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A BEST-ESTIMATE PLUS UNCERTAINTY TYPE ANALYSIS FOR COMPUTING ACCURATE CRITICAL CHANNEL POWER UNCERTAINTIES

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Abstract

This paper provides a Critical Channel Power (CCP) uncertainty analysis methodology based on a Monte-Carlo approach. This Monte-Carlo method includes the identification of the sources of uncertainty and the development of error models for the characterization of epistemic and aleatory uncertainties associated with the CCP parameter. Furthermore, the proposed method facilitates a means to use actual operational data leading to improvements over traditional methods (e.g., sensitivity analysis) which assume parametric models that may not accurately capture the possible complex statistical structures in the system input and responses.

Introduction

Best estimate plus uncertainty (BEPU) methods are widely used for nuclear safety analyses [1, 2, 3, 4, 5]. These methods use realistic codes to represent the physical phenomena that underlie the safety analyses. The use of best-estimate codes within the reactor technology, either for design or safety analysis, requires an understanding of the limitations and deficiencies associated with these codes.

It has been shown in [6] that the methodology based on Extreme Value Statistics (EVS) can lead to significant improvements in probabilistic safety analyses. A critical component of the EVS methodology introduces and includes the distinction between aleatory and epistemic uncertainties in the evaluation of the safety margins. The term aleatory refers to variations in underlying conditions whose effects cannot be predicted in advance and which result from a variety of operating or conditions states. These arise because the reactor core is subject to continuous change, such as refuelling, operation of the reactor regulating system, changes in thermal hydraulic conditions, etc. The timing and interaction of these changes cannot be predicted in advance, and this results in the state of the core being subject to random variations. The term epistemic refers to the state of, or lack of, knowledge of the underlying physical phenomena and an epistemic error refers to the difference between what a code or measurement is attempting to measure and the value actually obtained. Epistemic variation arises due to (for example) the accuracy of computer codes as well as accuracy of all the variables that are input into these codes. Also included as part of epistemic error is the accuracy of measurements that characterize different aspects of station operation. Epistemic error results in our perception of the reactor state differing from the true reactor state. Despite different origins, both epistemic and aleatory variations are analysed within the same probabilistic framework in [6] to derive the genuine a 95/95 tolerance interval. The results have shown to lead to significant improvements in the evaluation of the safety margins. This methodology has been successfully applied in the evaluation of the Neutron Overpower Protection (NOP) trip setpoint analysis [6, 7]. An important input in the NOP analysis is the calculation of the Critical Channel Powers (CCP) and uncertainties associated with the CCP parameter. The calculations of the CCP rely on best-estimate thermal hydraulic codes to model the response of the Heat Transport System (HTS) under the postulated reactivity failure event considered in the NOP analysis.

The objective of this paper is to describe a best-estimate plus uncertainty analysis methodology using a Monte-Carlo approach to develop the required statistical error models that capture the epistemic and aleatory errors associated with the CCP response variable. These resulting CCP error models provide a critical input in the evaluation of the 95/95 tolerance interval proposed in [6].

1. Background

1.1 NOP Trip Coverage Analysis

During a postulated power excursion, such as a Loss of Regulation (LOR) event, the reactor power may increase sufficiently to induce an unstable dry patch on the fuel sheath in a high power channel. This condition is commonly known as dryout[1]. Although the onset of fuel sheath dryout does not necessarily lead to fuel or fuel channel failures, elevated fuel temperatures can result in fuel element deformations and possibly, fuel centre-line melting and eventually pressure tube failure. Thus, the prevention of the onset of intermittent dryout has been used in the Canadian safety analysis industry as a conservative criterion for preventing fuel failures leading to radiological releases.

Determination of the dryout power or CCP is an important requirement in assessing the margins to dryout within the NOP analysis. A challenging aspect of a safety analysis is that the variable of interest (i.e., CCP) may not be a directly measurable quantity but computed by complex computational codes that are functions of input variables that define the initial boundary conditions (e.g., reactor header conditions, etc.) and those that define the properties and phenomenon of the system and code (e.g., size of the fuel elements, bundle weights, etc.).

In the NOP analysis, best-estimate codes are used to model the response of the heat transport system (HTS) under the postulated reactivity failure event leading to an LOR. A description of the CANDU's HTS is discussed in Section 1.2 and the method for calculating the CCPs to evaluate the margin to dryout in each channel is described in Section 2.1.

[1] The power at which dryout occurs is known as the Critical Channel Power (CCP)

1.2 CANDU Reactor Design

The CANDU (CANada Deuterium Uranium) reactors are pressurized heavy-water water reactors (PHWR). Heat removal from the fission process is accomplished in a CANDU reactor through a HTS as given in Figure 1. The HTS accomplishes the safety-related goal of cooling the fuel. The complete flow pattern of the HTS resembles that of a figure eight.

A particularly unique HTS is that of the design from Bruce Power's Nuclear Generation Stations (BNGS) where the HTS is a single closed loop but the core is physically divided into two separate hydraulic flow zones, referred to as the Outer Zone (OZ) and the Inner Zone (IZ) (see Figure 1). Hot coolant flow in the loop passes through the boilers, removing heat and reducing the coolant temperature, and then passes through the HT pumps, adding pump head to the pressure. After the HT pumps, the flow splits into the two flow zones. The fuel channels in the OZ are connected to a single Reactor Inlet Header (RIH) on each side of the loop (1 East and 1 West) only. The portion of the coolant flow that goes to the OZ goes directly to the OZ RIH at the boiler outlet temperature and pressure after the HT pumps and is completely separate from the IZ. There are a total of 480 fuel channels at BNGS.

The fuel channels in the IZ are connected to a single RIH on each side of the loop (1 East and 1 West) only, separate from the OZ RIH. The portion of the coolant flow that goes to the IZ does not go directly to the IZ RIH but first flows through a preheater heat exchanger which removes more heat from this portion of the coolant, further reducing the IZ coolant temperature. The flow goes from the preheater to the IZ RIH and is completely separate from the OZ. The fuel channels in the IZ therefore experience flow conditions coming into the channels that are lower in temperature due to heat removal by the preheater and also at a lower pressure as some pressure is lost as coolant passes through the preheater, relative to the fuel channels in the OZ. The two Zones join together downstream of the fuel channels and before the boilers, via a reactor outlet header on each side of the loop.

2. Critical Channel Power Analysis

2.1 Best-estimate Code for the Computation of the Critical Channel Powers

Estimates of dryout power for each fuel channel in a reactor core are computed using the thermal hydraulic code, *TUF* [10], through a series of iterative steady-state thermal hydraulic calculations. The initial boundary conditions and bundle power distributions corresponding to the reactivity failure event are used to calculate the channel flow and thermal hydraulic conditions along the channel. Based on the local thermal hydraulic conditions, the critical heat flux (CHF) at each axial node is determined and compared against the axial heat flux. The computed channel power is increased until the critical heat flux profile becomes tangential to the axial heat flux profile. The channel power corresponding to this condition is referred to as the Critical Channel Power (i.e., the channel power required to induce intermittent dryout).

2.2 A Monte-Carlo Method for the Development of CCP Error Models

The Monte-Carlo methodology for evaluating the uncertainties in CCP can be described, in principle, as follows:

Let: $\boldsymbol{X} = (X_1, X_2, \ldots, X_p)^T$ (1)

be a set of important random process and modelling variables required in the calculation of the CCP. Then $\boldsymbol{CCP} = (CCP_1, CCP_2, \ldots, CCP_{480})^T$ is the result of the CCP computations given $\boldsymbol{X}$, that is,

$$\boldsymbol{CCP} = g(\boldsymbol{X}) \tag{2}$$

where g represents the perfect understanding of the system (i.e., prediction of power required to induce intermittent dryout) using a thermal hydraulic code (described in Section 2.1). Most code imperfections are modelled as errors in the input variables. Methods based on data assimilation [4, 11] are utilized to capture the station specific phenomenon and properties such as that described in Figure 1. From above, $\boldsymbol{CCP}$ represents the column vector of Critical Channel Powers for $j = 1, 2, \ldots, 480$ fuel channels. Similarly, $\boldsymbol{ccp} = (\boldsymbol{ccp}_1, \boldsymbol{ccp}_2, \ldots, \boldsymbol{ccp}_{480})$ are the results of the computation from g using input variables $\boldsymbol{x}$, that is,

$$\boldsymbol{ccp} = g(\boldsymbol{x}) \tag{3}$$

where: $\boldsymbol{x} = \boldsymbol{x}_e(1 + \vartheta_x)$ (4)

represents the aleatory variation and $\boldsymbol{x}_e$ is a reference set of thermal hydraulic conditions.

Variables $\boldsymbol{x}_e$ represent true input values and are therefore unknown. These are estimated either by measurements or by available physics or thermal hydraulic codes. These estimates are denoted by $\boldsymbol{X}$ and are assumed to approximate $\boldsymbol{x}_e$ with a relative error, ε_x. This is given as follows:

$$\boldsymbol{X} = \boldsymbol{x}_e(1 + \varepsilon_x)$$

As described earlier, the errors ε_x are epistemic in their nature. We assume that these errors are known, or, could be evaluated based on the available validation data.

Estimates of $\boldsymbol{\varepsilon}_{ccp}$ and ϑ_{ccp} are required inputs into NOP trip set-point calculations. In the NOP analysis, it is of interest to evaluate the CCPs at conditions $\boldsymbol{x}_{\mathbf{e}}$ which lead to $\boldsymbol{ccp_e}$. This gives the following relationship describing the different errors associated with the CCP parameter for each channel *j:*

$$1+\boldsymbol{\varepsilon}_{\mathbf{ccp}_j} = \frac{g_j(x_{e_1}(1+\varepsilon_{x_1}),\ldots,x_{e_p}(1+\varepsilon_{x_p}))}{g_j(x_{e_1},\ldots,x_{e_p})} \tag{5}$$

$$1+\boldsymbol{\vartheta}_{\mathbf{ccp}_j} = \frac{g_j(x_{e_1}(1+\vartheta_{x_1}),\ldots,x_{e_p}(1+\vartheta_{x_p}))}{g_j(x_{e_1},\ldots,x_{e_p})} \tag{6}$$

Note that $\boldsymbol{x}$ represents a *true* variable, i.e., a variable that is known only if given perfect knowledge of the underlying physical system. As such, $\boldsymbol{x}$ is unobservable. When $\boldsymbol{x}$ is taken to be random, then $\boldsymbol{x}$ **is** referred to as an *aleatory* variable. Randomness, or the *aleatory* nature of $\boldsymbol{x}$, comes from the lack of knowledge of the conditions that lead to the actual values of $\boldsymbol{x}$. Note that since $\boldsymbol{x}$ represents true values, the random nature of $\boldsymbol{x}$ has nothing to do with the knowledge of the underlying physical phenomena. To define $\boldsymbol{\vartheta_x}$ it is helpful only if $\boldsymbol{\vartheta_x}$ can be written in some relatively simple analytical terms. Since $\boldsymbol{\vartheta_x}$ is a vector it may be that the components of $\boldsymbol{\vartheta_x}$ are dependent (i.e., the covariance matrix $Var(\boldsymbol{\vartheta_x})$ consist of non-zero off-diagonal entries).

The solution to equations (5) and (6) requires finding the probability distribution for $\boldsymbol{\varepsilon}_{ccp}$ and ϑ_{ccp} using the known parental errors. It can be shown that if the $\boldsymbol{X}$'s approximate the $\boldsymbol{x}$'s reasonably well, then we can substitute the known values of $\boldsymbol{X}$ to facilitate a means to approximate $\boldsymbol{\varepsilon}_{ccp}$ and ϑ_{ccp} [8]. Thus, for a given channel, the error in the CCP can be estimated using the following relationships:

$$1+\hat{\boldsymbol{\varepsilon}}_{\mathbf{ccp}\,j} = \frac{g_j(X_{e_1}(1+\varepsilon_{x_1}),\ldots,X_{e_p}(1+\varepsilon_{x_p}))}{g_j(X_{e_1},\ldots,X_{e_p})} \tag{7}$$

$$1+\hat{\boldsymbol{\vartheta}}_{\mathbf{ccp}\,j} = \frac{g_j(X_{e_1}(1+\vartheta_{x_1}),\ldots,X_{e_p}(1+\vartheta_{x_p}))}{g_j(X_{e_1},\ldots,X_{e_p})} \tag{8}$$

Performing this step many times, estimates of the probability distributions for $\boldsymbol{\varepsilon}_{ccp}$ and ϑ_{ccp} for each channel *j* can be obtained.

2.3 Development of the CCP Statistical Error Model

Using the Monte-Carlo method discussed in Section 2.2 estimates of $\boldsymbol{\varepsilon}_{ccp}$ and ϑ_{ccp} are readily obtained for further statistical analysis. The development of empirical models that clearly distinguish between the aleatory and epistemic variables and preserve the more complex structures of the errors are desirable for accurate NOP trip setpoint solutions. Examining the results of ϑ_{ccp}, a finer error structure is observed between different channels in the core for the aleatory errors.

These channels are defined as *Inner Zone* (*IZ*) and *Outer Zone (OZ*) channels as shown in Figure 1. For the aleatory variable, ϑ_{ccp} (a similar argument holds for ε_{ccp}), let

$$\vartheta_{ccp} = (\vartheta_p^{IZ}, \vartheta_q^{OZ})$$

where ϑ_p^{IZ} with *p=1,2,…,P* are all the channels in the inner zone region and ϑ_q^{OZ} with *q=1,2,…,Q* are all the channels in the outer zone region.

A *5-parameter* CCP statistical error model has been proposed to capture the observed phenomenon as follows:

$$\vartheta_p^{IZ} = \Phi_o + \Phi_o^{IZ} + \Phi_p^{IZ} \tag{9}$$

$$\vartheta_q^{OZ} = \Phi_o + \Phi_o^{OZ} + \Phi_q^{OZ} \tag{10}$$

where:

Φ_o = variation common to both inner and outer zone region channels;
Φ_o^{IZ} = variation common to all inner zone region channels;
Φ_o^{OZ} = variation common to all outer zone region channels;
Φ_p^{IZ} = variation unique to inner zone region channel p; and
Φ_q^{OZ} = variation unique to outer zone region channel q.

Based on available data, the results indicate that the 5 parameters: Φ_o, Φ_o^{IZ}, Φ_o^{OZ} , Φ_p^{IZ}, and Φ_q^{OZ} are well represented as normal and independently distributed random variables each with zero mean and standard deviations: σ_o^2, $\sigma_{oIZ}^2, \sigma_{oOZ}^2, \sigma_{IZ}^2$, and σ_{OZ}^2, respectively.

Thus, the variance of the CCP aleatory variable for each inner zone channel p is given as follows:

$$Var(\vartheta_p^{IZ}) = \sigma_{\mathrm{o}}^2 + \sigma_{\mathrm{oIZ}}^2 + \sigma_{\mathrm{IZ}}^2 \tag{11}$$

Similarly, for the outer zone region channel q:

$$Var(\vartheta_q^{OZ}) = \sigma_{\mathrm{o}}^2 + \sigma_{\mathrm{oOZ}}^2 + \sigma_{\mathrm{OZ}}^2 \tag{12}$$

Equation (11) indicates that the variability in ϑ^{CCP} for the inner zone channel can be described by a random variable that is common to both inner and outer zone region channels, σ_o^2, a random variable common to all inner zone channels, σ_{oIZ}^2, and a random variable that is unique to inner zone channel p, σ_{IZ}^2. The result is similar for outer zone region channels as given in Equation (12).

The covariance of the CCP aleatory variable for each inner zone region channels p_1 and p_2 is given by:

$$COV(\vartheta_{p_1}^{IZ}, \vartheta_{p_2}^{IZ}) = \sigma_{\mathrm{o}}^2 + \sigma_{\mathrm{oIZ}}^2 \tag{13}$$

where $p_1 \neq p_2$. Similarly, for outer zone region channels:

$$COV(\vartheta_{q_1}^{OZ}, \vartheta_{q_2}^{OZ}) = \sigma_{\mathrm{o}}^2 + \sigma_{\mathrm{oOZ}}^2 \tag{14}$$

where $q_1 \neq q_2$. Equation (13) indicates that the variability in ϑ_{ccp} for each inner zone region channel can be described by a variability that is common to both inner and outer zone region channels, σ_o^2 and a variability common to all inner zone region channels, σ_{oIZ}^2. The result is similar for outer zone region channels as given in Equation (14).

In similar fashion, the covariance of the CCP aleatory variable for each inner zone region channel p with each outer zone region channel q is:

$$COV(\vartheta_{p}^{IZ}, \vartheta_{q}^{OZ}) = \sigma_{\mathrm{o}}^2 \tag{15}$$

Using a method of moments, the 5 unknowns σ_o^2, σ_{oIZ}^2, σ_{oOZ}^2, σ_{IZ}^2, and σ_{OZ}^2 are estimated using equations (11) to (15). The solutions to the 5-parameter CCP error model have been shown to give non-negative estimates and model the data very well, as discussed further in Section 3.

3. CCP Uncertainty Analysis Results

Estimates of ϑ_{CCP} and ε_{CCP} are readily available using the Monte-Carlo method discussed in Section 2. This proposed method provides improvements over the deterministic based methods [11] by accurately capturing the statistical dependencies in the system inputs and responses when actual operational data is available (see Figure 2). The operational data define the initial boundary conditions in the calculation of CCPs and are used in place of Monte-Carlo simulations of these input variables. This approach accurately reflects the intricate inner and outer zone design of the HTS (see Figure 1), that is, uncertainties specific to each reactor header are reflected in the response variable (i.e., CCP). This approach eliminates the need to provide accurate estimates of the covariance matrix to describe the multivariate joint probability distributions for these variables that define the initial boundary conditions of the system.

Furthermore, an evaluation of the characteristics of ϑ_{CCP} and ε_{CCP} using tests for normality and independence is possible using results from the Monte-Carlo analysis. As an example[2], plots of histograms and qq-plots for ϑ_{CCP} associated with typical channels in the inner and outer zone are provided in Figures 3, and 4, respectively. The results of the qq-plots indicate that ϑ_{CCP} is well represented by normal distributions and support a normal assumption for modeling ϑ_{CCP} for all channels in core. Furthermore, statistics such as the mean error and standard deviations for ϑ_{CCP} are computable and illustrated in Figure 5.

Plots of the correlation coefficients are provided in Figure 6 for the CCP aleatory variable. These plots are used to evaluate the potential correlation structure in ϑ_{CCP}. The correlation results suggest that ϑ_{CCP} cannot be assumed to be independent but exhibit a correlation structure consistent with the specific flow distribution in the core design (e.g., outer zone channels and inner zone channels). These results indicate that a finer statistical error structure may exist in ϑ_{CCP} and warrant further investigation.

Using the error modelling methodology discussed in Section 2.3, the coefficients of the 5-parameter CCP error model are estimated and used to describe variations that are either common or unique to the inner and outer zone region channels. The randomness in each channel is simulated (i.e., using Monte-Carlo) based on the results of the 5-parameter model. The correlation coefficients are then computed and the results are compared against the actual raw data to test the adequacy of the 5-parameter model. These results are shown in Figure 6 and demonstrate that the proposed 5-parameter model captures the complex error structure observed in the data very well.

4. Summary

This paper presented an approach to model the error in the CCP parameter that is used as input into the NOP trip setpoint calculations. A key aspect of the error modelling is the separation of aleatory and epistemic errors. A 5-parameter CCP error model has been proposed to describe variations that are either common or unique to the inner and outer zone region channels. This proposed error model has been found to fit the data very well and facilitates the input of what would otherwise be a rather complex statistical structure into the NOP trip set-point computation.

The proposed Monte-Carlo method for error analysis provides improvements in the evaluation of the NOP trip coverage over the traditional methods which assume parametric models that may not accurately represent the statistical error structure.

[2] Note the same arguments and results hold for ε_{CCP}.

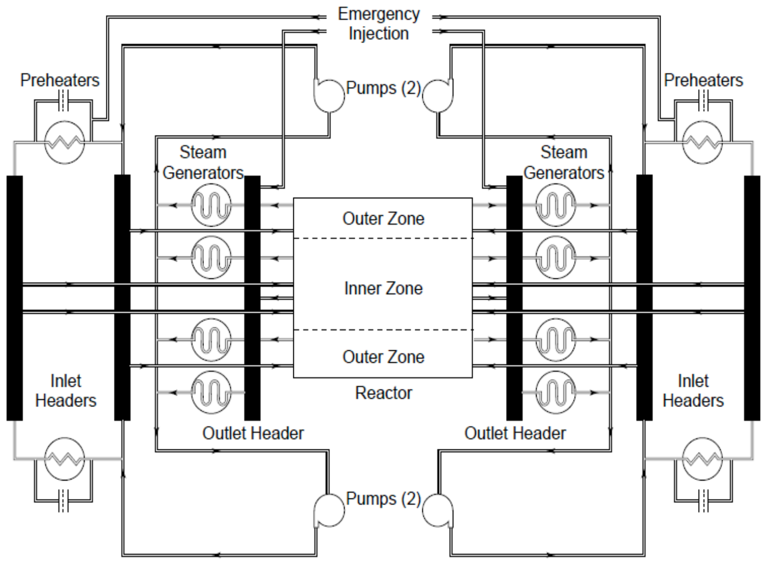

Figure 1 – Bruce NGS CANDU reactor with Inner and Outer Thermal hydraulic flow zones (images taken from [9]).

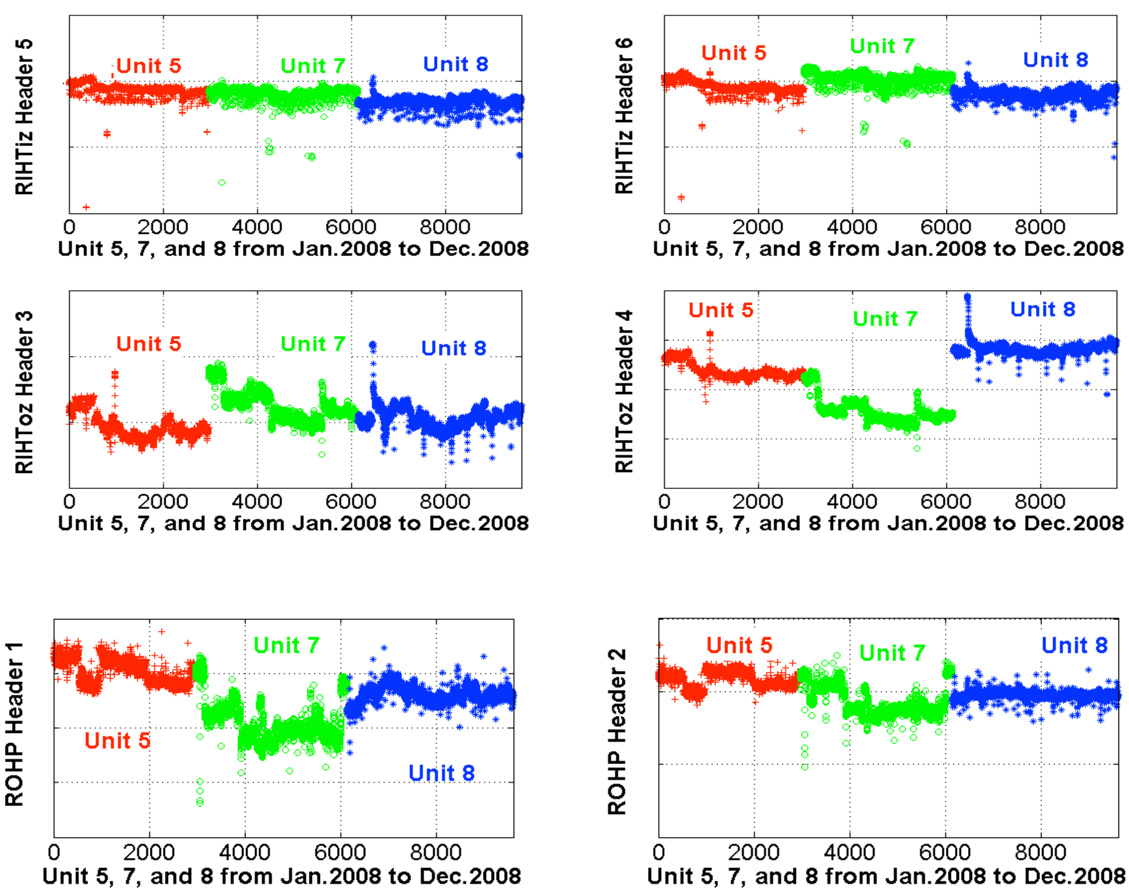

Figure 2 – Time series of raw operational data **TOP**: Reactor Inlet Header Temperatures (inner zone and outer zone); **BOTTOM:** Reactor Outlet Header Pressure (ROHP).

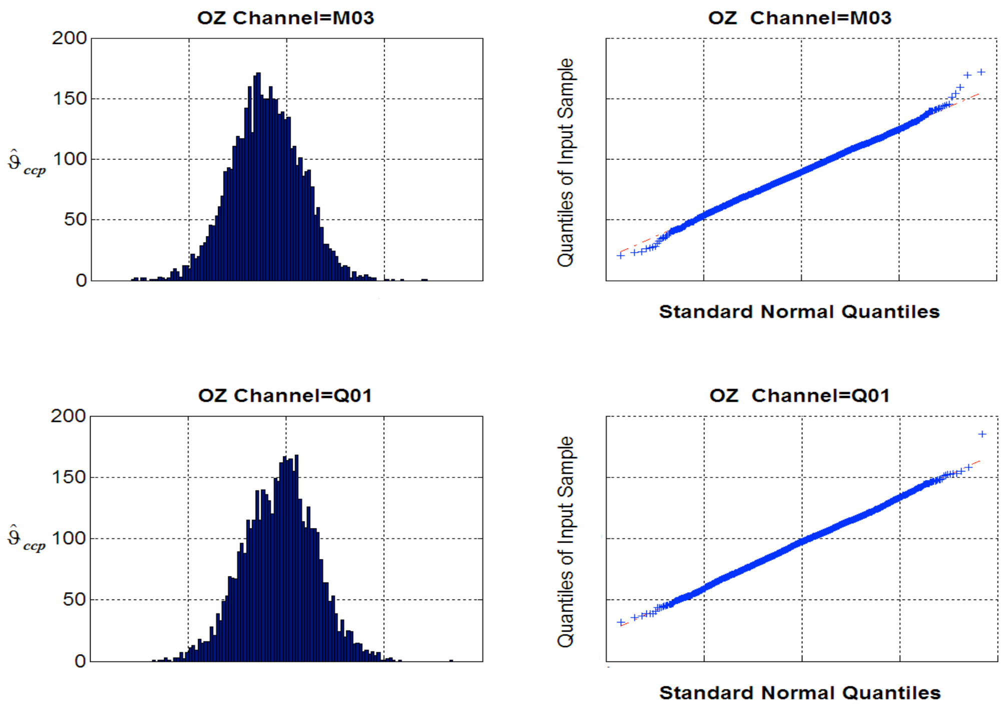

Figure 3 – Monte-Carlo analysis results: histogram and qq-plots of the aleatory error for channels in the outer zone (i.e., channels M03 and Q01).

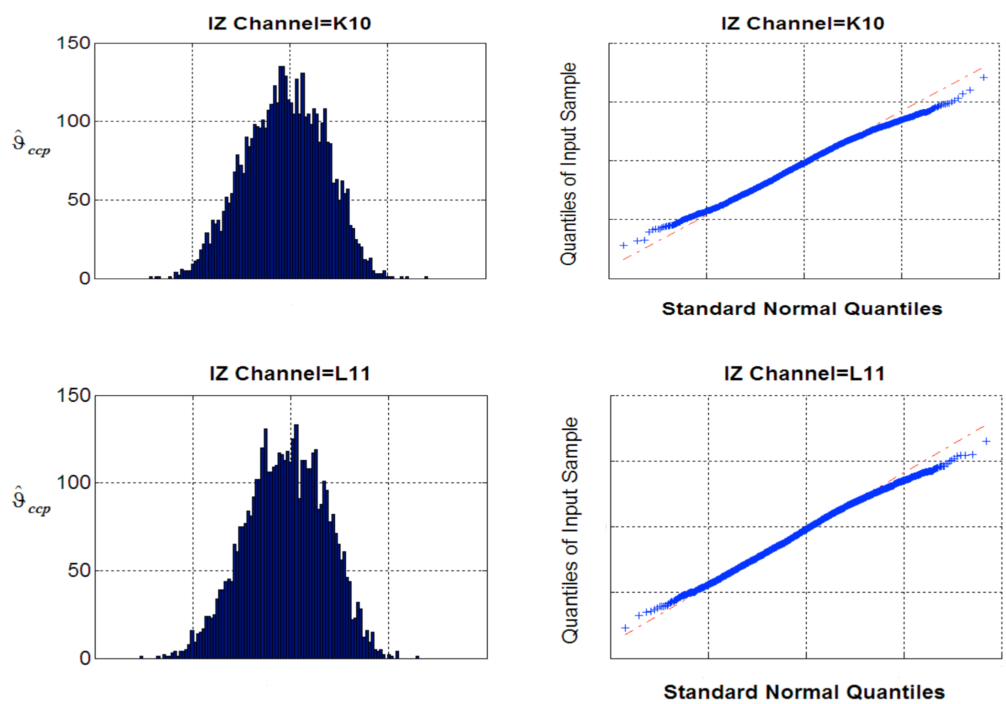

Figure 4 – Monte-Carlo analysis results: histogram and qq-plots of the aleatory error for channels in the inner zone (i.e., channels K10 and L11).

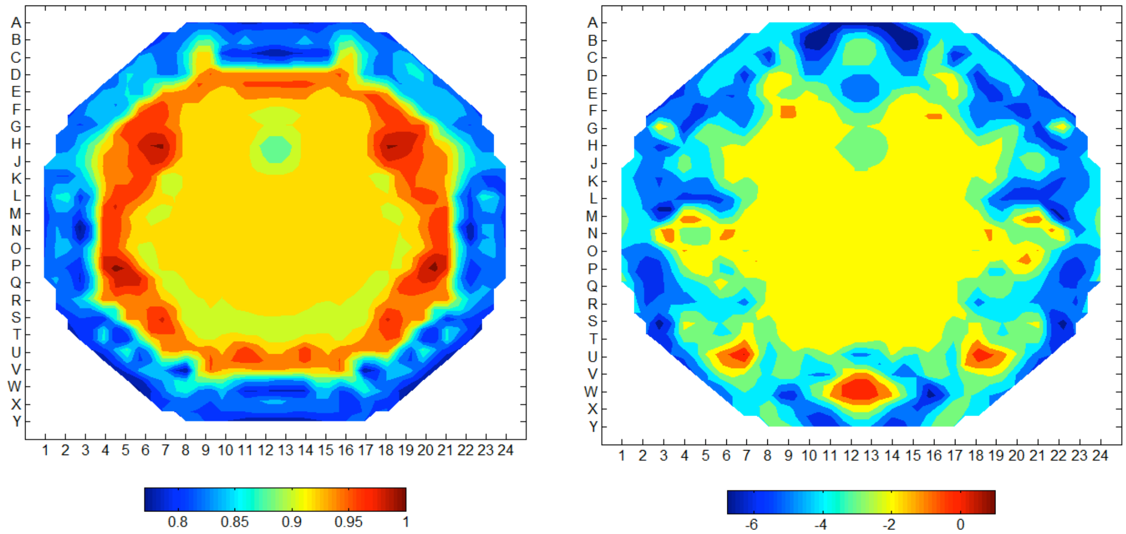

Figure 5 – **LEFT**: Standard deviations[3] of $\hat{\vartheta}_{ccp_j}$ for each channel *j* **RIGHT**: Mean error[4] of $\hat{\vartheta}_{ccp_j}$ for each channel *j*

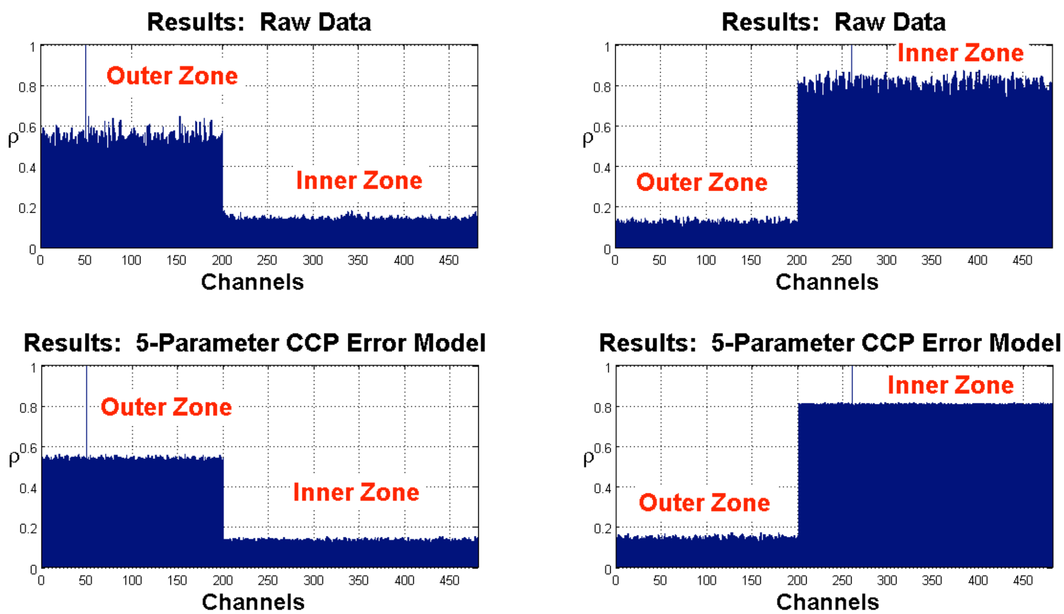

Figure 6 – **TOP:** Plots of correlation coefficients based on the actual raw data for each channel. **BOTTOM:** Correlation coefficients based on simulations from the results of the 5-Parameter CCP error model.

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[3] Values are normalized by the maximum variance of $\hat{\vartheta}_{ccp_j}$

[4] Values are normalized by the maximum mean error of $\hat{\vartheta}_{ccp_j}$

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