The 14th International Topical Meeting on Nuclear Reactor Thermalhydraulics, NURETH-14
Toronto, Ontario, Canada, September 25-30, 2011

NURETH14-230

ON A BEST-ESTIMATE APPROACH TO THE CALCULATION OF DRYOUT PROBABILITY DURING BWR TRANSIENTS

Carl Adamsson[1,2] and Jean-Marie Le Corre[1]
[1] Westingouse Electric Sweden, Västerås, Sweden.
[2] currently at Paul Scherrer Institut, Villigen, Switzerland
carl.adamsson@psi.ch ; lecorrjm@westinghouse.com

Abstract

A method is proposed whereby uncertainty of any dryout margin measure (figure of merit) may be quantified when the only experimental information available for validation is whether dryout has occurred or not. The method does not involve the heater temperature, except as a discrete dryout indicator. This is an advantage when analysing anticipated operational occurrences for which the acceptance criterion refers exclusively to the probability of dryout occurrence. The derived uncertainty provides a direct relation between the simulated dryout margin and the aforementioned probability. Furthermore, the method, which is based on logistic regression, has been designed to be consistent with more common parametric methods of uncertainty analysis that are likely to be used for other parts of a thermal hydraulic model. One example is provided where the method is utilized to assess statistical properties, which would have been difficult to quantify by other means.

1. Introduction

Best estimate safety analysis is usually carried out by demonstrating that some safety related parameter, e.g. temperature, does not, with sufficiently high probability, exceed some specific value. The probability is a result of the uncertainty analysis and is based on either parametric or non-parametric statistical analysis. The acceptance criteria, which are defined by the regulatory body in each country, usually require that the result of the uncertainty analysis, such as probabilities, shall be calculated with a certain degree of confidence. This accounts for the fact that the results are based on finite samples since the probability distributions are usually not known *a priori*.

The mathematical framework necessary to perform the uncertainty analysis is now rather well established [1]. It has been developed primarily for the analysis of severe accidents, e.g. a loss of coolant accident (LOCA), and the acceptance criteria that apply to these scenarios. There is, however, another class of events to which similar criteria apply but that has attracted less theoretical attention. These are the anticipated operational occurrences (AOO), which, by definition, are much more likely to occur than accidents. Since these events are relatively common, the acceptance criteria are more restrictive in order to ensure that the fuel is not damaged. The US NRC accepts the following criterion [2] and most other countries apply something similar:

> "For … CPR correlations, there should be a 95-percent probability at the 95- percent confidence level that the hot rod in the core does not experience a … boiling transition condition during normal operation or AOOs."

This criterion is very similar to e.g. the temperature criteria, which apply to less likely event classes. There is, however, an important difference. The dryout criterion for AOOs, as stated above, is formulated in terms of a parameter, e.g. the critical power ratio (CPR), that is not directly measurable during a transient. These numbers must be viewed as figures of merit, which may be useful as indicators of dryout margin but with a scale that cannot be related to any physical quantity [3]. If mechanistic models are used instead of correlations – something that starts to be feasible – the margin would probably be stated in terms of the liquid film thickness or flowrate instead. These are, in principle, measurable quantities but measurements are rarely available at the relevant conditions. The uncertainty analysis must instead be based on measurements of dryout power or wall temperature even though the acceptance criteria are formulated in terms of e.g. CPR. The present paper suggests a theoretical framework for estimating dryout probabilities based on typical steady-state and transient dryout measurements regardless of the specific definition of the figure of merit that is used as a measure of dryout margin.

2. Definitions

The result of a simulation is some measure of dryout margin (film-flow, film-thickness or some other arbitrarily defined measure, e.g. CPR). Given a specific value, M , of the simulated dryout margin we may interpret the dryout probability, $\pi(M)$ as the fraction of dryout tests that would indicate dryout if all possible tests consistent with the margin M could be experimentally evaluated. Thus we interpret the measurements as a stochastic variable, the dryout indicator $Y(M)$, which may take only two values: unity (dryout) or zero (not dryout). Note here that we view the result of the simulation, M , as a known value whereas the outcome of the experiment is treated as a stochastic variable. This is a matter of definitions but this approach reflects the fact that we cannot, in advance, know the outcome of an experiment, partly due to epistemic uncertainties (model deficiencies and simplifications), partly due to aleatory uncertainties (random or unspecified effects). The result of the simulation, on the other hand, is fully known and reproducible.

Were the dryout margin measure perfect we would have $Y(M)=1$ if and only if $M<0$ and $Y(M)=0$ for $M>0$. However, since no model is perfect, the boundary conditions are not known exactly and the two-phase flow and boiling processes themselves may be associated with uncertainties, the dryout indicator function will equal unity with probability $\pi(M)$ and zero with probability $1-\pi(M)$. That is, $Y(M)$ follows a Bernoulli distribution [4] with expectation value $E[Y(M)]=\pi(M)$.

There are many concepts that may be interpreted as measures of dryout margin, e.g. CPR and related concepts, which are, however, not easy to define precisely for transients [3]. The present theory is not intended to be limited to a single such concept. We will therefore discuss some general properties of a dryout margin measure that are required by the theory. An ideal margin measure would satisfy the following requirements:

1. At the onset of dryout the margin should equal zero. (Any other, known constant would work as well but we choose zero for simplicity.)

2. If dryout does not occur the margin should be greater than zero.

3. For a typical series of experiments that proceeds from a non-dryout to a dryout state (or vice-versa) the margin should be approximately linear with respect to the controlling parameters (power, flow, etc). It follows that the margin measure will extrapolate to negative values in the post-dryout heat transfer regime.

We have already concluded that an ideal margin measure cannot exist. The three requirements above will thus never be satisfied by any real dryout model. We may, however, imagine that there is such an ideal measure, M^*, which we will call the “true margin” and that the simulated margin, M, is related to the true margin as $M^* = M + \varepsilon$. We will refer to ε as the uncertainty, lumping the epistemic and aleatory effects. For a given value of the margin M we have

$$\pi(M) = \text{prob}(M^* < 0) = \text{prob}(M + \varepsilon < 0). \tag{1}$$

Note that the true margin, M^*, and the uncertainty, ε, are regarded as stochastic variables whereas the calculated margin, M, and the probability $\pi(M)$ are single-valued. We will also assume that the uncertainty ε does not exhibit any detectable trends or patterns, i.e. that it is reasonable to consider it an uncertainty with respect to the available simulation technology.

3. Application to typical steady-state database

Steady-state dryout experiments are typically performed in such a way that we may assume that the true margin is exactly zero for each experimental point (slowly increasing the power until the dryout power is found). The simulated margin is hence just the uncertainty, ε, that may, in this case, be directly observed. This assumes that the measurement uncertainty can be neglected in comparison with the model uncertainty.

It is common to assume, and sometimes to verify, that the uncertainty approximately follows a normal probability distribution. It is then sufficient to determine the mean, μ, and standard deviation, σ, of the uncertainty distribution in order to establish a relation between the simulated margin, M, and the dryout probability $\pi(M)$. Explicitly we have that

$$\pi(M) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{M - \mu}{\sqrt{2\sigma^2}}\right)\right] \tag{2}$$

This is the standard approach to uncertainty analysis, formulated in the terminology used in this paper.

4. Application to typical transient database

We have a set of transient measurements of heater rod temperature with known boundary conditions. The dryout margin, M, can be calculated based on the boundary conditions as a function of time and position and the measured rod temperatures determine whether dryout occurred or not for each transient. That is, we have a set of observations of the dryout indicator, $Y(M)$, for various, known, values of M and we want to establish a relation between M and the dryout probability, $\pi(M)$, based on these observations. Since the dependent variable, $Y(M)$, can take only the values one (dryout) or zero (not dryout) the relation $\pi(M)$ cannot be assessed by ordinary linear regression. The problem is preferably approached by the mathematical framework called logistic regression [5]. This type of regression is based on generalized linear models, whereby a simple linear model is postulated for a derived variable, connected to the probability through a so called link function. The canonical link function for the binomial distribution, and hence for $Y(M)$, is the logit [6], defined as

$$\mathrm{logit}(\pi) \equiv \ln\left(\frac{\pi}{1-\pi}\right) \tag{3}$$

A linear model for this function is hence reasonable. That is,

$$\mathrm{logit}(\pi) = aM + b\,, \tag{4}$$

where a and b are constants. This is equivalent to the following model for the dryout probability:

$$\pi(M) = \frac{\exp(aM+b)}{1+\exp(aM+b)} \tag{5}$$

This is the standard approach to logistic regression in a single variable. However, any monotonous, increasing function to the interval [0, 1] could be used as link function instead of the logit [5][6]. That is, the inverse of any cumulative distribution function (CDF):

$$\mathrm{CDF}^{-1}(\pi) = aM + b \tag{6}$$

Selecting the CDF of the normal distribution, gives the probit model:

$$\mathrm{probit}(\pi) \equiv \sqrt{2}\,\mathrm{erf}^{-1}(2\pi - 1) \tag{7}$$

The difference with eq. (3) is mainly that the probit is consistent with the common assumption that the steady-state error is normally distributed, i.e. with eq. (2).

For transients we do not have direct observations of the model uncertainty. We hence cannot estimate the mean and standard deviation as easily as for the steady-state case. The observations of the dryout indicator can, however, be exploited by a maximum likelihood technique to find estimates of the coefficients in the model. Explicitly, we maximize the log likelihood [7]

$$L = \sum_{i \in DO} \ln \pi_i + \sum_{i \notin DO} \ln(1 - \pi_i) \tag{8}$$

by finding optimal values of the constant a and b in eq. (6). Here DO is the set of experiments in which dryout was observed. For the probit model, the parameters a and b are related to the standard deviation and mean of the uncertainty ε as:

$$\sigma = \frac{1}{|a|} \tag{9}$$

$$\mu = -\frac{b}{|a|} \tag{10}$$

Note, however, that there is no reason to assume *a priori* that the mean and standard deviation thus estimated would agree with the corresponding estimates for the steady-state application of the same model. Since most dryout models have been developed based on steady-state measurements and transient applications involve more complex phenomena, one would, in general, rather expect the transient uncertainties to be larger.

5. Example: MEFISTO-T film flow analysis

5.1 The MEFISTO and MEFISTO-T codes

The MEFISTO code is a simplified sub-channel film-flow analyzer, specialized on dryout prediction in BWR fuel bundles [8]. The MEFISTO-T code is essentially the same model extended to transient applications [9]. Both codes will calculate the minimum film flowrate per unit wall perimeter (MFF) on all wetted surfaces in the fuel bundle. A special feature is that a negative MFF is calculated when the code predicts post-dryout conditions. A negative MFF does not have any physical meaning; it is merely a figure of merit that measures the distance to the dryout line and in that sense similar to and may replace a transient CPR value.

Since the MFF serves as a measure of dryout margin, corresponds to a measureable quantity (except when negative) and is the primary output of the film-flow analysis, it is natural to let the MFF replace the CPR concept in the safety analysis. All statistics and the calculation of dryout probability will hence be based on the MFF and there will be no reason to even define the CPR concept.

5.2 Analysis of steady-state performance

The MEFISTO code was validated with 1364 dryout experiments from the Westinghouse FRIGG loop [8]. The experiments were performed in full-scale quarter-bundles (24 rods) with pressure varying between 3.0 and 9.0 MPa and including a large number of lateral power distributions and three different axial power distributions. The dryout power was measured (by slowly increasing power until dryout occurred) and the MFF could hence be calculated at the experimental dryout power in each case (compare section 3).

The MEFISTO model was based on a discretization into 35 subchannels and 100 axial nodes. The high axial resolution was necessary in order to resolve the dryout locations in the vicinity of the spacer grids. The statistics of the calculated MFF values showed a mean (μ) close to zero and a standard deviation (σ) of about 0.04 kg/m/s. For the sake of comparison it can be mentioned this corresponded to a CPR standard deviation of about 4%.

5.3 Analysis of transient performance

The MEFISTO-T code was recently validated over a set of 294 transient dryout experiments from the Westinghouse FRIGG loop [9]. The mock-up bundle and test conditions were similar to the steady-state test program. Flow transients as well as power transients and combinations thereof were included (not fast pressure changes, though). Also the model was the same 35-subchannel, 10- nodes model that was used in the steady-state case.

For each experiment the measured heater rod temperatures were utilized to determine whether dryout occurred or not during the transient. From the experience with the steady-state code it was expected that dryout would occur approximately for MFF=0 but the transient results show a clear bias, with many experiments indicating dryout for MFF>0. Work is in progress to fully understand these results, but the biased MFF statistics is an interesting application of the methods developed in the present paper.

Applying the logistic regression based on the probit model as outlined in section 4 above gives the probability curve shown in Figure 1 together with the $Y(M)$ of the entire experimental database. Using the regression parameters and equations (9) and (10) we get a standard deviation σ= 0.034 kg/m/s and a bias μ=0.13 kg/m/s. The bias could have been quantified by simpler methods but the standard deviation could hardly have been calculated without using the presented method. It can be noted that the standard deviation is as low as in the steady-state case, which shows that the transient version of the model provides good precision but a lack of accuracy due to the apparent bias. The cause of this bias should identified and preferably eliminated before the MEFISTO-T code can be relied on for safety analysis but this is not the primary message of the present paper. Instead we want to point out that we have not even defined a transient CPR concept and hence base all the analysis of the transient model on the MFF concept as a measure of dryout margin. By the theory and examples provided in the present paper we also intend to show that it is fully possible to calculate the dryout probability and hence to base the safety analysis on the MEFISTO codes and the MFF concept without defining CPR.

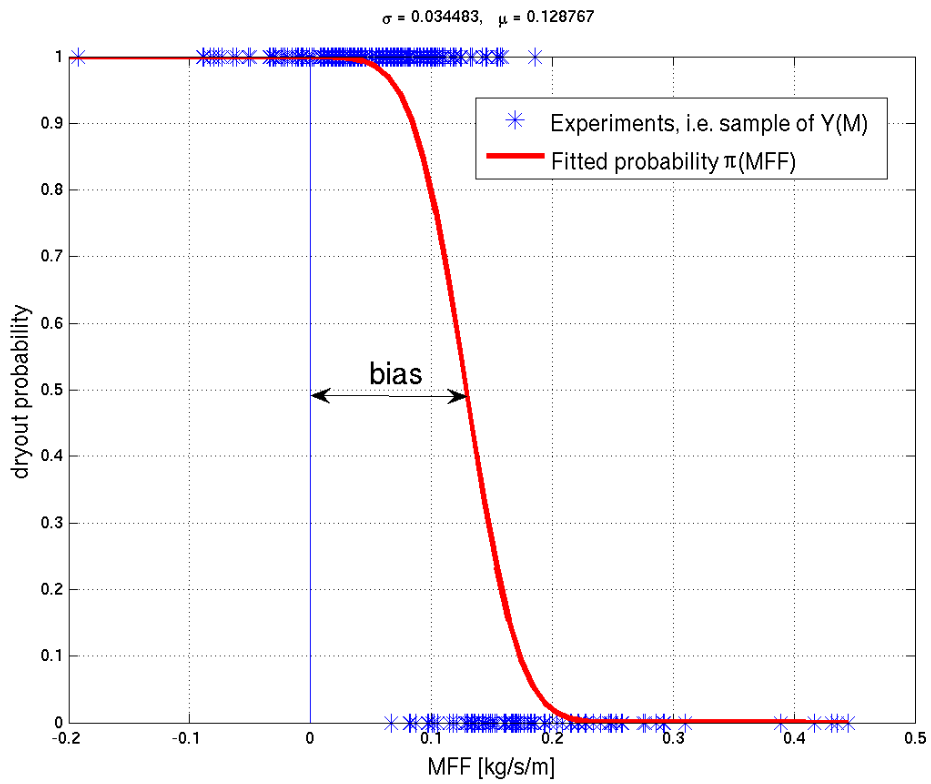

Figure 1 Logistic regression applied to a database of 294 transient dryout experiments simulated with the sub-channel film-flow model MEFISTO-T [9], which produces negative film-flow rates when dryout is expected to occur..

6. Discussion and conclusions

6.1 Local versus global margin and dependent probabilities

In section 3 and 4 we have tacitly assumed that margin for the fuel channel is the smallest simulated margin in both time and space. Having established a relation between any margin measure (e.g. the film flow rate) and the dryout probability we have, however, effectively a local and time-dependent dryout probability. An important question is to what degree these local probabilities are independent. It is quite obvious that the dryout probabilities at two points on a transient separated by short time interval cannot be fully independent. If they were, the total dryout probability would quickly increase with time, even in a zero transient. It follows that for a short enough time span the uncertainties must be completely correlated. It is relatively easy to show that the total dryout probability is then $\hat{\pi} = \max_t \left[\pi\left(M(t)\right)\right]$. In other words, it is sufficient to consider the maximum dryout probability during a transient, i.e. the minimum margin. A similar reasoning applies to spatial separation. The dryout probability at two nearby locations with the same film flow rate cannot be independent.

It is, however, questionable if maximal dependence can be assumed from one film to another, in particular if these films are located in different sub-channels, on different rods or even in different fuel assemblies. It can be noted that this problem is, in part, mitigated by the fact that dryout probabilities during a typical transient are negligible except for a rather small range in time and space. Within such a small range it is easier to motivate that uncertainties are correlated and that the over-all probability should be calculated based on the minimum margin.

It is hardly motivated to assume that the dryout probabilities in separate fuel assemblies are strongly correlated. In this case it seems more reasonable to assume independent probabilities. The safety criterion, on the other hand, refers to "the limiting channel". This could be interpreted as the channel with the highest dryout probability, which is equivalent to the assumption of completely correlated uncertainties. This is, in fact, the interpretation that is currently used by the industry for the calculation of safety margins. Typically a single fuel rod or a few potentially "limiting" rods are analyzed.

6.2 Steady-state versus transient uncertainties

As noted in section 4, the uncertainties based on the analysis of transient measurements may not agree with the uncertainties from the steady-state evaluation. There are several reasons why the steady-state and transient analysis may disagree. One is that transient simulations involve more complex physical phenomena and correspondingly more complex models. It is, for example, important to correctly model the pressure drop and void fraction in the channel when simulating a transient but these problems can be ignored when studying a single channel in steady-state and the flow-rate is known (measured). Other reasons may be a smaller database of transient measurements and the fact that logistic regression is based exclusively on the dryout classification whereas the steady-state statistics uses the information contained in the measured margin. In general, it seems that the correct approach would be to use the transient uncertainties when analysing transients and steady-state uncertainties only in steady-state or for transients that are slow enough that a quasi steady-state may be assumed, i.e. transients that can be evaluated in a steady-state code. On the other hand, if the transient database is small and does not fully cover the range of applications the transient standard deviation may be underestimated. An indication that this may be the case is if the estimated transient uncertainty is significantly smaller then the corresponding steady-state uncertainty without apparent reason.

6.3 Uncertainties in the boundary conditions

The theory outlined in the present paper concerns the uncertainty inherent in the dryout model or correlation. It is assumed that the boundary conditions used in the statistical evaluation are exact since, in practice, these boundary conditions are measured with much better precision than what can be expected from the model. In that sense the assumptions made are not problematic. In any realistic application, however, the boundary conditions would be calculated by other models, which are probably no more accurate than the dryout model itself.

In the final uncertainty analysis the contributions from all models must be evaluated together. Standard methods for this problem are available. One may, for example, assume that the various contributions to the total uncertainty are independent and sum up all the corresponding variances or use a non-parametric method. It is common, but not strictly necessary, to assume the individual as well as the total uncertainties are normally distributed. The outcome of the logistic regression as described in section 4 fits smoothly into this framework. In particular when the probit model is used, which corresponds to the common assumption of normality.

7. References

[1] G. B. Wallis, “Uncertainties and probabilities in nuclear reactor regulation,” *Nuclear Engineering and Design*, vol. 237, no. 15-17, pp. 1586-1592, Sep. 2007.

[2] Nuclear Regulatory Commision, “NUREG-0800, ‘Standard Review Plan for the Review of Safety Analysis Reports for Nuclear Power Plants, LWR Edition.’,” 2007. [Online]. Available: http://www.nrc.gov/reading-rm/doc-collections/nuregs/staff/sr0800/. [Accessed: 22-Jan-2011].

[3] D. C. Groeneveld, “On the definition of critical heat flux margin,” *Nuclear Engineering and Design*, vol. 163, no. 1-2, pp. 245-247, Jun. 1996.

[4] M. Evans, N. Hastings, and B. Peacock, *Statistical Distributions*, 3rd ed. Wiley-Interscience, 2000.

[5] S. W. Menard, *Logistic Regression: From Introductory to Advanced Concepts and Applications*. SAGE, 2009.

[6] R. Christensen, *Log-linear models and logistic regression*. Springer, 1997.

[7] D. W. Hosmer and S. Lemeshow, *Applied logistic regression*. John Wiley and Sons, 2000.

[8] C. Adamsson and J.-M. Le Corre, “Modeling and validation of a mechanistic tool (MEFISTO) for the prediction of critical power in BWR fuel assemblies,” *Nuclear Engineering and Design*, 2011 in press (doi:10.1016/j.nucengdes.2011.01.033).

[9] C. Adamsson and J.-M. Le Corre, “VIPRE-W / MEFISTO-T – A Mechanistic Tool for Transient Prediction of Dryout in BWR Fuel,” in *The 14th International Topical Meeting on Nuclear Reactor Thermal Hydraulics (NURETH-14)*, 2011.