

FINITE MIXTURE MODEL APPLIED IN THE ANALYSIS OF A TURBULENT BISTABLE FLOW ON TWO PARALLEL CIRCULAR CYLINDERS

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Abstract

This paper presents a study of the bistable phenomenon which occurs in the turbulent flow impinging on circular cylinders placed side-by-side. Time series of axial and transversal velocity obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel are used as input data in a finite mixture model, to classify the observed data according to a family of probability density functions. Wavelet transforms are applied to analyze the unsteady turbulent signals. Results of flow visualization show a predominantly two-dimensional behavior. A double-well energy model is suggested to describe the behavior of the bistable phenomenon in this case.

1. Introduction

Circular cylinders nearly disposed are found in several engineering applications. The turbulent flow impinging on two circular cylinders placed side-by-side presents a floppy and random phenomenon that changes the flow mode. This behavior is called in the literature as bistable flow and is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other, which generates two dominant Strouhal numbers, each one associated with one of the two wakes formed: the wide wake is associated with a lower Strouhal number and the narrow wake with a higher one. The study of the behavior of the bistability phenomenon in simplified geometries, as in the case of two tubes placed side-by-side, helps in understanding the parameters and variables that influence more complex geometries, as in the case of banks of tubes or rods, which are found in the nuclear and process industries, being the most common geometry used in heat exchangers. As flow induced vibration and structure-fluid interaction are very dependent of the arrangement or configuration of the cylinders and since bistability can be an additional excitation mechanism on the tubes, new studies are justified with new techniques to better understand and classify this phenomenon.

Finite mixture models are statistical tools applied in many knowledge areas to perform the PDF-estimation of an incomplete data problem, and recently the mixture of skew distributions has been found to be effective in the treatment of heterogeneous data with high asymmetry across subclasses. Through this approach bistable phenomenon can be considered as an incomplete data problem and experimental time series can be used as observed data. A maximum likelihood estimation (MLE) can be performed to know what the probability distribution function is more likely to have produced the observed data.

Wavelet transforms are a useful tool for the analysis of transient turbulent signals. Also, flow visualization techniques helps in the comprehension of the phenomena studied in laboratory conditions. The objective of the present work is to apply these techniques in experimental time series of bistable flow of the simplified geometry of two circular cylinders placed side-by-side to better comprehend the switching of the gap flow and to classify the data according to a representative PDF in a mixture model approach.

2. The bistable effect

The cross steady flow trough circular cylinder with same diameter (d) placed side-by-side can present a wake with different modes [1], depending on the pitch-to-diameter ratio p/d . For intermediate pitch ratios ($1.2 < p/d < 2.0$), the flow is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other, as shown schematically in Fig. 1a and Fig. 1b. This phenomenon generates two dominants vortex-shedding frequencies, each one associated with a wake: the wide wake is associated with a lower frequency and the narrow wake with a higher one. The switching of the gap flow, which is biased toward the cylinder, from one side to other at irregular time intervals, is therefore known as a flip-flopping regime or bistable flow regime [2]. Figure 1 presents a link between the wakes patterns (Figs. 1a and 1b) and a velocity measurement technique, performed by the hot wire anemometry technique (Fig. 1c). The velocity signals are measured downstream the cylinders, along the tangent to their internal generatrices, where one switching mode can be observed (modes 1 and 2). Previous studies show that this pattern is independent of Reynolds number, and it is not associated to cylinders misalignment or external influences, what suggest an intrinsically flow feature. The transition between the asymmetric states is completely random and it is not associated with a natural frequency. Through dimensional analysis it was observed that the mean time between the transitions is on order 10^3 times longer than vortex shedding period, and the mean time intervals between the switches decreases with the increasing of Reynolds number. There is no correlation between the bistable feature and the vortex shedding, due to the fact that Strouhal numbers are relatively independent from the Reynolds numbers [3]. Wavelet analysis is a useful tool for the study of transient turbulent signals [4, 5], and in special the switching phenomenon in two side-by-side cylinders [6].

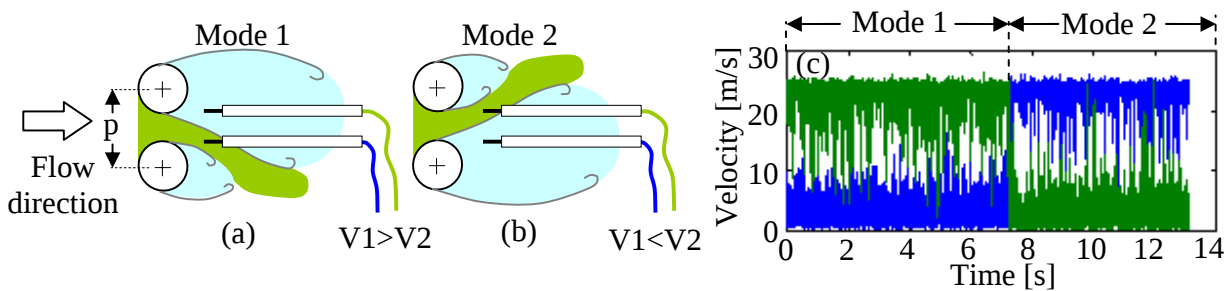


Figure 1 Bistability scheme for (a) mode 1 and (b) mode 2, and their respective characteristic hot wire anemometry signals (c).

3. Methodology

Time series of axial and transversal velocity obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel are used as input data in a finite mixture model, to classify the observed data according to a family of probability density functions (PDF).

In this context, the bistable phenomenon is considered as an incomplete data problem and PDF-estimation is performed using the observed data (time series), where the number of clusters was considered equal to two. An expectation-maximization (EM) algorithm [7] together with a Monte Carlo (MC) method is applied to estimate the maximum log-likelihood function [8] according to a skew Student's t PDF [9], to know what the probability distribution function is more likely to have produced the observed data. Finite mixture models have become more frequently used to provide a natural framework for unobserved heterogeneity in a population [10]. A finite mixture model using the Student's t distribution has been recognized as a robust extension of normal mixtures [11]. The joint time-frequency domain analysis was made through wavelet transform. Basically, a wavelet analysis is applied to time varying signals, where the stationarity hypothesis cannot be maintained. A discrete wavelet transform (DWT) is used to make a multilevel decomposition of a time signal in several bandwidth values, accordingly with the selected decomposition level. A continuous wavelet transform (CWT) is used to analyze the energy content of a signal through the so called spectrogram. Daubechies "db20" functions were used as bases of wavelet transforms.

3.1 Experimental technique

The velocity of the flow and its fluctuations, as well as the flow deviation angle, are measured by means of a DANTEC *StreamLine* constant hot-wire anemometry system, with a double hot wire probe (type DANTEC 55P71 Special), with a wire perpendicular to the flow, and a slant wire 45° with the probe axis. The measurements were performed aligning the probes along the tangent to the external generatrices of a cylinder (Fig. 2b). The aerodynamic channel used in the experiments is made of acrylic, with a rectangular test section of 0.146 m height, width of 0.193 m and 1.02 m of length (Fig. 2a). The air is impelled by a centrifugal blower of 0.64 kW, and passes through two honeycombs and two screens, which reduce the turbulence intensity to about 1% in the test section. Upstream the test section, placed in one of the side walls, a Pitot tube which measures the reference velocity of the non-perturbed flow. Data acquisition is performed by a 16-bit board (NATIONAL INSTRUMENTS 9215-A) with USB interface, which converts the analogical signal to digital series. The acquisition frequency of time series was of 1 kHz, and a low-pass filter of 300 Hz was used to avoid aliasing. The circular cylinders, with external diameter of 25.1 mm, are made of Polyvinyl chloride (PVC), and are rigidly attached to the top wall of test section. Their extremities are covered to avoid the possibility of acoustic resonance, which can be excited in one side closed cylinders (open cavities). The blockage ratio (β) is of 26 %, and recent studies show that the bistable behavior is identified until 51.8 % [12]. However, by increasing β , a gradual decrease in the number of changes occurs. Thus, no corrections were applied to eliminate the effect of the blockage ratio in the results. The probe support is positioned with 3D transverse system placed 200 mm downstream the outlets (Fig. 2c). The mean error of the flow velocity determination with a hot wire was about +/- 3%. The Reynolds number of the experiment is 2.27×10^4 , computed with the tube diameter and the reference velocity (14 m/s), and the pitch-to-diameter ratio is $p/d=1.26$.

3.2 Finite mixture model simulations

The finite mixture model simulations were performed using as input data the experimental time series. A skew Student's t PDF was used in the estimation the maximum log-likelihood function, which is view an inference problem. It is defined a Q function, or the expected complete data

log-likelihood function. An EM algorithm is an iterative method for the computation of the maximizer of the posterior density, which has two steps: the E-step, which is the current estimative of the Q function; and the M-step, which is the maximizer of the Q function. Due to difficulties in evaluating the E-step, caused by the complexity of the target distribution, which does not admit a close-form solution to the Q function, the iterations may be executed by a MC process, performing independent draws of the missing values from the conditional distribution and then approximating the Q function. In the M-step, the Q function is maximized over the parameters vector and a new estimative is calculated. As MC error is introduced at the E-step, the monotonicity property is lost. To assess the convergence of the algorithm, the MC sample size was increased with the number of iterations and its stability was monitored with a tolerance of 10^{-3} .

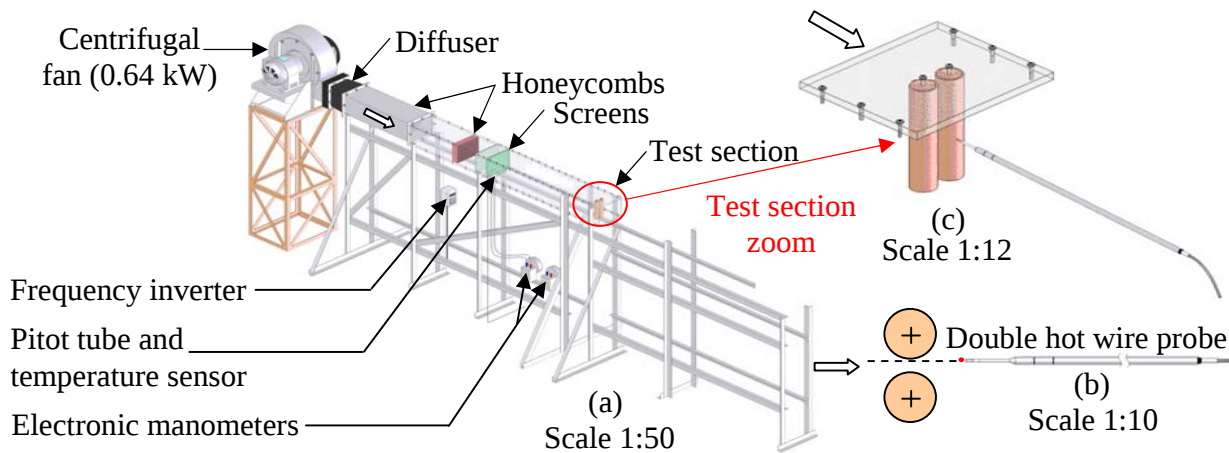


Figure 2 Schematic views: (a) aerodynamic channel, (b) test section and (c) probe position.

4. Results

The time series of axial velocity, transversal velocity and the angle of deviation of the flow are shown in Fig. 3a, Fig. 3b and Fig. 3c, respectively. Figure 3a shows several changes between two distinct velocity levels from the axial component, concerning to 3.0 m/s (wide near-wake - mode 1) and 18.6 m/s (narrow near-wake - mode 2). These changes are accompanied by the transversal component (Fig. 3b), and as the flow changes direction, from the wide near-wake to the narrow near-wake mode, the angle of incidence tends to have smaller fluctuations (Fig. 3c). The velocity components PDF present the predominance of two major states of energy, with different shapes (Fig. 3d and Fig. 3e). The angle of deviation of the flow tends to have smaller fluctuations when the flow direction changes from the wide near-wake to the narrow near-wake, and their PDF presents a concentration around a single value (Fig. 3f).

Figure 4 presents a joint analysis with DWT and CWT of the time series, where a multilevel decomposition in several bandwidth values, accordingly with the selected decomposition level, can be performed together with the analysis of their energy content, or spectrogram. The frequency intervals of the spectrograms vary from 10 to 150 Hz, with a bandwidth of 2 Hz. The time series were parameterized to present the same energy scale. Results show that there is an increase in energy content of velocity signals when the mean velocity is higher, with a relative spreading of frequencies (Figs. 4a and 4b), since 10 Hz until approximately 120 Hz. When the direction of the gap flow changes, to the wide near-wake, there is a decreasing in the energy

content followed by a higher concentration of the frequencies, until approximately 30 Hz, what shows that the wide near-wake has a higher frequency concentration, but with lower energy.

The spectrogram of the axial velocity signal shows this behavior more evident, while the spectrogram of the traverse velocity signal is presented in a more discreet. The spectrogram of the angle of deviation of the flow (Fig. 4c) has an opposite behavior, where the higher energy content is distributed along a large frequency band. It was observed that when the time series has higher variance, their energy content is higher.

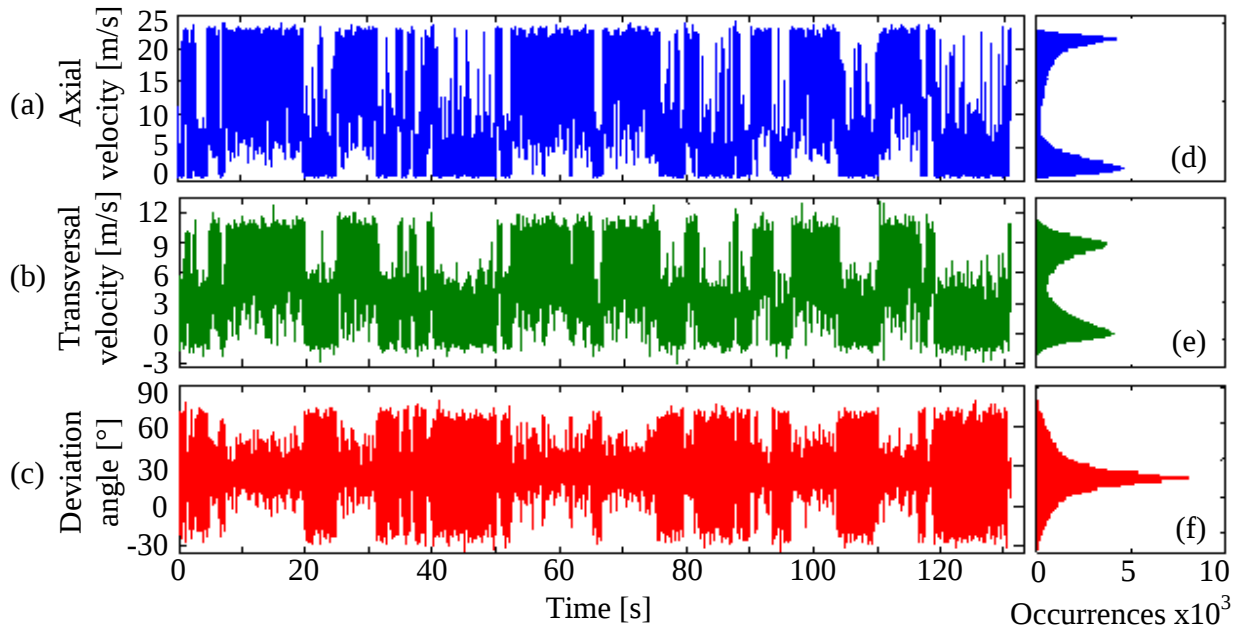


Figure 3 (a) Axial velocity signal, (b) transversal velocity signal and (c) deviation angle of the flow, with their respective PDF ((d), (e) e (f)).

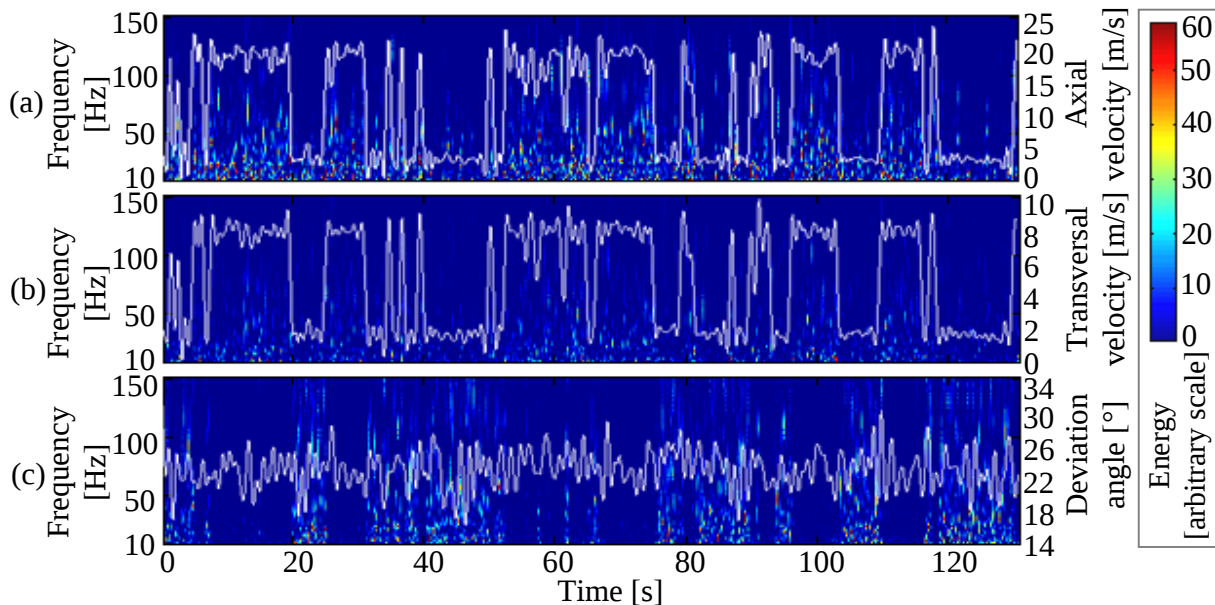


Figure 4 Joint analysis of the CWT and the DWT time series: (a) axial velocity, (b) transversal velocity and (c) deviation angle of the flow.

Dividing the velocity series in blocks and analyzing its dimensionless PDF, it is observed that they vary continuously between the two flow modes (Fig. 5). This figure is built with 125 intervals of 4096 points, with an overlap of 1024 points. Figures 5a and 5c show the results in a 3D view, while Figs. 5b and 5d are shown in color scale, for better identification of energy concentrations. The both velocity signals present a similar behavior. The Strouhal number computed with the gap velocity (18.9 m/s) and the tube diameter is $S = 0.1$ for the wide near-wake, with vortex shedding frequency (f_s) of 75 Hz. For the narrow near-wake, the $S = 0.31$, with $f_s = 230$ Hz.

The results with the finite mixture model simulation to the MLE estimation via EM algorithm is a parameter vector, with all the numerical values needed to express what skew Student's t PDF is more likely to have produced the observed data. For a visual comparison the number of intervals in each PDF variable in simulations was of 100. Figure 6 shows the results of the simulations, where is observed a goodness of fit. However, the fit for axial velocity signal presents small differences with the experimental PDF, which may be caused by the inference of same degrees of freedom for both flow modes. A search for individual degrees of freedom for each mode and also increasing the numbers of clusters could enable better fit results. The numerical results with the finite mixture model simulation are presented in Tab.1, where there is a remarkable difference between the skewness values between the velocity signals.

By analyzing the time variation of the flow modes presented in the PDF of Fig. 5, the probabilities of occurrences of the modes of the velocity signals show that there is no evident correlation between the changes with time (Fig. 7). However, both variables present a similar behavior.

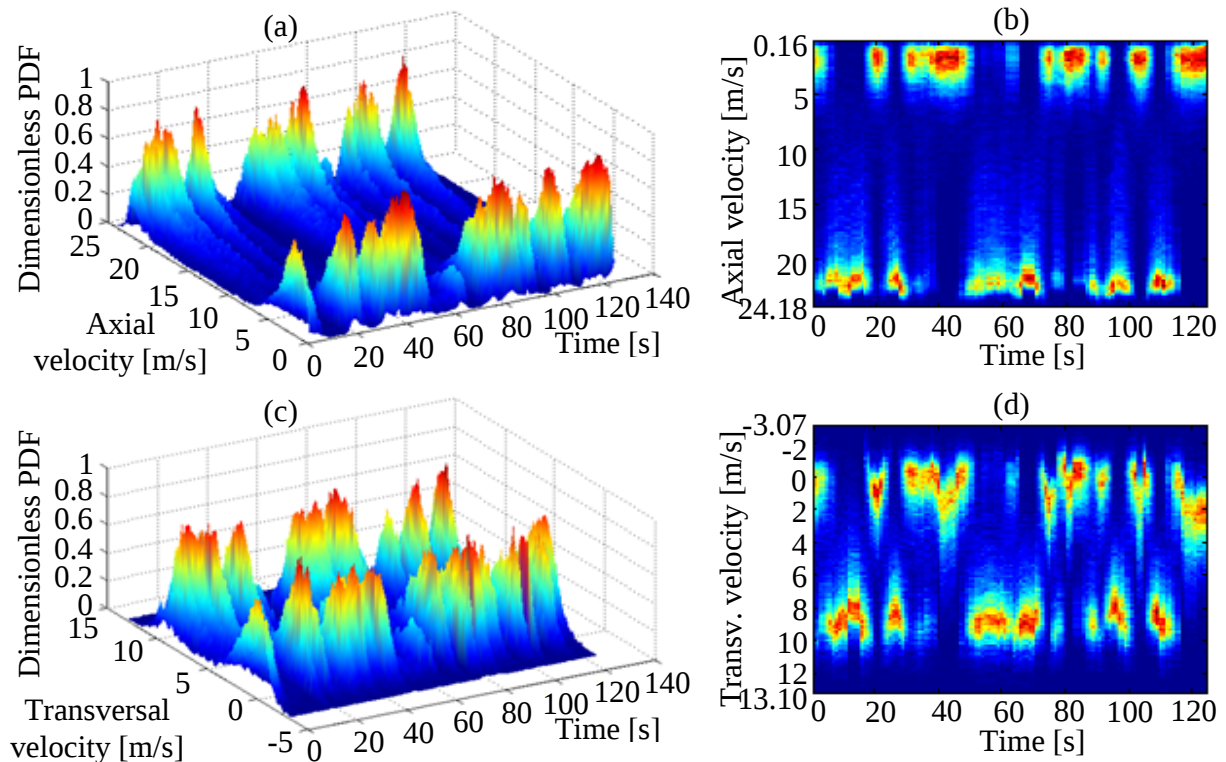


Figure 5 PDF results: (a - b) axial velocity. (c - d) transversal velocity.

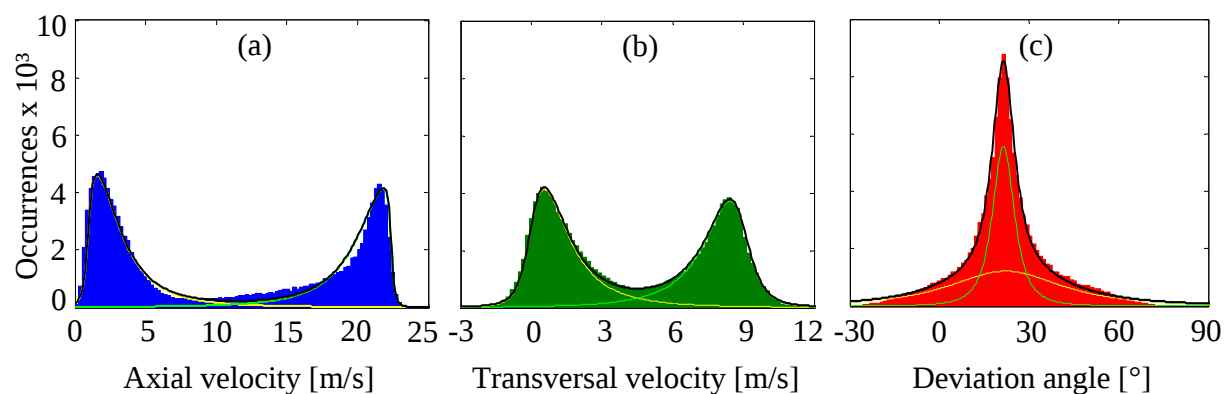


Figure 6 MLE simulations via EM algorithm. (a) axial velocity, (b) transversal velocity and (c) deviation angle of the flow.

Table 1 Numerical results of the MLE simulations of Fig. 6 via EM algorithm.

		Axial velocity [m/s]	Transversal velocity [m/s]	Deviation angle [°]
Mean	Mode 1	3.02	1.31	22.14
	Mode 2	18.67	7.89	22.64
Standard deviation	Mode 1	2.18	1.26	23.31
	Mode 2	3.94	1.81	7.44
Skewness	Mode 1	3.33	0.92	0.09
	Mode 2	-1.48	-1.29	0.52
Max. probability	Mode 1	0.49	0.52	0.32
	Mode 2	0.51	0.48	0.68

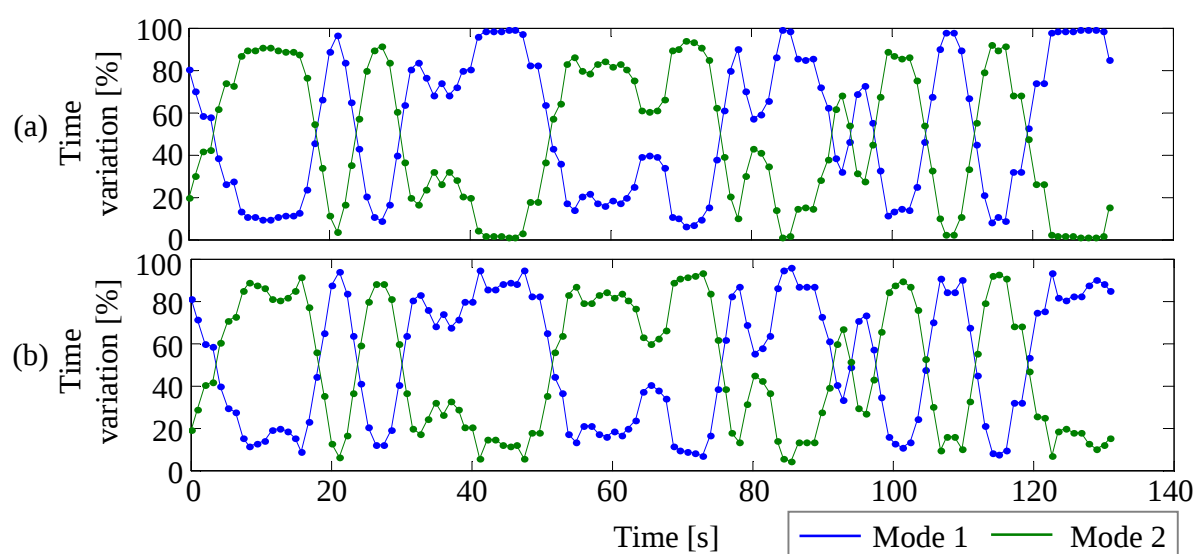


Figure 7 Time variation of the flow modes: (a) axial and (b) transversal velocity.

Flow visualizations performed in water channel with a Reynolds number of 7.5×10^3 , $p/d=1.26$ and $p/d=1.6$ are shown in Fig. 8, where the tubes has 60 mm of diameter [13]. Colored ink is injected in three different plans, according to the feature to be studied. In Fig. 8a and Fig. 8c is observed from a top plan view the formation of a large wake behind one of the tubes (red ink) and a narrow wake behind the other tube (blue ink), which refers to mode 1. After the switching of the gap flow (mode 2), the results are those in Fig. 8b and Fig. 8d, respectively. Figure 8e shows the result of the three simultaneous ink injection plans, from a top plan view, where the flow behavior is predominantly two-dimensional. The same conclusions can be drawn from frontal view visualization, with a slight elevation (Fig. 8f) and from a side view (Fig. 8g). This feature justifies a bivariate analysis of the PDF in the measurement plan.

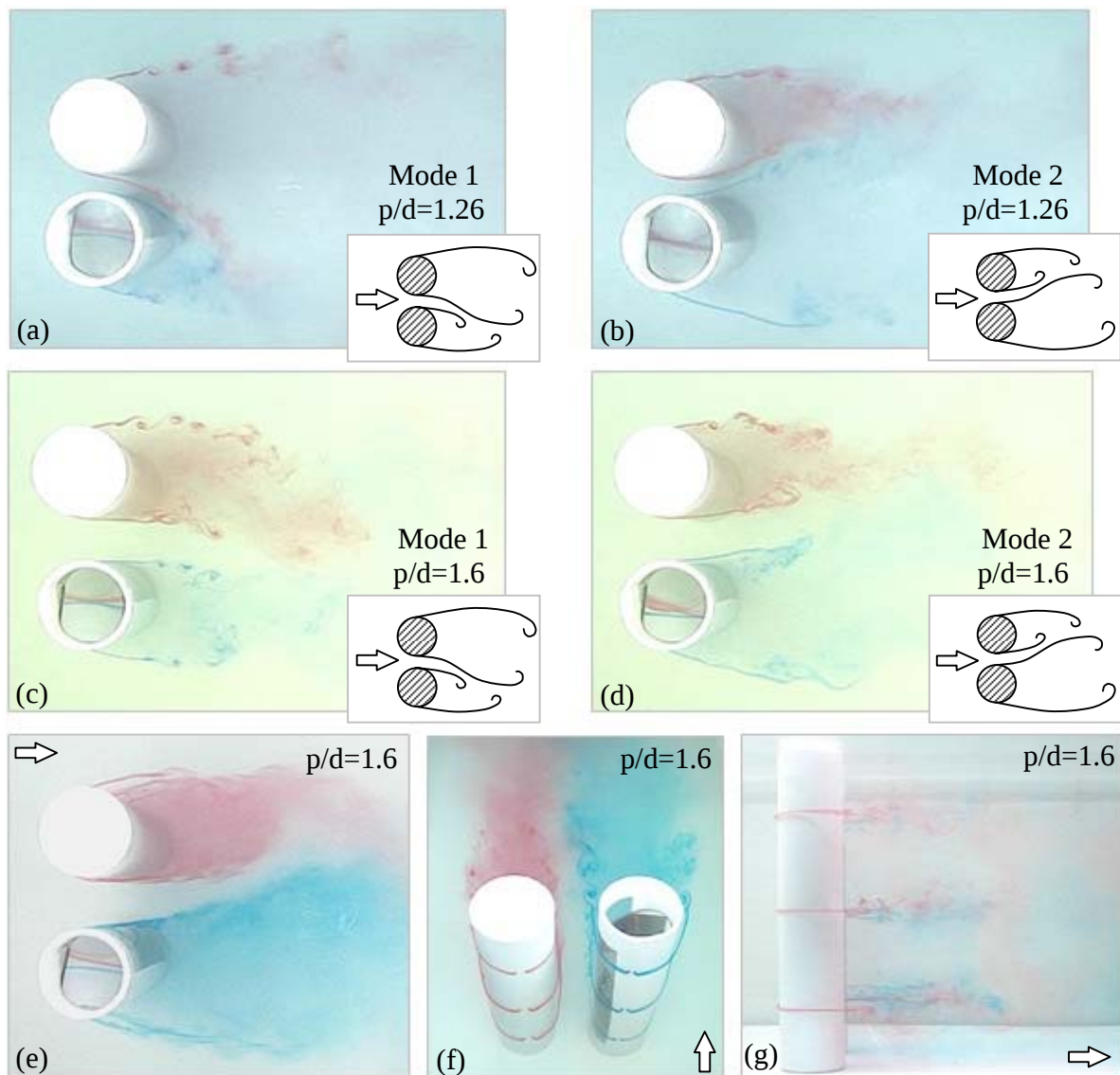


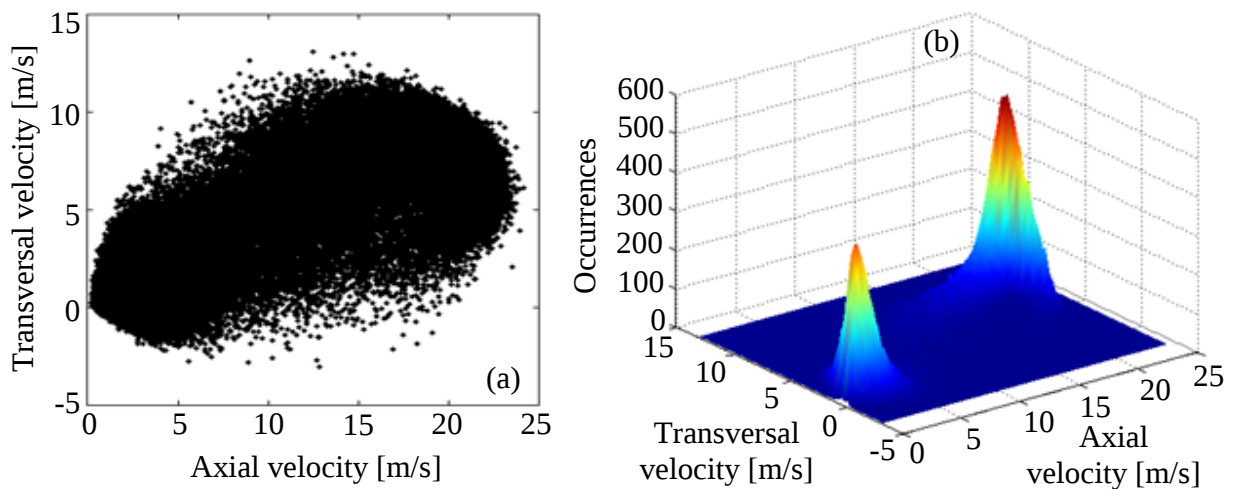
Figure 8 Flow visualizations in water channel of two side-by-side circular cylinders with $p/d=1.26$ and $p/d=1.6$. Top plan view: (a), (b), (c), (d) and (e). (f) Frontal view, with a slight elevation. (g) Side view. $Re = 7.5 \times 10^3$.

Figure 9a presents the results of the ordered pair of data, with both axial and transversal velocity components, where the points are dispersed over a large area. The bivariate velocity PDF presents better results (Fig. 9b), with the presence of two prominences. The higher velocity mode seems to be present in larger quantity in this case.

Reconstructions performed with DWT for levels 5 to 10 were performed for axial and transversal velocity components together, and show the path or temporal trajectory of the flow in the measurement plan (Fig. 10). As higher is the reconstruction level, the less is the data correlation in the plan. This feature can be correlated to the space phase concept, where a particular state of a dynamical system can be represented as a point in this phase space. As the time varies, the point moves along a path.

A double-well energy model (DWEM) is used to succinctly describe the bistable behavior of the system (velocity signals). Two wells of energy, related to the wide and narrow flow modes are separated by a barrier. The behavior of the time series (Figs. 11a, 11b and 11c) is well expressed by this model, which has two minima (M1 and M2) and a maximum point, or transition state (TS), as shown in Figs. 11d, 11e and 11f. This is an analysis of time series in Fig. 3, in the range between 14 and 25 seconds, where the PDF for each time interval is also presented in the graph and the transition between the probabilities concentrations is observed. For the bivariate case, the model is based on the probability distribution function of the problem, normalized by its maximum value (Eq. 1). Figure 12 shows the bivariate model, where an energy transition level (in gray color) is formed, which means that when the flow leaves one of the stable wells, it reaches the transition level and can move to the other well or back to the same again. The higher velocity mode presents a high nonsymmetrical shape (in cross-PDF terms), what suggests the presence of more than two flow modes, as stated by [7], who found that there was an intermediate flow of short duration in which the gap flow was oriented parallel to the free-stream flow.

$$DWEM = \frac{-\log[PDF(u, w)]}{\max(PDF(u, w))} \quad (1)$$



**Figure 9 (a) ordered pair of data points of the flow velocity components.
(b) bivariate velocity PDF.**

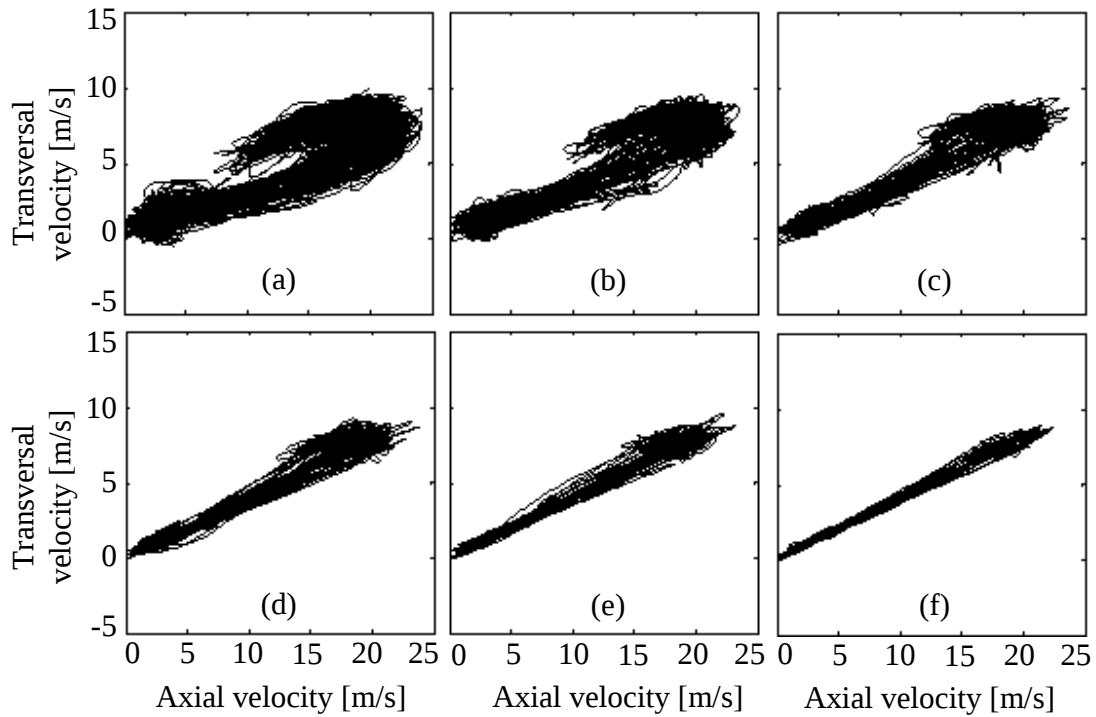


Figure 10 DWT for some reconstruction levels of the bivariate case. Reconstruction levels:
(a) $n = 5$, (b) $n = 6$, (c) $n = 7$, (d) $n = 8$, (e) $n = 9$, (f) $n = 10$.

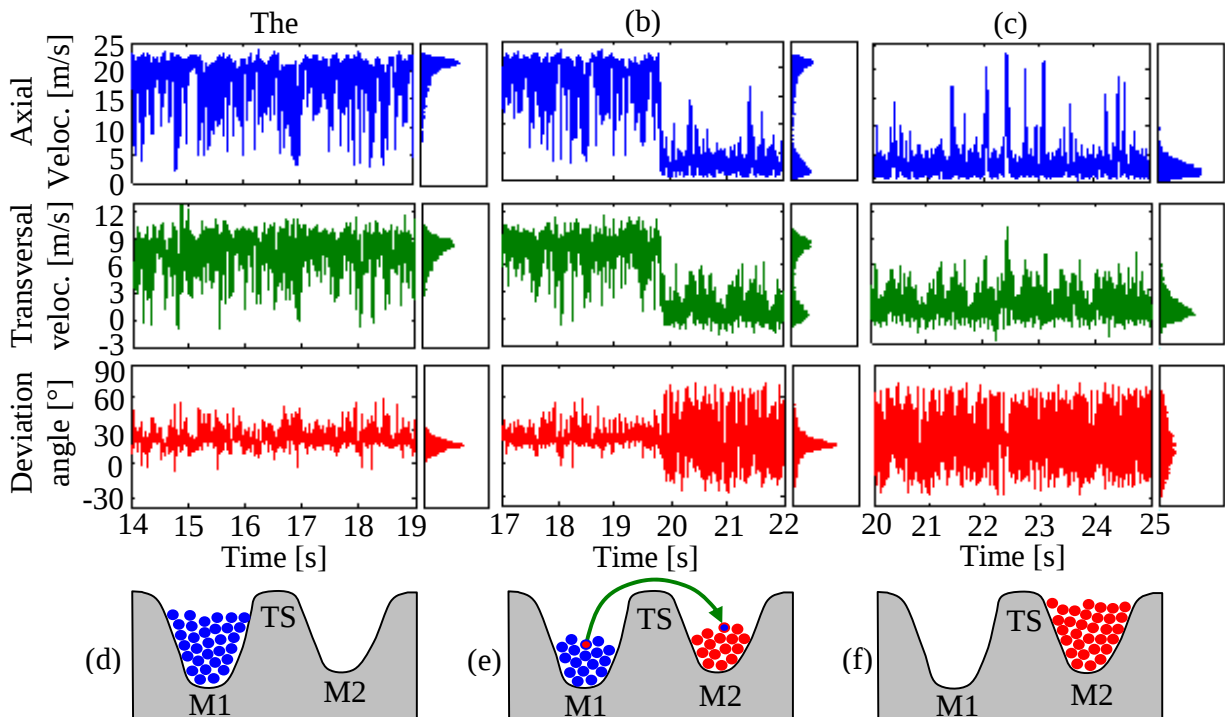


Figure 11 Time series of Fig. 3 in the range between 14 and 25 seconds: (a) mode 1, (b) transition between flow modes and (c) mode 2, with their respective PDF and double-well energy models ((d), (e) and (f)).

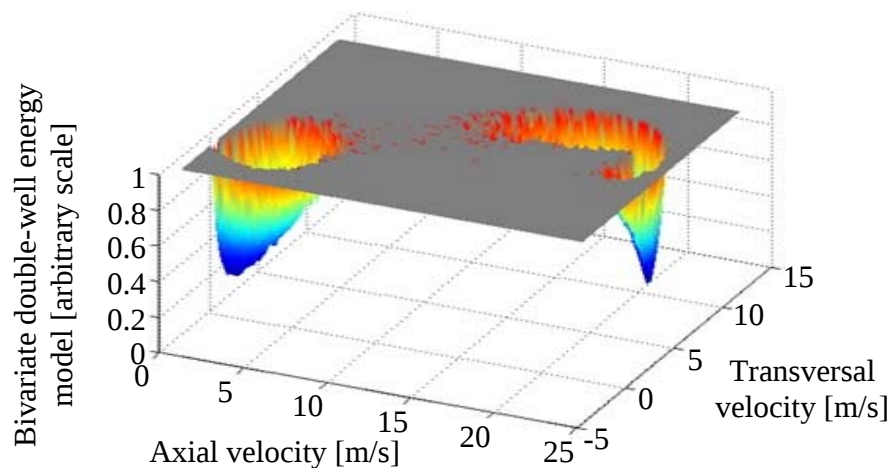


Figure 12 Bivariate double-well energy model of the bistable behavior of the axial and transversal velocity components.

5. Conclusions

This work presents a study about the bistable phenomenon which occurs in turbulent flows impinging on two side-by-side circular cylinders. A finite mixture model tool is applied, where an expectation-maximization algorithm performs the maximum likelihood estimation according to a skew Student's t PDF with aid of a Monte Carlo method, to know what the probability distribution function is more likely to have produced the observed data. Results with the hot wire anemometry technique show that in the changes between the flow modes the increase in the axial velocity component is accompanied by increased of the transverse component, and their PDF present the predominance of two major states of energy, with different shapes. The angle of deviation of the flow tends to have smaller fluctuations when the flow direction changes from the wide near-wake to the narrow near-wake, and their PDF presents a concentration around a single value. The wide near-wake, which has a higher frequency concentration, has a higher mean velocity than the narrow near-wake, but it has lower energy. From the spectrograms of the velocity signals, the energy content is higher and more spread in frequency domain when the mean velocity increases. The spectrogram of the angle of deviation of the flow has an opposite behavior, where the higher energy content is distributed along a large frequency band. When the time series has higher variance, their energy content is higher. This analysis could be possible through the use of wavelet transforms, which shown to be valuable tools for the analysis of transient turbulent signals. The finite mixture model was valuable to determine the numerical values of the shape of the both PDF modes and the results show a goodness of fit. The numerical results of the simulations present a remarkable difference between the skewness values between the velocity signals. Comparisons between the probabilities of occurrences of the modes of the velocity signals show that there is no evident correlation between the changes with time. Flow visualizations show that the flow behavior is predominantly two-dimensional. An energy model was adopted to describe the bistable behavior of the velocity components, where the higher velocity mode, which seems to be present in larger quantity than the lower one, has a high nonsymmetrical shape, what suggests the presence of more than two flow modes. New simulations are in progress to perform the bivariate fit of the phenomenon, as well as a search for individual degrees of freedom for each flow mode and increasing the numbers of clusters for the univariate case.

6. Acknowledgements

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