BASIC EQUATIONS OF INTERFACIAL AREA TRANSPORT IN GAS-LIQUID TWO-PHASE FLOW

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Abstract

The rigorous and consistent formulations of basic equations of interfacial area transport were derived using correlation functions of characteristic function of each phase and velocities of each phase. Turbulent transport term of interfacial area concentration was consistently derived and related to the difference between interfacial velocity and averaged velocity of each phase. Constitutive equations of turbulent transport terms of interfacial area concentration were proposed for bubbly flow. New transport model and constitutive equations were developed for churn flow. These models and constitutive equations are validated by experimental data of radial distributions of interfacial area concentration in bubbly and churn flow.

I. Introduction

The two-phase flow phenomenon is playing an important role about safety issues of a nuclear reactor. In order to analyze two-phase flow phenomena, various models such as homogeneous model, slip model, drift flux model and two-fluid model have been proposed. Among these models, the two-fluid model is considered the most accurate model because this model treats each phase separately considering the phase interactions at gas-liquid interfaces. In two-fluid model, averaged conservation equations of mass, momentum and energy are formulated for each phase. The conservation equations of each phase are not independent each other and they are strongly coupled through interfacial transfer terms of mass, momentum and energy through gas-liquid interface. Interfacial transfer terms are characteristic terms in two-fluid model and are given in terms of interfacial area concentration (interfacial area per unit volume of two-phase flow) and driving force. Therefore, the accurate knowledge of interfacial area concentration is quite essential to the accuracy of the prediction based on two-fluid model and a lot of experimental and analytical studies have been made on interfacial area concentration. Recently, more accurate and multidimensional predictions of two-phase flows are needed for advanced design of nuclear reactors. To meet such needs for improved prediction, it becomes necessary to give interfacial area concentration itself by solving the transport equation. In view of above, recently, intensive researches have been carried out on the models, analysis and experiments of interfacial area transport throughout the world. The extensive review on such researches was recently carried out [1,2]. Formulation and modeling of basic transport equation of interfacial area concentration and constitutive equations of the transport equation have been carried out by various researchers. Morel [3] derived the transport equation of averaged interfacial area concentration as given by

$$\frac{\partial \overline{a_i}}{\partial \mathbf{t}} + \nabla \bullet \overline{a_i \mathbf{V_i}} = \overline{a_i (\mathbf{V_i} \bullet \mathbf{n_{Gi}}) \nabla \bullet \mathbf{n_{Gi}}} \tag{1}$$

Here, $\overline{}$ denotes averaging and $\overline{\mathbf{V_i}}$ is the averaged velocity of interface. The research group directed by Prof. Ishii in Purdue university derived the transport equation of interfacial area concentration of averaged interfacial area concentration based of the transport equation of number density function of bubbles [4]. It is given by

$$\frac{\partial \overline{a_i}}{\partial t} + \nabla \bullet \overline{a_i} \overline{V_i} = \frac{2}{3}\left(\frac{\overline{a_i}}{\alpha}\right)\left(\frac{\partial \alpha}{\partial t} + \nabla \bullet \alpha \overline{\overline{v_G}}\right) + \sum_{j=1}^{4} \phi_j + \phi_{ph} \tag{2}$$

where α is void fraction and $\overline{\overline{v_G}}$ is averaged velocity of gas phase. Here, the terms in right hand side of Eq.(2) represent the source and sink terms due to change in bubble volume, bubble coalescence and break up and phase change. The constitutive equations are given by [5]based on detailed mechanistic modeling.

In strict meaning, Eqs. (1) and (2) are "conservation equations of interfacial area concentration". They are not "transport equations of interfacial area concentration" by themselves. In order to express the transport of interfacial area concentration, it is necessary to formulate the conservation equation of interfacial velocity at the gas-liquid interface. However, at present, most researchers used equations (1) and (2) as "transport equation of interfacial area" assuming interfacial velocity is identical to average velocity of gas phase.

In the present paper, a new and rigorous modeling of basic transport equation and constitutive equations of turbulent transport terms of interfacial area concentration was carried out. Further modeling of turbulent transport terms of interfacial area concentration was carried out for bubbly flow and churn flow. Based on this modeling, radial distributions of interfacial area concentration in bubbly and churn flow are predicted with successful agreements of experimental data.

2. Local Instant Formulation of Interfacial Area Concentration

Interfacial area concentration is defined as interfacial area per unit volume of two-phase flow. Therefore, the term "interfacial area concentration" is usually used in the meaning of averaged value and denoted by $\overline{a_i}$. The transport equation of interfacial area concentration is given in averaged form in terms of averaged interfacial area concentration, $\overline{a_i}$. However, for the rigorous derivation of the transport equation, it is desirable to formulate interfacial area concentration and its transport equation in local instant form. Kataoka et al. [6,7] and Morel [3] derived the local instant formulation of interfacial area concentration as follows. As Shown in Fig.1, interface of gas and liquid is mathematically given by

$$f(x,y,z,t)=0 \tag{3}$$

Using this function, local instant interfacial area concentration (denoted by a_i) is formulated by

$$a_i=|\mathrm{grad}\, f(x,y,z,t)|\delta(f(x,y,z,t)) \tag{4}$$

where $\delta(w)$ is the delta function which is defined by

$$\int_{-\infty}^{\infty} g(w)\delta(w - w_0)dw = g(w_0) \tag{5}$$

where g(w) is an arbitrary continuous function.

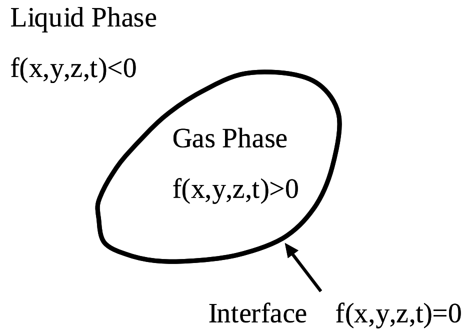


Fig.1 Mathematical representation of interface

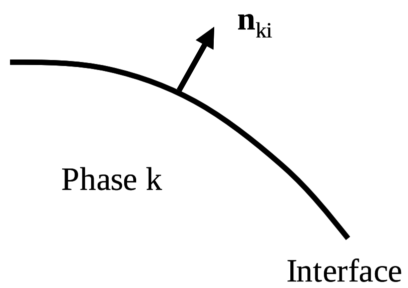


Fig.2 Unit normal outward vector of phase k

In relation to local instant interfacial area concentration, characteristic function of each phase (denoted by ϕ_k) is defined by

$$\phi_G = h(f(x,y,z,t)) \quad \text{(gas phase)} \tag{6}$$

$$\phi_L = 1 - h(f(x,y,z,t)) \quad \text{(liquid phase)} \tag{7}$$

where suffixes G and L denote gas and liquid phase respectively. ϕ_k is the local instant void fraction of each phase and takes the value of unity when phase k exists and takes the value of zero when phase k doesn't exist. Here, h(w) is Heaviside function which is defined by

$$\begin{aligned} h(w) &= 1 \quad (w>0) \\ &= 0 \quad (w<0) \end{aligned} \tag{8}$$

Heaviside function and the delta function are related by

$$\delta(w) = \frac{dh(w)}{dw} \tag{9}$$

Using above equations, the derivatives of characteristic function are related to interfacial area concentration as follows.

$$\mathrm{grad}\,\phi_k = -\mathbf{n}_{ki} a_i \quad (k = G, L) \tag{10}$$

$$\frac{\partial \phi_k}{\partial t} = \mathbf{v}_i \bullet \mathbf{n}_{ki} a_i \quad (k = G, L) \tag{11}$$

Here, $\mathbf{n}_{ki}$ is unit normal outward vector of phase k as shown in Fig.2 and $\mathbf{v}_i$ is the velocity of interface. Using above-mentioned relations, it is shown that local instant interfacial area concentration is given in term of correlation function of characteristic function as [8]

$$a_i = -\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\{\frac{\partial}{\partial \mathbf{r}}\phi_k(\mathbf{x}) - 2\phi_k(\mathbf{x}+\mathbf{r})\frac{\partial}{\partial \mathbf{r}}\phi_k(\mathbf{x})\}\sin\theta d\theta d\eta \tag{12}$$

Here, $\partial/\partial\mathbf{r}$ is directional differentiation of characteristic function $\phi_k(\mathbf{x})$ in $\mathbf{r}$ direction. Using the definition of differentiation, Eq. (12) can be rewritten by

$$a_i = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\} + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})}{|r|}\sin\theta d\theta d\varphi \tag{13}$$

3. Basic Equation of Interfacial Velocity and Transport Equation of Interfacial Area

If there is no phase change or rate of phase change at interface is sufficiently small, the interfacial velocity is equal to the velocity of each phase at interface by

$$\mathbf{v}_i = \mathbf{v}_{Li} = \mathbf{v}_{Gi} \tag{14}$$

In what follows, the most simple case is considered where Eq.(14) is valid. One considers two spatial locations denoted by **x** and **x**+**r**, which are close to each other. There are two cases where gas liquid interface exists between **x** and **x**+**r**. They are shown in Figs 3. Since two locations are very close to each other, it is assumed that only one interface can exist.

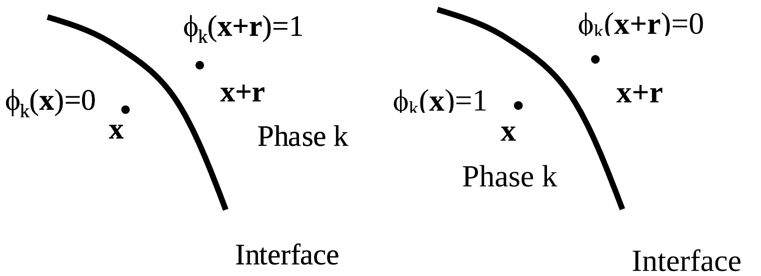


Fig.3 The case where interface exists between **x** and **x**+**r**. (**x** is in phase k)

In each case, interfacial velocity is approximated by the velocity of phase k at the point where phase k exists. When the displacement, **r,** is sufficiently small, the interfacial velocity, $\mathbf{V}_i$ is given by

$$\mathbf{V}_i = \lim_{|\mathbf{r}|\to 0}[\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}\mathbf{v}_k(\mathbf{x}) + \{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})\mathbf{v}_k(\mathbf{x}+\mathbf{r})] \tag{15}$$

Combining Eqs.(13) and (15) and averaging, one obtains.

$$\overline{\mathbf{V}_i a_i} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|\mathbf{r}|\to 0}\frac{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}\mathbf{v}_k(\mathbf{x})} + \overline{\{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})\mathbf{v}_k(\mathbf{x}+\mathbf{r})}}{|\mathbf{r}|}\sin\theta d\theta d\varphi \tag{16}$$

Here, averaged interfacial velocity, $\overline{\mathbf{V}_i}$ is defined by

$$\overline{\mathbf{V}_i} = \frac{\overline{\mathbf{V}_i a_i}}{\overline{a_i}} \tag{17}$$

Averaging Eq.(12), one obtains.

$$\overline{a_i} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|\mathbf{r}|\to 0}\frac{\overline{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x}+\mathbf{r})\}} + \overline{\{1-\phi_k(\mathbf{x})\}\phi_k(\mathbf{x}+\mathbf{r})}}{|\mathbf{r}|}\sin\theta d\theta d\varphi \tag{18}$$

Using, Eqs.(16) and (18), the difference between averaged interfacial velocity, $\overline{\mathbf{V}_i}$ and average velocity of phase k, $\overline{\overline{\mathbf{v}_k}}$ is given by

$$\begin{aligned}
(\overline{V_i} - \overline{\overline{v_k}})\overline{a_i} &= \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{\overline{\phi_k(x)\{1-\phi_k(x+r)\}\{v_k(x)-\overline{\overline{v_k(x)}}\}} + \overline{\{1-\phi_k(x)\}\phi_k(x+r)\{v_k(x+r)-\overline{\overline{v_k(x)}}\}}}{|r|}\sin\theta d\theta d\varphi \\
&= \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{-\overline{\phi_k(x)\phi_k(x+r)\{v_k(x)-\overline{\overline{v_k(x)}}\}} - \overline{\phi_k(x)\phi_k(x+r)\{v_k(x+r)-\overline{\overline{v_k(x)}}\}}}{|r|}\sin\theta d\theta d\varphi \\
&= \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{-\overline{\phi_k(x)\{\phi_k(x+r)-\overline{\phi_k(x+r)}\}\{v_k(x)-\overline{\overline{v_k(x)}}\}} - \overline{\{\phi_k(x)-\overline{\phi_k(x)}\}\phi_k(x+r)\{v_k(x+r)-\overline{\overline{v_k(x)}}\}}}{|r|}\sin\theta d\theta d\varphi \\
&\quad - \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{\overline{\phi_k'(x)\phi_k(x+r)v_k'(x+r)} + \overline{\phi_k(x)\phi_k'(x+r)v_k'(x)}}{|r|}\sin\theta d\theta d\varphi
\end{aligned} \tag{19}$$

Here, $\phi_k{}'$ and $\mathbf{v}_k{}'$ are fluctuating terms of local instant volume fraction and velocity of phase k which are given by

$$\mathbf{v}_k{}' = \mathbf{v}_k - \overline{\overline{\mathbf{v}}}_k , \qquad \phi_k{}' = \phi_k - \overline{\phi}_k \tag{20}$$

In deriving Eq.(19), Following relations were used.

$$\overline{\phi_k(x)\{v_k(x)-\overline{\overline{v_k(x)}}\}} = 0 , \qquad \overline{\phi_k(x+r)\{v_k(x+r)-\overline{\overline{v_k(x+r)}}\}} = 0 \tag{21}$$

Equation (19) indicates that the difference between averaged interfacial velocity, $\overline{\mathbf{V}_i}$ and average velocity of phase k, $\overline{\overline{\mathbf{v}}}_k$ is given in terms of correlations between fluctuating terms of local instant volume fraction and velocity of phase k which are related to turbulence terms of phase k. The governing equation of the correlation term given by Eq.(19) is derived based on the local instant basic equations of mass conservation and momentum conservation of phase k [9]. In these conservation equations, tensor representation is used. Einstein abbreviation rule is also applied. When the same suffix appear, summation for that suffix is carried out except for the suffix k denoting gas and liquid phases.

$$
\begin{aligned}
&\frac{\partial}{\partial t}\{\overline{(\overline{\mathbf{V}}_i-\overline{\overline{\mathbf{v}}}_k)a_i}\}+\frac{\partial}{\partial x_\beta}(\overline{(\overline{\mathbf{V}}_i-\overline{\overline{\mathbf{v}}}_k)a_i}\,\overline{\overline{v}}_{k\beta})\\
&=\frac{1}{\rho_k}\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\overline{\phi_k'\phi_{kr}\frac{\partial P_{kr}'}{\partial x_\alpha}}+\overline{\phi_k\phi_{kr}'\frac{\partial P_k'}{\partial x_\alpha}})\sin\theta d\theta d\varphi\\
&-\frac{1}{\rho_k}\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}\{\overline{\phi_k'\phi_{kr}\frac{\partial}{\partial x_\beta}(\tau_{k\alpha\beta r}'+\rho_k\overline{\overline{v_{k\alpha r}'v_{k\beta r}'}})}+\overline{\phi_k\phi_{kr}'\frac{\partial}{\partial x_\beta}(\tau_{k\alpha\beta}'+\rho_k\overline{\overline{v_{k\alpha}'v_{k\beta}'}})}\}\sin\theta d\theta d\varphi\\
&-\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\overline{\phi_k'\phi_{kr}F'_{k\alpha r}}+\overline{\phi_k\phi_{kr}'F'_{k\alpha}})\sin\theta d\theta d\varphi\\
&-\frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\frac{\overline{\phi_k'\phi_{kr}}}{\overline{\phi}_{kr}}\frac{1}{\rho_k}\overline{P_{ki}'n_{k\alpha i}a_{ir}}+\frac{\overline{\phi_k\phi_{kr}'}}{\overline{\phi}_k}\frac{1}{\rho_k}\overline{P_{ki}'n_{k\alpha i}a_i})\sin\theta d\theta d\varphi\\
&+\frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\frac{\overline{\phi_k'\phi_{kr}}}{\overline{\phi}_{kr}}\frac{1}{\rho_k}\overline{\tau_{ki}'n_{k\alpha i}a_{ir}}+\frac{\overline{\phi_k\phi_{kr}'}}{\overline{\phi}_k}\frac{1}{\rho_k}\overline{\tau_{ki}'n_{k\alpha i}a_i})\sin\theta d\theta d\varphi\\
&+\frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\frac{\overline{\phi_k'\phi_{kr}}}{\overline{\phi}_{kr}}\overline{\overline{v_{k\alpha r}'v_{k\beta r}'}}\,\overline{n_{k\alpha i}a_{ir}}+\frac{\overline{\phi_k\phi_{kr}'}}{\overline{\phi}_k}\overline{\overline{v_{k\alpha}'v_{k\beta}'}}\,\overline{n_{k\alpha i}a_i})\sin\theta d\theta d\varphi\\
&+\frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}\{\frac{\partial}{\partial x_\beta}(\overline{\phi_k'\phi_{kr}v_{k\alpha r}'v_{k\beta r}'})+\frac{\partial}{\partial x_\beta}(\overline{\phi_k\phi_{kr}'v_{kr}'v_{k\beta}'})\}\sin\theta d\theta d\varphi\\
&+\frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\overline{\phi_k'\phi_{kr}v_{k\beta r}'}\frac{\partial\overline{\overline{v}}_{k\beta r}}{\partial x_\beta}+\overline{\phi_k\phi_{kr}'v_{k\beta}'}\frac{\partial\overline{\overline{v}}_{k\beta}}{\partial x_\beta})\sin\theta d\theta d\varphi\\
&-\frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}\{\overline{v_{k\alpha r}'(\overline{\overline{v}}_{k\beta r}+v_{k\beta r}')\phi_{kr}\frac{\partial}{\partial x_\beta}\phi_k'}+\{\overline{v_{k\alpha}'(\overline{\overline{v}}_{k\beta}+v_{k\beta}')\phi_k\frac{\partial}{\partial x_\beta}\phi_{kr}'}\}\sin\theta d\theta d\varphi\\
&+\frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}\{\overline{v_{k\alpha r}'\phi_{kr}\frac{\partial}{\partial x_\beta}(\phi_r v_{k\beta}'+\phi_k'\overline{\overline{v}}_{k\beta r})}+\overline{v_{k\alpha}'\phi_k\frac{\partial}{\partial x_\beta}(\phi_{kr}v_{k\beta r}'+\phi_{kr}'\overline{\overline{v}}_{k\beta})}\}\sin\theta d\theta d\varphi
\end{aligned}
\tag{22}
$$

In the derivation, following simplified representation of quantities at the location **x**+**r.**

$$\phi_k(\mathbf{x}+\mathbf{r}) = \phi_{k\mathbf{r}} \quad , \quad A_k(\mathbf{x}+\mathbf{r}) = A_{k\mathbf{r}} \tag{23}$$

As shown above, the rigorous formulation of governing equation of interfacial velocity is derived. Then, the most strict formulation of transport equations of interfacial area concentration is given by conservation equation of interfacial area concentration (Eq.(1) or Eq(2)) and conservation equation of interfacial velocity (Eq.(22)). As shown in Eq.(22), the conservation equation of interfacial velocity consists of various correlation terms of fluctuating terms of velocity and local instant volume fractions. These correlation terms represent the turbulent transport of interfacial area, which reflects the interactions between gas liquid interface and turbulence of gas and liquid phases. Equation (22) rigorously represents such turbulence transport terms of interfacial area concentration. Accurate predictions of interfacial area transport can be possible by solving the transport equations derived here.

IV. Constitutive Equations of Turbulent Transport of Interfacial Area

As shown in the previous section, the rigorous formulation of transport equation of interfacial area concentration are given by conservation equation of interfacial area concentration (Eq.(1) or Eq(2)) and conservation equation of interfacial velocity (Eq.(22)). However, Eq.(22) consists of complicated correlation terms of fluctuating terms of local instant volume fraction, velocity, pressure and shear stress. The detailed knowledge of these correlation terms is not available. Therefore, solving Eq.(22) together with basic equations of two-fluid model is difficult at present. More detailed analytical and experimental works on turbulence transport terms of interfacial area concentration are necessary for solving practically Eq.(22). From Eq.(19), Interfacial velocity is related to averaged velocity of phase k (gas phase or liquid phase)by following equation.

$$\overline{\overline{\mathbf{V}_i}}\,\overline{a_i} = \overline{\overline{\mathbf{v}_k}}\,\overline{a_i} - \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi} \lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\overline{\phi'_k\phi_{k\mathbf{r}}\mathbf{v}'_{k\mathbf{r}}} + \overline{\phi_k\phi'_{k\mathbf{r}}\mathbf{v}'_k})\sin\theta d\theta d\varphi \tag{24}$$

When one considers bubbly flow and phase k is gas phase, Eq.(24) can be rewritten by

$$\overline{\overline{\mathbf{V}_i}}\,\overline{a_i} = \overline{\overline{\mathbf{v}_G}}\,\overline{a_i} - \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi} \lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\overline{\phi'_G\phi_{G\mathbf{r}}\mathbf{v}'_{G\mathbf{r}}} + \overline{\phi_G\phi'_{G\mathbf{r}}\mathbf{v}'_G})\sin\theta d\theta d\varphi \tag{25}$$

From Eqs.(12) and (18) the term, $\phi'_G/|\mathbf{r}|$ is related to the fluctuating term of interfacial area concentration at the location, **x** and the term, $\phi_{G\mathbf{r}}\mathbf{v}'_{G\mathbf{r}}$ is the fluctuating term of gas phase velocity at the location, **x**+**r.** Also, the term, $\phi'_{G\mathbf{r}}/|\mathbf{r}|$ is related to the fluctuating term of interfacial area concentration at the location, **x**+**r** and the term, $\phi_G\mathbf{v}'_G$ is the fluctuating term of gas phase velocity at the location, **x.** Then, the second term of right hand side of Eq.(25) is considered to correspond to turbulent transport term due to the turbulent velocity fluctuation. In analogous to the turbulent transport of momentum, energy (temperature) and mass, the correlation term described above is assumed to be proportional to the gradient of interfacial area concentration which is transported by turbulence (diffusion model). Then, one can assume following relation.

$$-\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi} \lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\overline{\phi'_G\phi_{Gr}\mathbf{v}'_{Gr}} + \overline{\phi_G\phi'_{Gr}\mathbf{v}'_G})\sin\theta d\theta d\varphi = -D_{ai}\,\mathrm{grad}\,\overline{a_i} \tag{26}$$

Here, the coefficient, D_{ai} is considered to correspond to turbulent diffusion coefficient of interfacial area concentration. In analogy to the turbulent transport of momentum, energy (temperature) and mass, this coefficient is assumed to be given by

$$D_{ai} \propto |\mathbf{v}'_G| L \tag{27}$$

Here, L is the length scale of turbulent mixing of gas liquid interface and $|\mathbf{v}'_G|$ is the turbulent velocity of gas phase. In bubbly flow, it is considered that length scale of turbulent mixing of gas liquid interface is proportional to bubble diameter, d_B and length scale of the turbulent velocity of gas phase is proportional to the turbulent velocity of liquid phase. These assumptions were confirmed by experiment and analysis of turbulent diffusion of bubbles in bubbly flow [10]. Therefore, turbulent diffusion coefficient of interfacial area concentration is assumed by following equation.

$$D_{ai} = K_1|v'_L|d_B = 6K_1\frac{\alpha}{a_i}|v'_L| \tag{28}$$

Here, α is the averaged void fraction and $|\mathbf{v}'_L|$ is the turbulent velocity of liquid phase. K_1 is empirical coefficient. For the case of turbulent diffusion of bubble, experimental data were well predicted assuming K_1=1/3. For the case of turbulent diffusion of interfacial area concentration, there are no direct experimental data of turbulent diffusion. However, the diffusion of bubble is closely related to the diffusion of interfacial area (surface area of bubble). Therefore, as first approximation, the value of k for bubble diffusion can be applied to diffusion of interfacial area concentration in bubbly flow.

Equations (28) is based on the model of turbulent diffusion of interfacial area concentration. In this model, it is assumed that turbulence is isotropic. However, in the practical two-phase flow in the flow passages turbulence is not isotropic and averaged velocities and turbulent velocity have distribution in the radial direction of flow passage. In such non-isotropic turbulence, the correlation terms of turbulent fluctuation of velocity and interfacial area concentration given by Eq.(27) is largely dependent on anisotropy of turbulence field. Such non-isotropic turbulence is related to the various terms consisting of turbulent stress which appear in the right hand side of Eq.(22). Assuming that turbulent stress of gas phase is proportional to that of liquid phase and turbulence model in single phase flow, turbulent stress is given by

$$\overline{\overline{\mathbf{v}'_L\mathbf{v}'_L}} = -\varepsilon_{LTP}\{\nabla\overline{\overline{\mathbf{v}_L}} + {}^t(\nabla\overline{\overline{\mathbf{v}_L}})\} + \frac{2}{3}k\delta_{ij} \tag{29}$$

Here, ε_{LTP} is the turbulent diffusivity of momentum in gas-liquid two-phase flow. For bubbly flow, this turbulent diffusivity is given by various researchers [11] proposed the following correlation.

$$\varepsilon_{LTP} = \frac{1}{3}\alpha d_B v'_L \tag{30}$$

Based on the model of turbulent stress in gas-liquid two-phase flow and Eq.(22), it is assumed that turbulent diffusion of interfacial area concentration due to non-isotropic turbulence is proportional to the velocity gradient of liquid phase. For the diffusion of bubble due to non-isotropic turbulence in bubbly flow in pipe, Kataoka et al. [11] proposed the following correlation based on the analysis of radial distributions of void fraction and bubble number density.

$$J_B = K_2 \alpha d_B n_B \frac{\partial \overline{\overline{v_L}}}{\partial y} \tag{31}$$

Here, J_B is the bubble flux in radial direction and n_B is the number density of bubble. y is radial distance from wall of flow passage. K_2 is empirical correlation and experimental data were well predicted assuming K_2=10. In analogous to Eq.(31), it is assumed that turbulent diffusion of interfacial area concentration due to non-isotropic turbulence is given by following equation.

$$J_{ai} = K_2 \alpha d_B a_i \frac{\partial \overline{\overline{v_L}}}{\partial y} \tag{32}$$

Here, J_{ai} is the flux of interfacial area concentration in radial direction. Equation (32) can be interpreted as equation of bubble flux due to the lift force due to liquid velocity gradient.
As shown above, turbulent diffusion of interfacial area concentration due to non-isotropic turbulence is related to the gradient of averaged velocity of liquid phase and using analogy to the lift force of bubble, Eq.(26) can be rewritten in three dimensional form by

$$-\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi} \lim_{|\mathbf{r}|\to 0}\frac{1}{|\mathbf{r}|}(\overline{\phi'_G \phi_{Gr} \mathbf{v}'_{Gr}} + \overline{\phi_G \phi'_{Gr} \mathbf{v}'_G})\sin\theta d\theta d\varphi = -D_{ai}\mathrm{grad}\overline{a_i} + C\alpha a_i(\overline{\overline{\mathbf{v}_G}} - \overline{\overline{\mathbf{v}_L}})\times \mathrm{rot}(\overline{\overline{\mathbf{v}_L}}) \tag{33}$$

Empirical coefficient C in the right hand side of Eq.(33) should be determined based on the experimental data of spatial distribution of interfacial area concentration and averaged velocity of each phase. However, at present, there are not sufficient experimental data. Therefore, as first approximation, the value of coefficient C can be given by Eq.(32)

$$C = K_2 d_B / u_R \qquad \text{(based on Eq.(32))} \tag{34}$$

Using Eqs(22) and (33), transport equation of interfacial area concentration Eq.(2) can be given by following equation for gas-liquid two-phase flow where gas phase is dispersed in liquid phase for bubbly flow.

$$\frac{\partial \overline{a_i}}{\partial t} + \mathrm{div}(\overline{a_i}\overline{\overline{\mathbf{v}_G}}) = \mathrm{div}(D_{ai}\mathrm{grad}\overline{a_i}) - \mathrm{div}\{C\alpha a_i(\overline{\overline{\mathbf{v}_G}} - \overline{\overline{\mathbf{v}_L}})\times \mathrm{rot}(\overline{\overline{\mathbf{v}_L}})\} + \frac{2}{3}\frac{\overline{a_i}}{\alpha\rho_G}\left(\Gamma_G - \alpha\frac{D\rho_G}{Dt}\right) + \phi_{CO} + \phi_{BK} \tag{35}$$

Here, D/Dt denotes material derivative following the gas phase motion and turbulent diffusion coefficient of interfacial area concentration, D_{ai} is given by Eq.(28). Coefficient of turbulent diffusion of interfacial area concentration due to non-isotropic turbulence, C is given by Eq.(34). The third term in the right hand side of Eq.(35) is source term of interfacial area concentration due to phase change and density change of gas phase due to pressure change. Γ_G is the mass generation rate of gas phase per unit volume of two-phase flow due to evaporation. ϕ_{CO} and ϕ_{Bk} are sink and source term due to bubble coalescence and break up These source and sink terms are given by Hibiki and Ishii [3] based on detailed analysis and experiment of interfacial area concentration.

Here, in the most simplified case, we consider two-phase flow in pipe under steady state and developed region without phase change, where coalescence and break up of bubbles are negligible. Under such assumptions, transport equation of interfacial area concentration based on turbulent transport model, Eq.(35) can be simplified and given by following equation.

$$K_1 d_B |v'_L| \frac{1}{R-y}\frac{\partial}{\partial \mathbf{y}}\left((R-y)\frac{\partial \overline{a_i}}{\partial y}\right) + K_2 \alpha d_B \overline{a_i}\, \frac{1}{R-y}\frac{\partial}{\partial \mathbf{y}}\left((R-y)\frac{\partial \overline{\overline{V_L}}}{\partial y}\right) = 0 \tag{36}$$

Here, R is pipe radius and y is distance from pipe wall. Kataoka's model for turbulent diffusion of interfacial area concentration, (Eqs.(28) and (34)) was used.

For churn flow, additional turbulence void transport terms appear due to the wake of large babble as schematically shown in Fig.4.

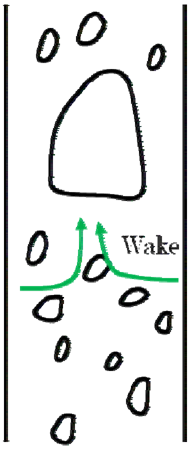


Fig.4 .Wake in Churn Flow Regime

For interfacial area transport due to wake of churn bubble, interfacial area is transported toward the center of pipe. The flux of interfacial area concentration in radial direction J_{ai}, due to churn bubble is related to the terminal velocity of churn bubble. The flux of interfacial area concentration toward the center of pipe is large at near wall and small at the center of pipe. Then, it is simply assumed to be proportional to the distance from pipe center. Finally, the flux of interfacial area concentration in radial direction, J_{ai} due to churn bubble is assumed to be given by

$$J_{ai} = K_{Cai}\frac{R-y}{R}\{0.35\sqrt{gD}\}\overline{a_i} \tag{37}$$

Then, transport equation of interfacial area concentration based on turbulent transport model in churn flow is given by

$$\frac{1}{R-y}\frac{\partial}{\partial \mathbf{y}}\left((R-y)K_1 d_B |v'_L|\frac{\partial \overline{a_i}}{\partial y}\right) +_c \frac{1}{R-y}\frac{\partial}{\partial \mathbf{y}}\left((R-y)^2 K_{Cai}\frac{0.35\sqrt{gD}}{R}\overline{a_i}\right) = 0 \tag{38}$$

In order to predict radial distribution of interfacial area concentration using Eq.(36) or Eq.(38), radial distributions of void fraction, averaged liquid velocity and turbulent liquid velocity are needed. These distributions were already predicted based on the turbulence model of two-phase flow for bubbly flow and churn flow [12].

Using transport equation of interfacial area concentration for bubbly flow (Eq.(36) and churn flow (Eq.(38), the radial distributions of interfacial area concentration are predicted and compared with experimental data. Serizawa et al., [1] measured distributions of void fraction, interfacial area concentration, averaged liquid velocity and turbulent liquid velocity for vertical upward air-water two-phase flow in bubbly and churn flow regimes in round tube of 60mm diameter. Void fraction and interfacial area were measure by electrical resistivity probe and averaged liquid velocity and turbulent liquid velocity were measured by anemometer using conical type film probe with quartz coating. Their experimental conditions are

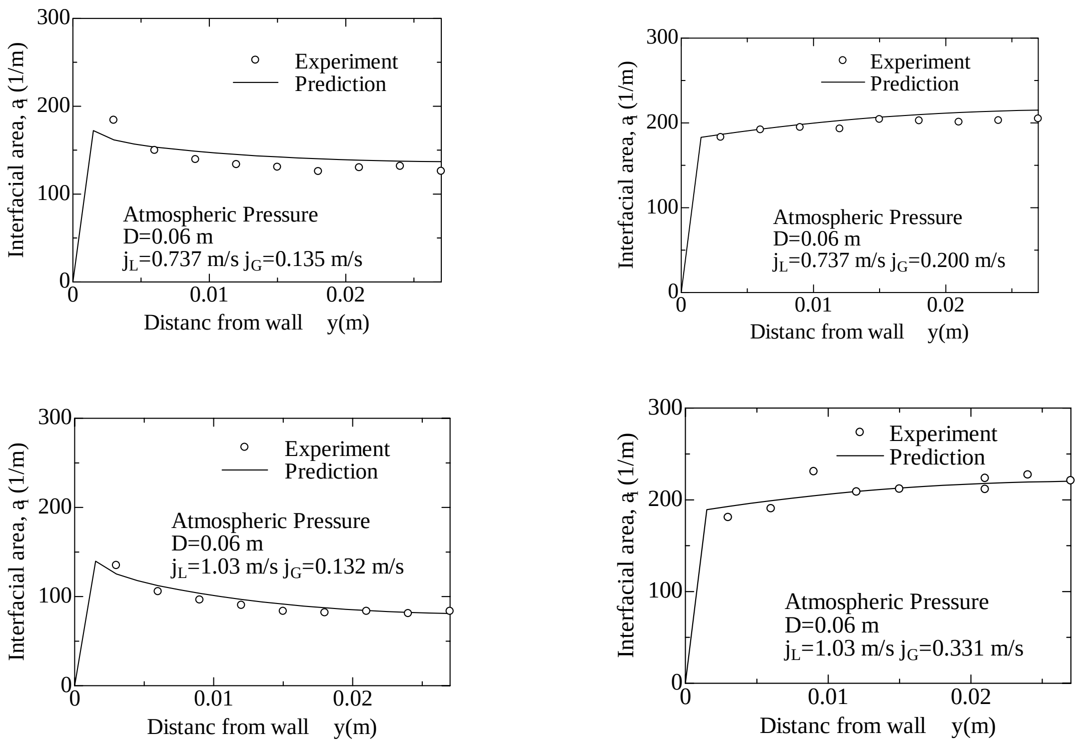
Liquid flux, J_L: 0.44 - 1.03 m/s

Gas flux, J_G: 0 - 0.403 m/s

As bubble diameter, 4mm is used based on experimental observation. For empirical coefficient, K_{cai} is assumed to be 0.01 based on experimental data. The condition of flow regime transition from bubbly to churn flow is given in terms of averaged void fraction, $\overline{\alpha}$ based on experimental results which is given by

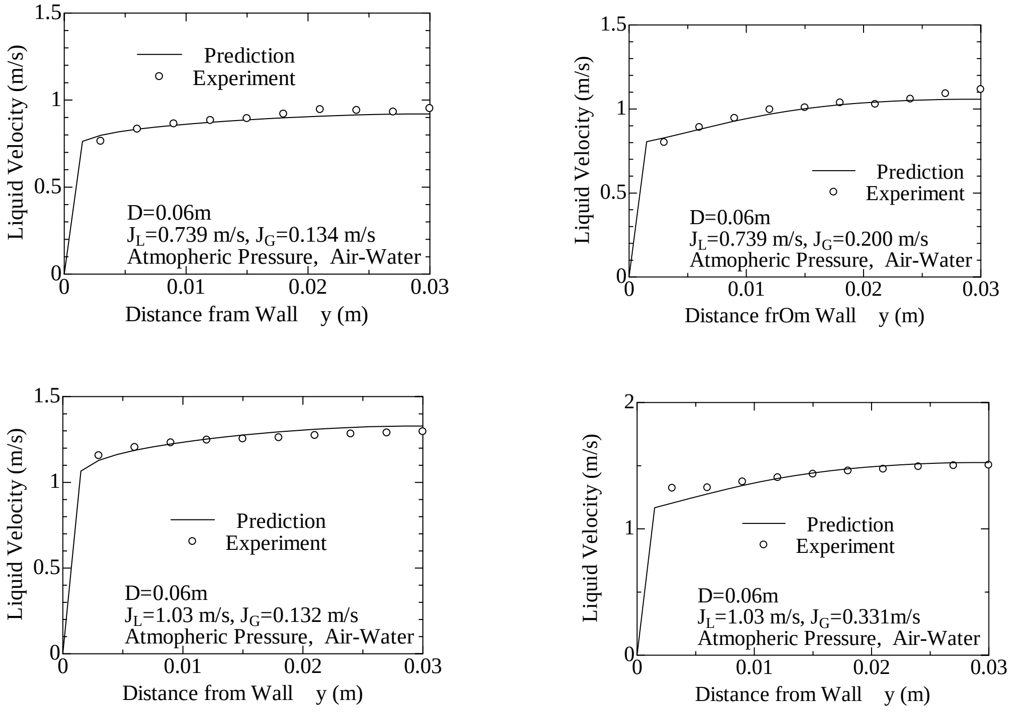
$$\overline{\alpha} = 0.2 \tag{15}$$

Figure 5 shows some examples of the comparison between experimental data and prediction of radial distributions of interfacial area concentration in bubbly flow and churn flow. In bubbly flow regime, distributions of interfacial area concentration show wall peak of which magnitude is larger for larger liquid flux whereas distributions interfacial area concentration show core peak. The prediction based on the present model well reproduces the experimental data. Figures 6 and 7 show the comparison between experimental data and prediction of radial distributions of averaged liquid velocity and turbulent velocity for bubbly and churn flow in the same flow conditions as Fig5 [13]. As shown in these figures, turbulent structures are also well predicted by present model both for bubbly and churn flow. The analyses and experiments of void fraction distributions showed the similar trends as those of interfacial area concentration (wall peak in bubbly flow and core peak in churn flow). The details of the results were presented at elsewhere [13].



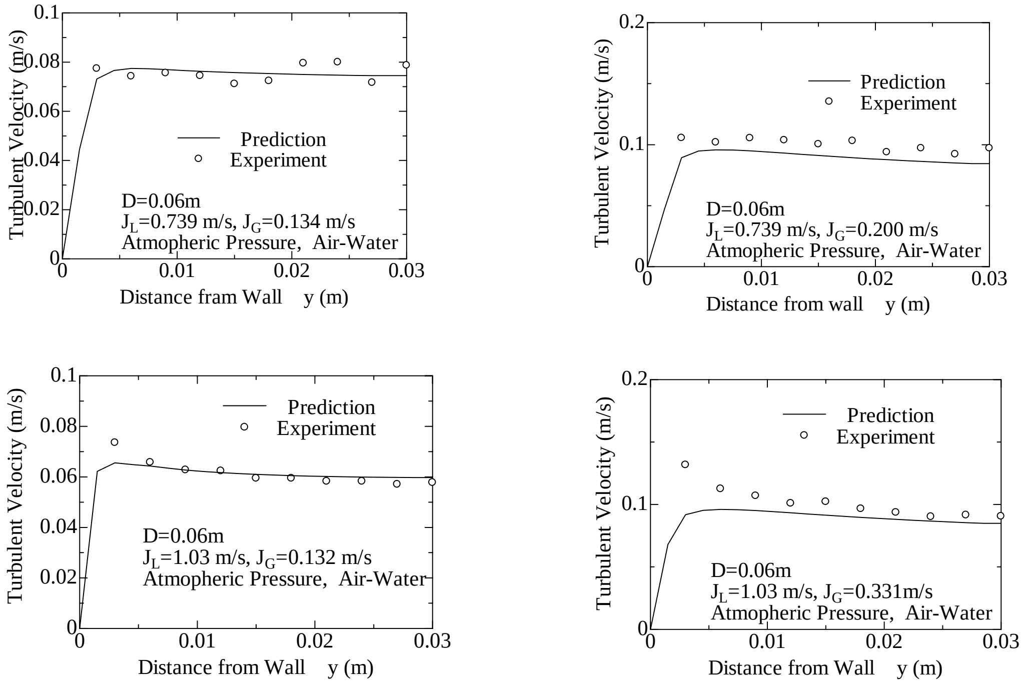
Bubbly Flow Churn Flow

Fig.5. Distributions of Interfacial Area Concentration for Bubbly and Churn Flow



Bubbly Flow Churn Flow

Fig.6. Distributions of Averaged Liquid Velocity for Bubbly and Churn Flow



Bubbly Flow Churn Flow

Fig.7. Distributions of Turbulent Liquid Velocity for Bubbly and Churn Flow

V. Conclusions

A rigorous formulation of basic transport equations of interfacial area concentration is derived. Interfacial velocity is formulated by spatial correlation of characteristic function and velocity of each phase. The constitutive equations of turbulent diffusion and lateral migration of interfacial area concentration were obtained both for bubbly and churn flow. Prediction of radial distributions of interfacial area concentration well agreed with experimental data both for bubbly and churn flow.

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