(1/\rho)(\partial \rho / \partial T)_p)$
μ	Viscosity, kg/ms
ν	Kinematic viscosity, m^2/s
ρ	Density, kg/m^3
δ	Wall buffer layer thickness
τ	Shear stress, N/m^2
τ_w	Wall shear stress, N/m^2

Subscripts

b	Denotes properties evaluated at the bulk temperature
in	Denotes properties evaluated at the inlet fluid temperature
pc	Denotes properties evaluated at the pseudocritical temperature
w	Denotes properties evaluated at the wall temperature

Supercript

-	Denotes property is integrated over the range between the wall and bulk temperature
+	Denotes dimensionless thickness of buffer layer in universal wall coordinate form, $\delta^+ = \rho_b \tau_w^{1/2} \delta / \mu_b$

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