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# GENERALIZED QUASI-ONE-DIMENSIONAL MODEL OF NON-HOMOGENEOUS TWO-PHASE FLOW AND CRITERION FOR DENSITY WAVE OSCILLATION IN PARALLEL CHANNELS WITH RISER

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## Abstract

A one-dimensional analytical model (with quasi-1D (or q1D) corrections to the homogeneous flow representation) estimated in a lumped parameter fashion has been developed to describe the threshold of density wave oscillations (DWO) in a parallel heated channel with riser. The heated channel consists of a single-phase part occurring up to the point of bubble detachment and a two-phase part comprising the riser. The two-phase region is described by the drift-flux model that accounts for drift velocity and subcooled boiling. In the proposed model, corresponding pressure drops include friction, acceleration, drift, and local pressure drops linked with the inlet and the outlet of the system, respectively. Using perturbation theory, we have linearized conservation law equations around the steady-state operating conditions. The result is a set of two ordinary differential equations with coefficients representing physical properties and model parameters. Using the theory of linear attenuating oscillator, generalized expression (criterion) is obtained for the stability thresholds of DWO for wide range regime and geometric conditions. The current study shows that this generalized criterion is a function of not only traditional homogeneous parameters represented in the classical studies (e.g., Morozov-Gerliga, Ishii-Zuber, Guido et al.), but also new parameters, such as q1D corrections, riser effect, and density, enthalpy, and pressure drop, which are described with drift effects and phase-shift of the model parameters. It is shown here that the new generalized criterion approaches the classical ones in the limiting cases.

**Keywords**
Lumped parameter, quasi-1D, linear model, density wave instability threshold

## Introduction

Thermal-hydraulic instability (i.e. density wave oscillations or DWO) is one of the general types of such self-starting macro-oscillation processes with boiling flow [1] in a system of parallel channels with a constant pressure drop. Pulsations of parameters in the system of parallel channels and oscillations in the loop of natural circulation (e.g., in a nuclear power plant) propagate with a typical space-time periodical changes in density and others parameters. These pulsations can disrupt the normal functioning and controllability of a system, create thermal stresses, and, most importantly, reduce the working thermal margin of the core or steam generator. This is why obtaining analytical criteria of DWO describing such oscillation threshold is of considerable theoretical and practical interest.

In addition to the fundamental works using transfer functions and other approaches of control theory [2-4], recently, the novel approaches of ‘ad initio’ investigation (i.e., founded on the analytical results from the ‘first principles’ founded on the conservation equations of mass, energy and momentum) have been used in this line of research [5, 6]. Compared to the practical solutions proposed by the classical researchers, the 'ad initio' approaches are capable of providing a deeper conceptual and succinct insight into the dynamics under investigation. Although the simple and concise nature of 1D approach stimulated a large number of investigations, 1D approach has been criticized because they have resulted in some brittle models; thus, the current study aims to infuse more flexibility into the 1D approach by accounting for spatial-temporal nonhomogeneity of the two-phase nonequilibrium flow.

Therewith, the study of thermal-hydraulic characteristics in two-phase flows and their stability by means of one–dimensional (1D) mathematical formulations currently remains one of the most popular approach applied for analytical models and analysis of contemporary industrial design and nuclear engineering [1,2]. In the terms of the 1D thermal-hydraulic control theory [1,2], the dynamic response of the system in the circuit from the perturbations (i.e., enthalpy – density – pressure drop – flow rate – enthalpy) reflects the essence of the feedback control mechanism caused by the usual 1D physical effects, but also prevailed over the 2D and 3D non-homogeneity effects. These non-homogeneous parameter distributions (e.g., transversal void fraction profile in subcooling boiling flow) can cause a visible effect on the pressure drop, heat transfer [7-12] and instability boundary [13]. The current model is based on homogeneous two-phase flow description taking into account two principal corrections: 1) various non-homogeneous distribution in transversal direction of local mass (void fraction) and velocity in the channel on the friction forces and heat transfer [7, 9-12] and 2) additional effects of drift-flux on momentum, mass and energy equations with correct descriptions of the phase-shift parameters. The purpose of this paper is to present: 1) the suggested lumped parameter approach of constructing generalized criteria for DWO threshold of boiling channel with riser; 2) transversal parameter distributions therewith taken into account [7-12], 3) demonstration of the fact that the new generalized DWO criterion (for high and low subcooling conditions, including low mass velocity effects) tends to classical ones [1-6] in the limiting cases.

After the number of transformations (similar to [5, 6]) of the original quasi-1D mass, energy and momentum equations, the resulting set of nonlinear ordinary differential equations is then linearized around the steady state and reduced to a dimensionless form. Thus we obtain classical equations of linear attenuating oscillator [14] using them later to obtain generalized analytical criteria for DWO onset with coefficients depending on geometrical and regime conditions. This generalized DWO criteria describe thermal-hydraulic oscillations onset within the wide range of both qualities and flow rates and possess more fundamental and physically-mathematically strict description.

## 1. Integral forms of general mass forces and form-factors introducing

Although, at present, DWO phenomena is studied well enough [1], its analytical models remain uncompleted, in particular, concerning the influence of the radial momentum and heat flow non-uniformity in the channel cross-section. Really, if, for example, a noticeable density change due to radial void fraction distribution causes a considerable influence on local shear stress and its wall values, then noticeable changes in channel friction losses and heat transfer follow as compared with the uniform void fraction case [7,9]. And this, in its turn, changes region boundary of density wave's oscillation. Moreover, in case of subcooled boiling, non-uniform saddle-shape void fraction profile dominating at channel wall and zero void fractions at the channel centre are also typical for the point of bubble detachment [7], this considerably increasing heat transfer coefficient and the placement of this point in the channel.

On detecting effects of parameters non-uniform distribution and their description, in particular, saddle-shape void fraction profile and its connection with shear stresses [7], it is possible to account for such non-homogeneities in instability models grounding on generalized quasi-1D approach. A simple and descriptive approach has been suggested to construct generalized three-dimensional integral relationships for the local and subchannel wall friction, heat and mass transfer coefficients [12]. The approach is based on the boundary layer approximations using Reynolds' flux concept and a generalized substance transfer coefficient. The model takes into account both the effect of non-uniform flow and density profiles as the effect of the geometry (annular and subchannel type [9, 10]). This model is based on momentum, mass and energy conservation equations in the quasi-1D approximation with correction of friction forces (and heat transfer (HT)) by taking into account nonhomogeneous distribution of local mass and velocity on the bases of generalization of Schukin's mass forces conception about variable local acceleration [15].

With this approach, one can formulate the integral analytical expressions for the wall friction factor, heat and mass transfer coefficients accounting for the contribution of various complementary effects. It is these average substance profile components that are complementary effects to the one-dimensional model. They include not only the density (in the mixed convection problem), but also other components in the momentum, heat and mass transfer processes, and their sources and sinks of the substance in the channel flow cross section. Unlike Petukhov-Popov's relationship [16], the integral forms deduced in the papers [11, 12] are typified by a more general character and are characterized by an additive form of notation of the effects under consideration. This is significant for the quasi-1D criteria to assess the contribution of the 2D effects in question. From the mathematical point of view, the present approach can be reduced to the Fourier method- separating variables while constructing closure relationships for thermal-hydraulic non-homogeneous flow: for friction, heat and mass transfer coefficients. This makes possible to obtain a generalization of becoming already classic in the heat transfer theory Lyon's integral and of its successive improvements [8, 11, 16].

Integral transformations applied to the conservation equations of energy and motion makes it possible to transit correctly from the 3D case to the q1D one [12]. The influence of radial variables non-uniformity is realized via integral correction coefficients (form-factors) arising from the corresponding components of the conservation laws and responsible of these component variations in radial direction. As a matter of fact, it is form-factors accounting for flow non-homogeneity, which represent q1D corrections in the obtained generalized integral equations [12]. The q1D model operates with two-dimensional corrections (taking into account 2D distributions) materializing generalized mass forces relative to the right part of q1D of momentum equation (see Table 1: ***e(=1,2,3,4)-*** effects, with definitions and designations).

Form- factors $K_{e\varphi}$ in its physical and mathematical sense reflect the influence of generalized mass forces distributed in the channel cross-section on the shear stress distribution and result in the possibility of its considerable deviation from the typical one (linear function in the simplest case). The derived analytical relationships, being in general case non-linear integral-differential equations, are to be completed with the appropriate model representing the involved physical phenomena (beginning from the turbulent models of substance transfer in one and two-phase non-equilibrium flows and ending with models of substance radial and axial transfer), and also of development appropriate numeric solving procedures. However, under the number of additional simplifications and hypotheses removing non-linearity, it is possible to obtain solution in quadratures [9] (or simplified correlations [7]) keeping generalizing and heuristic properties of the integral forms obtained above. Thus basing on q1D theory from available one-dimension models and methods we come to a possibility of generalization in a way of taking into account spatially non-uniform mass forces of various nature [9-12][*)], in general notation, see Table 1, for example rows *e=1¸4*.

**Table 1.** Definitions for the components average values $\Phi_{e\varphi}$ and form-factors $K_{e\varphi}$ in equations of substance transport: velocity ($j \equiv w$), enthalpy ($j \equiv h$) at azimuthally uniform flow.

| $e$ | Averaged transfer components $\Phi_{e\varphi}$, definitions | Form-factors for the variable $K_{e\varphi}$ |
|---|---|---|
| 1 | $\Phi_\rho = \int_0^1 \tilde{\rho} \boldsymbol{R}_\gamma \boldsymbol{dR}$, where $\tilde{r} = r / r'_w$, $\boldsymbol{R}_\gamma = \gamma \boldsymbol{R}^{\gamma-1}$; | $\boldsymbol{K}_\rho = 1 - \int_0^{\boldsymbol{R}} \tilde{\rho} \boldsymbol{R}_\gamma \boldsymbol{dR} \Big/ \left(\Phi_\rho \Re_\tau\right)$; |
| 2 | $\Phi_{\boldsymbol{zw}} = \int_0^1 \tilde{\rho} \boldsymbol{w}^+ \dfrac{\partial \boldsymbol{w}^+}{\partial \boldsymbol{Z}} \boldsymbol{R}_\gamma \boldsymbol{dR}$, $g = 1 - plain\ gap$, $g = 2 - round\ pipe$; | $\boldsymbol{K}_{\boldsymbol{zw}} = 1 - \int_0^{\boldsymbol{R}} \tilde{\rho} \boldsymbol{w}^+ \dfrac{\partial \boldsymbol{w}^+}{\partial \boldsymbol{Z}} \boldsymbol{R}_\gamma \boldsymbol{dR} \Big/ \left(\Phi_{\boldsymbol{zw}} \Re_\tau\right)$; |
| 3 | $\Phi_{\boldsymbol{yv}} = \int_0^1 \tilde{\rho} \boldsymbol{v}^+ \dfrac{\partial \boldsymbol{w}^+}{\partial \boldsymbol{R}} \boldsymbol{R}_\gamma \boldsymbol{dR}$, $\Re_\tau = \int_0^{\boldsymbol{R}} \rho \boldsymbol{w} \boldsymbol{R}_\gamma \boldsymbol{dR} / \boldsymbol{G}_{\boldsymbol{A}}$; | $\boldsymbol{K}_{\boldsymbol{yv}} = 1 - \int_0^{\boldsymbol{R}} \tilde{\rho} \boldsymbol{v}^+ \dfrac{\partial \boldsymbol{w}^+}{\partial \boldsymbol{R}} \boldsymbol{R}_\gamma \boldsymbol{dR} \Big/ \left(\Phi_{\boldsymbol{yv}} \Re_\tau\right)$; |
| 4 | $\Phi_{\omega \boldsymbol{w}} = \dfrac{1}{\boldsymbol{Sr}_*} \int_0^1 \tilde{\rho} \dfrac{\partial \boldsymbol{w}^+}{\partial \tilde{\boldsymbol{t}}} \boldsymbol{R}_\gamma \boldsymbol{dR}$, $\boldsymbol{Sr}_* = \dfrac{\boldsymbol{t}_0 \boldsymbol{w}_*}{\boldsymbol{r}_1}$. | $\boldsymbol{K}_{\omega \boldsymbol{w}} = 1 - \dfrac{1}{\boldsymbol{Sr}_*} \int_0^{\boldsymbol{R}} \tilde{\rho} \dfrac{\partial \boldsymbol{w}^+}{\partial \tilde{\boldsymbol{t}}} \boldsymbol{R}_\gamma \boldsymbol{dR} \Big/ \left(\Phi_{\omega \boldsymbol{w}} \Re_\tau\right)$. |

Here $R=r/r_1$ -is the current radius, $r_1$- tube radius, $v$ and $w$ - radial and axial velocities, $\Re_t$ - weight function, $w^+ = w/w_*$, $w_*$ - friction velocity, $t_0$- characteristic time ($t_t << t_0 < t_{tr}$, where $t_t$- time of turbulent pulsation, $t_{tr}$ – transport time through the channel).

The most simple and obvious is the assumption of asymptotic closeness of uniform and non-uniform variables, as well as $1 >> \sum_e \Phi_{ew} \boldsymbol{K}_{ew} / \gamma$. Then correlation coefficients in terms of the form-factors representing the "measure" of this non-uniformity can be simplified, as

$$\Psi_{fr} = \frac{\lambda}{\lambda_{ho}} = \int_0^1 \frac{\langle\tilde{\rho}\rangle_{hoR} \cdot \Re_{ho\tau}}{\tilde{\rho}_{ho} \tilde{\nu}_{0T} R^{\gamma-1}} dR \Big/ \int_0^1 \left(1 - \frac{1}{\gamma} \sum_e \Phi_{ew} K_{ew}\right) \frac{\langle\tilde{\rho}\rangle_R \cdot \Re_\tau}{\tilde{\rho} \tilde{\nu}_T R^{\gamma-1}} dR \approx 1 + \frac{1}{\gamma} \sum_e \Phi_{ew} K_{ew}, \quad (1)$$

where $_{ho}$ –homogenous model, for turbulent model $\tilde{n}_T = 1 + n_t / n_w$ relative cinematic viscosity. Then the correlation for Stanton number (using form-factors) may be represented as

$$\Psi_{ht} = \frac{St}{St_{ho}} \approx 1 + \frac{1}{\gamma} \sum_e \Phi_{eh} K_{eh}. \quad (2)$$

And the simpler algebraic approximations can be seen in **APPENDIX A** (fitted ones – from the experimental data [7]).

## 2. Derivation of the criterion for threshold of density wave oscillation in parallel channels with riser

At the mathematical modeling the boiling channel is divided into three control volumes with friction and local pressure drops. The top control volume is the riser, the middle one is two-phase flow region and the bottom one is single phase flow region with changeable length between them. Moreover, the following assumptions are accepted: 1) subcooled boiling length is included in the boiling region, 2) the heat flux is constant, 3) the friction losses are concentrated at the inlet and outlet of the channel, 4) the temperature and flow quality (with drift flux model correction) is linear in the single phase and two-phase region, respectively.

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*) not only density (in mixing convection), but also other components in the momentum and HT processes, and their sources and sinks of the substance in the channel flows (with acceleration due to different physical effects).

To solve this problem, we use the scheme (see Fig. 1) applicable for a system of parallel, hydraulically identical and heated channels with riser representing an up-flow branch of a natural circulation loop. It has three control volumes with local pressure drops: (1) single-phase flow part from the inlet ($k_{in}$) of the channel to the bubble detachment point $z_d=z'$, (2) two-phase flow part from the bubble detachment point $z_d$ to the end of the heater part ($k_{e1}$), and (3) riser part with two-phase flow ($k_{e2}$). Boundary $z_d$ between single phase and boiling parts changes in accordance with flow conditions and the oscillation process.

To obtain analytical solutions the above assumptions are used in the original mass, energy, and momentum equations with condition that the corresponding friction pressure drops there are related to the channel inlet and outlet points, respectively. Then by integrating mass and energy equations along the single- and two-phase sections heights one get the following system of non-linear ordinary differential equations (ODE).

$$\frac{d}{dt}\int_{z_{in}}^{z_d} Ar\,dz = G_{in} - G_d\,, \tag{3}$$

$$\frac{d}{dt}\int_{z_{in}}^{z_d} A\rho h\,dz = G_{in}h_{in} - G_d h_d + q''\Pi z_d\,, \tag{4}$$

$$\frac{d}{dt}\int_{z_d}^{L+L_r} A\rho\,dz = G_d - G_{e2}\,, \tag{5}$$

$$\frac{d}{dt}\int_{z_d}^{L+L_r} A\rho h\,dz = G_d h_d - G_{e2}h_{e2}D_{rr} + q''\Pi(L - z_d) \tag{6}$$

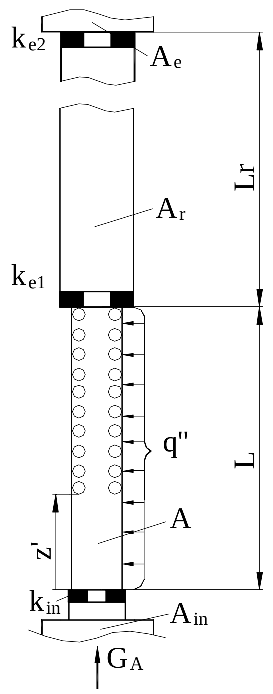


Figure 1 Schematic representation of a heated channel with riser (HCwR)

with the following approximation of the state equation

$$\frac{\bar\rho_f}{\bar\rho} = \begin{cases} (1 + x_{e1}\rho'''/2)D_{ro}, & if \quad h_f > h_d \\ \bar\rho = \bar\rho_f = (\rho_{fin} + \rho')/2, & if \quad h_f \le h_d \end{cases} \tag{7}$$

here $\rho''' = (\rho' - \rho'')/\rho''$, $z_d=z'$ and $z_b=L-z_d$ – length of non-boiling and two-phase boiling parts, $h$ denotes bulk flow enthalpy at the single-phase and boiling parts with appropriate subscript, $G_d$ and $G_{e2}$ – flow rates at the boiling boundary and the riser exit, respectively.

After a number of simple transformations made to eliminate auxiliary variables $G_d$ and $h_d$, to linearize and normalize variables relative to steady state values and scales $t_{sc} = r'LA/G_{in0}$ of the time and enthalpy $h_{cs} = Q/(G_{in0})$ one gets the following linear system ODE with coefficients representing physical properties and model parameters

$$\frac{d\delta\tilde z_d}{d\tilde t} = \frac{Zu}{\tilde{\tilde\rho}_f}\left[\left(b - \frac{1}{Ja_d}\right)\delta\tilde z_d + c\delta\tilde h_{e2}\right], \qquad \tilde t = \frac{t}{t_{sc}}, \qquad \tilde{\tilde r}_f = \frac{\bar r_f}{r'}, \qquad \bar\rho_f = \frac{\rho_{in} + \rho_d}{2}, \tag{8a-d}$$

$$\frac{d\delta\tilde{h}_{e2}}{d\tilde{t}} = \frac{-D_{rr}}{M}\left\{\left[1+\frac{1}{D_{rr}}\left(1+\frac{S_a\chi}{2}\right)-\frac{MZu}{2\tilde{\bar{\rho}}_f}\left(b-\frac{1}{Ja_d}\right)\right]\delta\tilde{z}_d+\left[1-c\left(\frac{MZu}{2\tilde{\bar{\rho}}_f}+\frac{\chi}{2D_{rr}}\right)\right]\delta\tilde{h}_{e2}\right\}, \quad (9)$$

where the following approximations and designations are used for character parameters

$$e_a = \frac{1+c}{2(1+c/2)}, \quad e_r = \tilde{A}_r\tilde{L}_r Zu, \quad M = \frac{e_a(e_r+c)}{Zu(1+c)}, \quad \chi = Zu - Ja_d, \quad S_a = a_d - b, \quad (10a\text{-}e)$$

$$\bar{h}_d = \frac{h_{in}+h_d}{2}, \text{ moreover } \quad h_d \neq f(t), \quad \bar{h}_b = \frac{h_d+h_{e2}}{2}, \quad h_{e2} = h_{e1}(\tilde{t},\tilde{\vartheta}_r)D_{rr}, \quad (11a\text{-}d)$$

$a_d$ and $J_r$ - detachment point coefficient and riser enthalpy transport delay, see §4. Also here $D_{rr}$ and $D_{ro}$ are corrections, taking into account flow enthalpy and density defects due to drift flux transport, see Appendix B. This system can be interpreted as describing enthalpy variation at the single- and two-phase parts in interaction with the other dynamic model parameters via the state equation (7) and boundary condition of constant HCwR pressure drop, see § 4. The presence in calculation scheme in Fig.1 of regions with different cross-sections and taking into account new effects and correlations widens the realm of practical application of the suggested method as compared with the previous works [4-6].

Momentum equation written in increments allows completing the system of feed-back links (using of linear analytical model with eqs. (8a), (9)), as shown in fig.2, in terms of linear control theory.

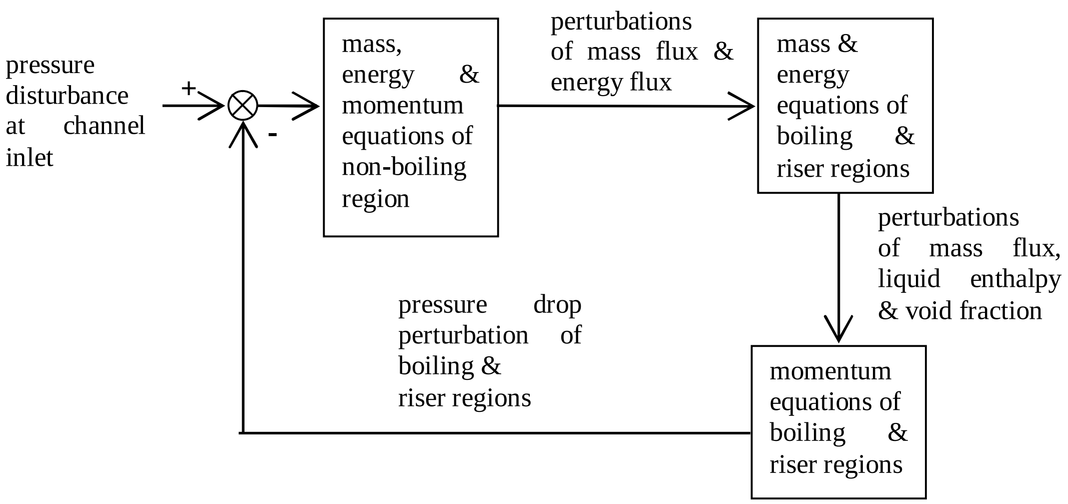


Figure 2 General block-diagram of the physical two-phase flow model of HCwR.

From methodical standpoint, eqs. (8a) and (9) generalize the models [3-7] by phase-shift disturbances $S_a$, subcool boiling $Ja_d^*$, density influence, as well as including vapor drift flux $D_{rr}$, and friction and hydrostatic losses.
Linear system ODE (8a), (9) can be written in canonic form as

$$\frac{dd\tilde{z}_d}{d\tilde{t}} = a_1 \cdot d\tilde{z}_d + a_2 \cdot d\tilde{h}_2, \quad (12)$$

$$\frac{dd\tilde{h}_2}{d\tilde{t}} = a_3 \cdot d\tilde{z}_d + a_4 \cdot d\tilde{h}_2, \quad (13)$$

where

$$a_1 = \frac{Zu}{\tilde{\tilde{r}}_f}\left(b - \frac{1}{Ja_d}\right), \qquad a_4 = \frac{-D_{rr}(1+c)Zu}{e_a(e_r+c)}\left[1 - c\left(\frac{e_a(e_r+c)}{\tilde{\tilde{r}}_f(1+c)} + \frac{c}{2D_{rr}}\right)\right], \qquad \text{(14a,b)}$$

$$a_2 = \frac{Zu \cdot c}{\tilde{\tilde{r}}_f}, \qquad a_3 = \frac{-(1+\chi)Zu}{e_a(e_r+\chi)}\left\{1 + \frac{S_a\chi}{2} + D_{rr}\left[1 - \frac{e_a(e_r+\chi)}{\tilde{\tilde{\rho}}_f(1+\chi)}\left(b - \frac{1}{Ja_d}\right)\right]\right\}. \qquad \text{(15a,b)}$$

Thus, coefficients $a_i$ (of canonic form in physical variables) are correlated via dimensionless parameters with geometrical and thermal-hydraulic conditions of two-phase flow in the HCwR.

## 3. Linear attenuating oscillator and generalized DWO criteria

General solution of the linear system ODE (12), (13) is

$$X = X_0 \exp(e\tilde{t}), \qquad \text{(16)}$$

which after simple algebraic transformations results in eigenvalues second order problem with characteristics equation for classic [14] linear attenuating oscillator

$$e^2 + C \cdot e + D = 0, \qquad \text{or} \qquad \begin{vmatrix} e - a_1 & -a_2 \\ -a_3 & e - a_4 \end{vmatrix} = (e - a_1)(e - a_4) - a_2 a_3 = 0, \qquad \text{(17a,b)}$$

where coefficients take form

$$C = -(a_1 + a_4), \qquad D = a_1 a_4 - a_2 a_3. \qquad \text{(18a,b)}$$

Accepting $C=0$ in accordance with the oscillations linear theory [8], one gets a linear conservative system with the periodic solution in form of harmonic oscillations with the dimensionless frequency

$$\omega^{*2} = \left(\frac{\tilde{\omega}}{Zu}\right)^2 = \frac{2(1+\chi)}{e_a(e_r+\chi)}\left\{b - \frac{1}{Ja_d} - \frac{c}{D_{rr}}\left[1 + D_{rr} + \chi\left(\frac{a_d}{2} + \frac{1}{Ja_d}\left(1 - \frac{a_d}{b}\right)\right)\right]\right\}. \qquad \text{(19)}$$

Thus, $C=0$ condition serves as a mathematically precise criteria for the DWO threshold, which in physical variables of the present model takes form:

$$c^2 - 2\left[\frac{1}{c}\left(D_{rr} + \frac{e_a}{\tilde{\tilde{r}}_f Ja_d}\right) - e_a\left(D_{rr} + \frac{b}{\tilde{\tilde{r}}_f c} + \frac{1}{2e_a}\right)\right]c -$$
$$- 2\left[\frac{1}{c}\left(D_{rr} + \frac{e_a e_r}{\tilde{\tilde{r}}_f Ja_d}\right) - e_a e_r\left(D_{rr} + \frac{b}{\tilde{\tilde{r}}_f c}\right)\right] = 0. \qquad \text{(20)}$$

This is the most general form in the linear approximation of the threshold DWO criteria on the basis of two phase drift flux model for channels with risers with constant pressure drop between collectors, generalizing (in comparison to [5, 6]) the list of geometrical conditions, and considered physical effects, including low mass velocity and subcooled boiling.

# 4. Phase shift and amplification parameters determination in the DWO-model

Independent general solutions of basic ODE system (12), (13) are

$$d\tilde{z}_d = \tilde{a}_{zd}\sin(wt + \tilde{J}_z) \qquad \delta\tilde{h}_{e2} = \tilde{a}_{he}\cos(\omega t + \tilde{\vartheta}_h) \qquad (21a,b)$$

with phase shift $\vartheta_{hz}$ (see Fig.3a at zero initial phase angles $J_z = J_h = 0$)

$$\tilde{\vartheta}_{hz} = arctg(a_{hz}); \qquad a_{hz} = \frac{\tilde{a}_{he}c}{\tilde{a}_{zd}b} \qquad (22a,b)$$

where $\tilde{a}_{zd}$ and $\tilde{a}_{he}$ are relative amplitudes of boiling boundary and enthalpy oscillations.
In accordance with superposition property (in linear theory at the oscillations threshold), solutions for the remaining parameters of oscillation system can be written as the linear combination of harmonic functions [14]. In particular, mass flow solution at the channel inlet can be written using (21a,b) solutions, as

$$d\tilde{G}_{in} = Zu(bd\tilde{z}_d + cd\tilde{h}_{e2}), \quad \text{or} \quad \delta\tilde{G}_{in} = \tilde{a}_{zd}bZu\sqrt{1+a_{hz}^2}\sin\left(\omega t + \tilde{\vartheta}_{hz}\right). \qquad (23a,b)$$

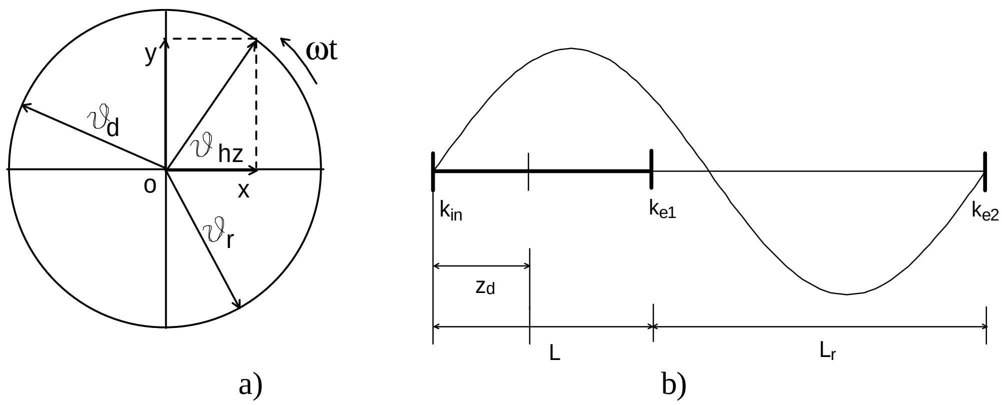


Figure 3 a) Circle diagram for harmonic solution: $y = \delta\tilde{z}_d$ and $x = \delta\tilde{h}_{e2}$ for $\vartheta_z = \vartheta_h = 0$;
b) Example of instantaneous mass flow distribution (or of any sin-parameter in the $L+L_r$ length) at the DWO threshold with zero phase-shift at the inlet.

Longitudinal density wave motion (see Fig.3b) leads to a description of mass flow phase shift at the detachment point $\vartheta_d$ and at the raiser exit $\vartheta_r$ as

$$\tilde{J}_d = \tilde{\bar{r}}_f \frac{Ja_d}{Zu}\tilde{w}, \qquad \tilde{J}_r = \frac{\tilde{A}_r\tilde{L}_r\tilde{w}}{C_0(1+c)+\tilde{A}_r b_G}, \qquad (24a,b)$$

with relative amplification coefficients

$$a_d = \frac{b\sqrt{1+(a_{hz})^2}\cos\left(\tilde{\vartheta}_{hz} - \tilde{\vartheta}_d\right)}{1-(b/\tilde{\omega})\sqrt{1+(a_{hz})^2}\sin\left(\tilde{\vartheta}_{hz} - \tilde{\vartheta}_d\right)}, \quad a_{jr} = 1-(1-b)\left(1-\cos\tilde{\vartheta}_r + \frac{a_d}{\tilde{\omega}}\sin\tilde{\vartheta}_r\right) \qquad (25a,b)$$

## 4.1 Momentum equation and generalization of the basic parameters *b* and *c*

In the quasi-static approximation perturbation qualities of pressure drops and mass fluxes are small in comparison with their steady-state values. Then the general form of integral momentum conservation equation along heated channel with riser (HCwR) can be written as:

$$0=\left(\Delta P-\Delta P_{fr}-\Delta P_{gr}-\Delta P_{acc}D_{ra}-\Delta P_{loc}\right)\Big|_{z_{in}}^{z_2} \tag{26}$$

where $\Delta P=P_{in}-P_{e2}$ is the difference between the inlet and exit (riser) pressures, and pressure drops with subscripts (*fr ,gr, acc, loc)* friction, gravitation, acceleration, and local ones, and $D_{ra}$ is correction, taking into account flow acceleration to drift flux transport of the true quality at the interface, see Appendix B.

Two-phase flow momentum equation in the considered channel plays the central role in closing the oscillation model and serves to determine mass flow variations $\delta\tilde{G}_{in}$ at the channel inlet and for writing it via dependant variables and appropriate amplification coefficients ***b*** and ***c***. Then at the zero boundary and initial conditions we have, considering pressure friction drop, hydrostatic pressure and local friction ($F_i$- components), the resulting desirable closing relations for amplification coefficients ***b*** and ***c***

$$b=\frac{2(1+F_b)}{\tau_0(1+F_m)},\quad \tau_0=2\left(1+\frac{K_{in}+K_{e2}-2}{K_{e1}+2}\right),\quad F_m=2\frac{F_1+F_3+F_4+2(D_{ra}-1)}{\tau_0(K_{e1}+2)}, \tag{27a-c}$$

$$F_b=\frac{F_3+a_{jr}(K_{e2}+2D_{ra}+F_4)-2-\tilde{a}_{hz}\sin\tilde{\vartheta}_r-F_2}{K_{e1}+2},\qquad c=\frac{1+F_c}{\tau_0(1+F_m)}, \tag{28a,b}$$

$$F_c=\cos\tilde{\vartheta}_r-1+\frac{2\cos\tilde{\vartheta}_r(F_5-1)+K_{e2}+2(D_{ra}+F_6)}{K_{e1}+2},\qquad \frac{c}{b}=\frac{1+F_c}{2(1+F_b)}, \tag{29a,b}$$

$$\tilde{\lambda}_d=\frac{\lambda L}{d_e},\qquad F_1=\tilde{\lambda}_d\frac{Ja_d}{Zu},\qquad F_2=(a_d+a_{dx})\left(F_1-\tilde{\lambda}_b\frac{\chi}{Zu}\right)-\tilde{\lambda}_b\left(1+\frac{\chi}{2}\right), \tag{30a-c}$$

$$\tilde{\lambda}_b=\tilde{\lambda}_d\Psi_{fr},\qquad F_3=\tilde{l}_b\left(\frac{c}{Zu}+\tilde{A}_r\tilde{L}_r\right),\qquad F_4=\tilde{l}_b\tilde{A}_r\tilde{L}_r, \tag{31a-c}$$

$$F_5=\frac{1}{2}\left(F_3+\frac{Fr}{(1+\chi)^2}\left(1+\frac{\chi}{Zu}\right)\right),\qquad F_6=\frac{1}{2}\left(F_4+\frac{Fr}{(1+\chi)^2}\right), \tag{32a,b}$$

$$a_{dx}=\frac{\rho''}{\rho_{fd}(1+Ja_d^*)^2}\frac{X_dZu}{(1-x_{fd})},\qquad X_d=\frac{\alpha_d(C_0+\tilde{V}_{gj})}{1-\alpha_d(1-\rho''/\rho')}. \tag{33a,b}$$

These relations (33a,b) play the role of corrections for flow rate increment in subcool boiling enthalpy $Ja_d^*$ *or* $(x_{fd})$ at the bubble detachment point basing on Saha-Zuber [17] recommendations.

It is the $F_i$ components that define additional pressure drops by means of quasi-one-dimensional corrections of the non-uniform flow structure $\psi_{fr}$ (see Appendix B, [7]) and new drift flux boiling model corrections $D_{ro}$, $D_{rr}$, and $D_{ra}$. Essential new feature of the model is that parameters linked with phase-shift influencing the corresponding parameter variation $a_d$, $a_{dx}$ $a_{jr}$ and $\vartheta_{hz},\vartheta_d,\vartheta_r$ are taken into account as well as the drop component via friction $F_\lambda$ hydrostatic *Fr,* and local pressure drops.

### 4.2 Limit cases and simplifications of the present model

Non-linear influence of variables in both parts of relations with trigonometric functions turns them into goniometric equations that somewhat hampers their direct application as criteria. There are various means of nonlinearity evaluation and obtained solutions simplification; however it involves considerable additional calculations which go beyond this paper frame. That is why a short analysis of the obtained solutions asymptotic behavior is given below:

1. Assumptions $C_0 \to 1$ and $V_{gj} \to 0$ and $Ja_d^* \to 0$ reduce the case to a two-phase homogeneous model with subcooled zone neglecting. Friction loss, hydrostatic head, and riser influence space shifts disturbances of flow rate, enthalpy and density remain accounted for.
2. Friction loss and hydrostatic head neglecting simplifies the system only by local pressure drop, leaving riser influence space shifts of flow rate, enthalpy and density accounted for.
3. Riser space shifts of flow rate, enthalpy and density influence neglecting reduce the model to almost Su et al [6] results. It becomes clear that there are mistakes in Su and other basic ODE system coefficients deduction.
4. Finally, the most simplifications are for channel system without riser, the absence of friction loss, hydrostatic head and space shifts disturbances also simplifies. In this case, as can be seen from eq. (20), the model with its solutions reduces to those in [5].

Returning to eq.(20), one can see that in general case components in eq. (20) can be written as a linear with $\tau_0$ (see eq. (27b) with $D_{rr} = D_{ro} = D_{ra} = 1$), and the second (and more) power function $b = b(Zu, Ja,\ D_{rr}(Zu, Ja))$ and $c = c(Zu, Ja,\ D_{rr}(Zu, Ja))$, see equations from § 4.1 and Appendix B, after simple transformations one have sixth (and more) order equation of the DWO criteria. The result obtained proves the highest possible multinomial order. It can be formulated in the form of the following lemma.

***Lemma.*** The highest order of polynomial that determines DWO criteria, relative to the main variables *Zu* and *Ja,* can exceed the power six.

This formal result is practically useful in finding solutions and for grounding the assumptions made at getting simple relations as a threshold DWO criteria and it shows the possibility of two instability regions described by the third order equation*). Experimental research for DWO boundaries carried out in a wide range of conditions [18] serves as a confirmation to it. In these experiments the existence of two instability regions (with flow rate oscillations in parallel steam channels) with a complex form was confirmed and they were named as the first and the second type oscillations for the low and high quality respectively.

The analytical form of the proposed criterions appears to be rigorous and universal; thus, it is capable to demonstrate the most striking differences between the proposed method and the extant ones in the limited space of this report. Unfortunately, the limited space precludes a more applied discussion and graphical representation of the results.

Assumption of the possibility of linear approximation application for evaluation of contribution of additional $F_i$ effects simplifies significantly this technique and facilitates concise presentation of some initial verification results – see Appendix C.

*) In the work [13] it was shown that the criteria of static instability is a quadratic form relative to the model main variables *Zu* and *Ja*, while the static instability boundary curves are hyperbolas.

## 5. Summary

A one-dimensional analytical lumped parameter model (with quasi-1D corrections to the homogeneous flow representation) has been developed to describe the DWO threshold in a parallel heated channel with riser. The developed model is based on homogeneous two-phase flow description taking into account three principal corrections: 1) various non-homogeneous distribution in transversal direction of local mass (void fraction) and velocity in the channel on the friction forces and heat transfer (with help suggested earlier form-factors [9-12]), 2) additional effects of drift-flux on momentum, mass and energy equations and 3) phase shifts in searched variables and space delays in density waves parameters.

As the considered linear thermal-hydraulic problem of the two-phase density wave oscillation is reduced to a canonic equation for classic [14] linear attenuating oscillator with coefficients as a function of geometry and flow character, the solutions obtained can be regarded as the benchmark ones. Its analytical form renders good opportunities for the obtained solution analysis. Using the theory of linear attenuating oscillator implicit generalized expression (criterion) is obtained for the stability thresholds of DWO and frequency of oscillations.

These new results essentially widen and enrich both empirical DWO threshold criteria of Gerliga-Morozov, Ishii-Zuber - type models and the analytical ones like Guido-Converti-Clausse not only in respect of accounting for parameters non–uniformity but also as regards the results' mathematical analysis on the linear attenuating oscillator basis. Asymptotic transitions to models like Guido-Converti-Clausse and Su-Dounan-Fukuda-Yujun having more cumbersome and strict assumptions are pointed out.

### Nomenclature

$A$ – cross section area;
a, b, c, e– coefficients in equations (8)-(19);
$C_0$- distribution parameter;
$d_e$ – equivalent channel diameter;
$D_{ra}$, $D_{rr}$, $D_{ro}$- component of drift correlations;
$g$ – free fall acceleration;
$G_A$, $G_{in0}=A \cdot G_A$ – mass flux, flow rate;
$h$ – flow specific enthalpy;
$h_{fg}$ – latent heat evaporation;
$k$ – pressure drop coefficient;
$K_{ej}$ – form factor, see Table 1;
$L$ and $L_r$ – heater and raiser lengths';
$P$ – pressure;
$q^2$- heat flux;
$r$, $r_1$ – radial coordinate, radius of pipe;
$R=r/r_1$ – dimensionless coordinate;
$Q_0=q^2 L\Pi$– channel's heat power, $\Pi=\pi d_e$;
$t$, $t_{tr}$ – time, transit time;
$x$ –relative enthalpy of flow;
$x_f=(h'-h_f)/h_{fg}$- relative enthalpy of fluid;
$X$ – true quality of flow;
$V_{gj}$- drift velocity;
$v$, $w$ – radial velocity, axial velocity;
$z$ – axial coordinate;
$\alpha$ - void fraction;
$\delta$ - increment;
$\lambda$ - friction coefficient;
$\rho$ - density; $\rho''' = (\rho' - \rho'')/\rho''$
$\tau$ – relative pressure drop parameter;
$f_{ej}$ - component average value, see Table 1;
$\psi_{fr} = \lambda/\lambda_{ho}$ - non-uniformity parameter;
$Ar$ – Archimedes number;
$Ja_d = (h_{fd} - h_{in})\, r'''/h_{fg}$ – Jacob number;
$Fr_G=gL\rho^2_f/G^2_A$ – Froude number;
$Re$ – Reynolds number;
$St$ – Stanton number;
$Zu = Q_0 \rho''' / (G_{ino} h_{fg})$ – Zuber number.

### Indexes:

*acc*- acceleration; *b* – boiling; *d*- bubble departure point; *e* – effect index (see Table 1); *ex* (or 2) – outlet; *f* – fluid; *fg* – phase change; *fr*- friction; *g*–vapor; *gr*–gravitation; sc-scale; *h* – to enthalpy; *ho* – homogenous; *in* –inlet; *loc* – local; *r*- riser; *R* – radial; *w*- to axial velocity, wall; $g$ - channel form; $'$, $^2$ - saturated fluid, vapor; — average; ~ relative.

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## APPENDIX A – simplified closure q1D corrections for friction and heat transfer

From the practical view point of stability analysis it is need the simplified closure q1D corrections for friction and heat transfer based on generalized form-factors (Table 1 type) for wide range of regime conditions. For the steady-state subcooled boiling channel flow there is very strong radial nonhomogeneous (saddle-shape typical) void fraction profile. Then it is needed to take into account only density (void fraction) form factor placed in the first row of the Table 1. Such nonhomogeneous effect can be taken into account as it is done in the paper [7], or in the following more direct fashion. For shear stresses we have

$$\tilde{t}\ (R) = R\left(1-\Phi_r K_r\right)\ ,\quad \Phi_r = \frac{\langle\tilde{r}\rangle g r_w r_1}{g\, t_w} = \frac{\langle\tilde{r}\rangle g r_1}{g\, w_*^2} = \frac{\langle\tilde{r}\rangle}{g\ \mathrm{Fr}_*},\quad K_r = 1 - \frac{\langle\tilde{r}\rangle_R}{R^g\langle\tilde{r}\rangle}. \qquad \text{(A1a-c)}$$

Here relative density distribution is a function of void radial profile

$$\tilde{r}(R) = r(R)/r_w = \left[a(R)r'' + (1-a(R))r'\right]/r' = 1 - a(R)(1 - r''/r'). \qquad \text{(A2)}$$

Then with the following simplified assumption about a relative density difference $\langle\Delta\tilde{r}\rangle$ between wall zone and core of flow (with making designation $\langle\ \rangle$ *and* $\langle\ \rangle_R$ clear)

$$\langle\langle\tilde{r}\rangle - \frac{\langle\tilde{r}\rangle_R}{R^g}\rangle = \langle\Delta\tilde{r}\rangle \cong \langle\tilde{r}\rangle\left(1 - \int_0^1 \frac{\int_0^R \tilde{r} R_g\, dR}{\int_0^1 \tilde{r} R_g\, dR \int_0^R R_g\, dR} R_g\, dR\right), \qquad \text{(A3)}$$

we get friction coefficient as

$$\frac{8}{l\ \mathrm{Re}} = \int_0^1 \frac{r}{\langle r\rangle}\left[\int_R^1 \frac{R}{\tilde{r}\tilde{n}_T}\left(1 - \frac{\langle\Delta\tilde{r}\rangle}{g\ Fr_*}\right) dR\right] R_g\, dR\ ,\qquad Fr_* = \frac{w_*^2}{g r_1} = \frac{l}{8}\frac{\mathrm{Re}^2}{Ar},\qquad Ar = \frac{\langle\Delta\tilde{r}\rangle g r_1^3}{n_f^2}. \qquad \text{(A4a-c)}$$

In the linear approximation using reference function $l_{ho}$ we get

$$\Psi_{fr} = \frac{l}{l_{ho}} = \left(1 + \frac{8}{l}\frac{\langle\Delta\tilde{r}\rangle Ar}{g\ \mathrm{Re}^2}\right)\frac{\mathrm{Re}_{ho}}{\mathrm{Re}}\Bigg|_{\mathrm{Re}\to\mathrm{Re}_{ho}} \approx 1 + \frac{8}{l_{ho}}\frac{\langle\Delta\tilde{r}\rangle Ar}{g\ \mathrm{Re}^2} \qquad \text{(A5)}$$

In the frame of the acceptable approximations the friction coefficient (A5) is the most generalized implicit analytical relationship taking into account hydrostatic component. It is obvious that, if we have $\langle\Delta\tilde{r}\rangle \to 0$, $\mathrm{Re} \to \mathrm{Re}_{ho}$, then $\lambda \to \lambda_{ho}$.

Stanton number at the point of bubble detachment may correlate with friction factor through the analogy factor

$$\mathrm{St}_d = \frac{\lambda}{8} A_\tau^q,\qquad A_\tau^q = \left\{1 + 5\sqrt{\frac{\lambda}{8}}\left[\mathrm{Pr} - 1 + \ln\left(1 + 0.83(\mathrm{Pr} - 1)\right)\right]\right\}^{-1},\ \text{for } 1<\mathrm{Pr}<6, \qquad \text{(A6a,b)}$$

where $A_\tau^q$ - Reynolds analogy factor between momentum and energy transfer.

## APPENDIX B – drift-flux corrections

In the drift flux model, the void fraction $\alpha$ is a function of the true quality $X$ total mass flux $G_A=\langle\rho w\rangle$, phase distribution parameter $C_0$ and a drift velocity $V_{gj}$ as

$$\alpha=\frac{X\rho'}{C_0\left[X\rho'+(1-X)\rho''\right]+\tilde{\tilde{V}}_{gj}\rho''}, \qquad \tilde{\tilde{V}}_{gj}=\frac{\bar{V}_{gj}\rho'}{G_A}. \tag{B1a,b}$$

Using these basic correlations of two-phase drift flux model it is not difficult to get their correlations with homogeneous two-phase flow parameters

$$\frac{r'}{r_m}=\frac{1}{\tilde{r}_m}=\frac{1}{\tilde{r}_{ho}}\left(1+\frac{\left[1-C_0\left(1-\tilde{r}_{ho}\right)\right]Xr'''-\left(1-\tilde{r}_{ho}\right)\left(C_0+\tilde{\tilde{V}}_{gj}\right)}{C_0+\tilde{\tilde{V}}_{gj}+\left(C_0-1\right)Xr'''}\right)=\frac{D_{ro}}{\tilde{r}_{ho}}, \qquad \rho'''=\frac{\rho'-\rho''}{\rho''}. \tag{B2a,b}$$

This can be written out for the considered conditions, particularly, for subcooled boiling or high quality boiling, and for low or high mass velocity flow.

There are two limiting cases for different two-phase flows

$$\left.\frac{1}{\tilde{r}_m}\right|_{C_0\to 1}=\frac{1}{\tilde{r}_{ho}}\left(1-\frac{\left(1-\tilde{r}_{ho}\right)\cdot\tilde{\tilde{V}}_{gj}}{1+\tilde{\tilde{V}}_{gj}}\right), \quad \left.\frac{1}{\tilde{r}_m}\right|_{\bar{V}_{gj}\to 0}\cong\frac{1}{\tilde{r}_{ho}}\left(1+\frac{Xr'''-C_0\left(1+Xr'''\right)\left(1-\tilde{r}_{ho}\right)}{C_0+\left(C_0-1\right)Xr'''}\right), \tag{B3a,b}$$

from where the form of density-drift correction factors -$D_{ro}$ can be seen clearly.

Correction factors for the enthalpy-drift and the momentum-drift take the following forms

$$D_{rr}=1+\frac{r'-r_m}{r_m}\frac{r'r''}{r'-r''}\frac{\left(h''-h_f\right)\cdot\bar{V}_{gj}}{G_A h_m}=1+\left(\frac{D_{ro}}{\tilde{r}_{ho}}-1\right)\frac{r''}{r'-r''}\frac{\left(1+x_f\right)\cdot\tilde{\tilde{V}}_{gj}}{h_m/h_{fg}}=1+\left(\frac{D_{ro}}{\tilde{r}_{ho}}-1\right)\frac{\left(1+x_f\right)\cdot\tilde{\tilde{V}}_{gj}}{Zu} \tag{B4}$$

and

$$D_{ra}=1+\frac{r'-r_m}{r_m-r''}\frac{\bar{V}^2{}_{gj}\,r'r''}{G_A^2}=1+\tilde{\tilde{V}}_{gi}^2\left(\frac{D_{ro}}{\tilde{r}_{ho}}-1\right)\Big/\left(1+r'''-\frac{D_{ro}}{\tilde{r}_{ho}}\right). \tag{B5}$$

Using them the correction is made for the corresponding homogenous parameters in two-phase flow energy and momentum equations (taking into account the drift flux interface transfer of the density, enthalpy, and momentum). As we can see, the contribution of these correlations to the property of the homogeneous mixture increases with the deviation from homogeneous conditions: distribution parameter $C_0$, drift velocity $V_{gj}$ and relative fluid subcooling $x_f$.

## APPENDIX C – some results of the verification

Obviously, the generalized DWO criterion (20) may be used in form

$$Ja_d=Zu-\left(B+\sqrt{B^2+C_e}\right), \tag{C-1}$$

here $$B=\frac{1}{c}\left(D_{rr}+\frac{e_a}{\tilde{\bar{r}}_f\,Ja_d}\right)-e_a\left(D_{rr}+\frac{b}{c\tilde{\bar{r}}_f}-\frac{1}{e_a}\right), \tag{C-2}$$

$$C_e=\frac{D_{rr}}{c}-e_a e_r\left(D_{rr}+\frac{1}{c\tilde{\bar{r}}_f}\left(b-\frac{1}{Ja_d}\right)\right), \tag{C-3}$$

where $b$ and $c$ are introduced above by formulas (22)-(25) and (27)-(33). However in the linear approximation (when $F_i$<<$1$) we have serious simplifications:

$$b \cong \frac{2(1+F_b-F_m)}{\tau_0} \approx \frac{2(1\pm\Delta_b)}{\tau_0}; \qquad c \cong \frac{1+F_c-F_m}{\tau_0} \approx \frac{1\pm\Delta_c}{\tau_0}; \qquad \tau_0 = 2\left(1+\frac{k_{in}-2}{k_e+2}\right). \qquad \text{(C-4)}$$

Then, assuming quasi-one-dimensional and other corrections to be equal to one, $D_{r0}=D_{rr}=D_{ra}=\tilde{\tilde{\bar{\rho}}}_f=1$, the following graphic illustrations of experimental and analytical data can be obtained:

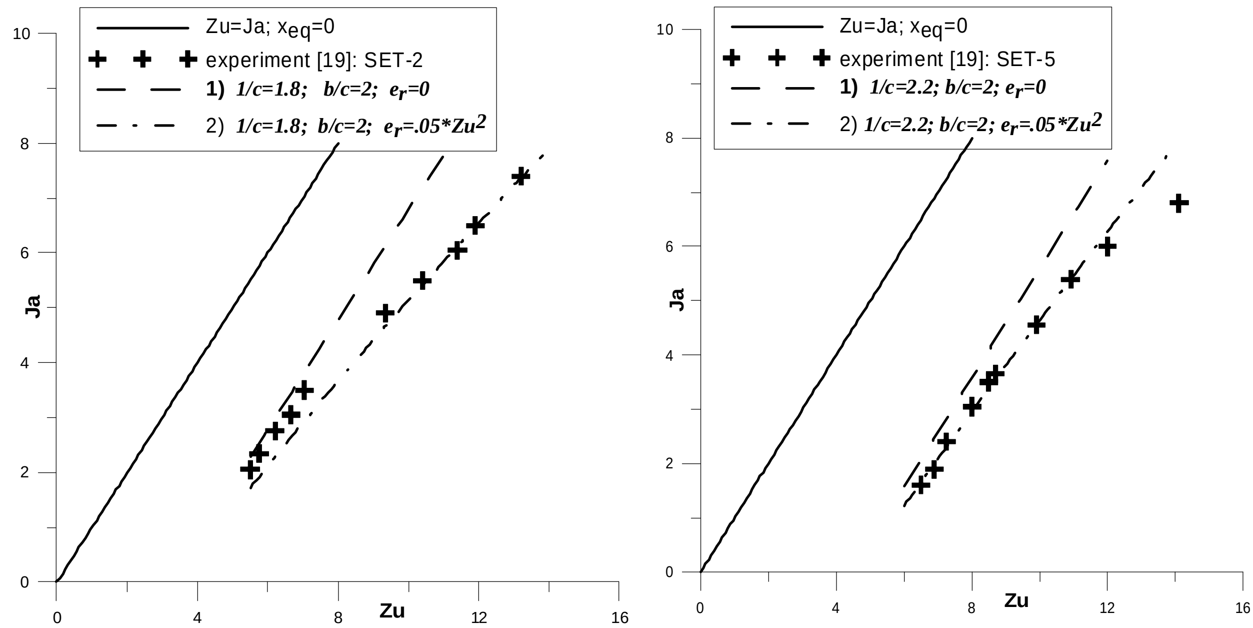


Figure 4 The model results compared with SET-2 and SET-5 [19] data.

It should be noted that the really existing section between the outlet of heated unit and the inlet of the upper part of the by-pass (respectively, points C and D in Fig. 1 - Schematic of the boiling loop [19]) in the experimental facility [19] provides signal time delay and acts as a riser. However, its geometrical characteristics are not presented in [19], so below its effect is taken into account parametrically.

As it can be seen, values calculated by the simplified homogeneous model (dashed lines - 1) are quite different from experimental data [19], while 5% riser additive with Zuber number square law (chain-dotted lines - 2) showed the best approximation to the experimental data. This testifies to the riser effect on DWO threshold.