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CFD ANALYSIS ON HEAT TRANSFER IN LOW PRANDTL NUMBER FLUID FLOWS

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Abstract

Use of Computational Fluid Dynamics (CFD) code is helpful for designing liquid metal cooled nuclear reactor systems. Before using any CFD code proper evaluation of the code is essential for simulation of heat transfer in liquid metal flow. In this paper, a review of the literature on the correlations for liquid metal heat transfer is carried out and a comparison with experimental results is performed. CFD analysis is carried out using PHOENICS-3.6 code on heat transfer in molten Lead Bismuth Eutectic (LBE) flowing through tube. Turbulent flow analyses are carried out for the evaluation of the CFD code. The CFD results are compared with the available correlations. Assessment of various turbulence models and correlations for turbulent Prandtl number in the tube geometry are carried out. From the analysis it is found that, the CFD prediction can be improved with modified turbulent Prandtl number in the turbulence models.

Introduction

The renewed interest in lead bismuth coolant for various nuclear energy systems can be attributed to the inherent safety features of the coolant like, high boiling point, inertness to water and air etc. LBE is being actively considered as the main coolant for Accelerator Driven Sub-critical Systems (ADSS) and other high temperature nuclear reactor systems. The Compact High Temperature Reactor (CHTR), which is being designed in Bhabha Atomic Research Centre (BARC), is also LBE natural circulation cooled reactor [1]. An extensive thermal hydraulic analysis is required for the design of the reactor. The analyses can be divided mainly in two types: a) Heat transfer analysis, required for the design of complete core cooling systems. The basic objective of the heat transfer analysis is to find the appropriate correlations for the prediction of heat transfer coefficient in LBE flow. b) Detailed analysis of three-dimensional thermal-hydraulic behaviour, where application of CFD codes is inevitable.

As far as experimental liquid metal heat transfer studies are concerned, a lot of work was carried out in 50s and 60s. Lubarski and Kaufman [2], Kutalteladze et al. [3] and LBE handbook [4] briefed most of the experimental studies. Kirrilov and Ushakov [5] have given a chronology of liquid metal studies in Russia from 1940 to 1975. A number of experimental studies carried out on heat transfer in liquid metals: mercury, sodium, sodium-potassium and lead-bismuth flowing through tube and annulus geometries are briefed in the above literatures. There are several empirical and semi-empirical correlations available for liquid metal heat transfer, an assessment of which is required to find the suitable correlation for the CFD validation.

This paper deals with the assessment of some heat transfer correlations for LBE flow in pipe. Assessment of PHOENICS-3.6 [6], a commercial CFD code for prediction of heat transfer in liquid

LBE flowing through tube is another part of the paper. Comparative studies carried out with different turbulence models and average turbulent Prandtl number are also discussed.

1. Review of liquid metal heat transfer studies in tube

1.1 Fully developed flow heat transfer

Studies with constant wall heat flux are mostly performed so far because it is the most common phenomena in practical applications. After a literature survey, few correlations are selected for assessment with experimental data on LBE. Table 1 gives the brief of the heat transfer correlations for liquid metals.

Table 1: Literature review of heat transfer correlations in liquid metal flow in pipe with uniform wall heat flux

Reference	Correlations	Range of Application	Remarks
Lyon [7]	$Nu = 7.0 + 0.025\left(\frac{Pe}{Pr_t}\right)^{0.8}$	$0 < Pr < 0.1$ and $4x10^3 < Re < 3x10^6$,	Based on analogy of turbulent momentum and energy transfer
Lubarski and Kaufman [2]	$Nu = 0.625 Pe^{0.4}$	$0 < Pr < 0.1$ and $10^4 < Re < 10^5$,	Based on the experimental studies on various liquid metals
Ibragimov et al. [8]	$Nu=4.5+0.014Pe^{0.8}$	$0 < Pr < 0.1$ and $10^4 < Re < 10^5$,	Experimental
Notter and Sleicher [9]	$Nu=6.3+0.0167Pe^{0.85}Re^{0.8}$	$0.004 < Pr < 0.1$ and $10^4 < Re < 10^6$,	Numerical analysis
Kirrilov and Ushakov [5]	$Nu=5.0+0.025Pe^{0.8}$	$0 < Pr < 0.1$ and $10^4 < Re < 10^5$,	Experimental, with special measure for LBE purification
Cheng & Tak [10]	$Nu=A+ 0.018Pe^{0.8}$ Where, $A=\begin{cases} 4.5 & Pe \leq 1000 \\ 5.4-9x10^{-4}Pe & 1000 \leq Pe \leq 2000 \\ 3.6 & Pe \geq 2000 \end{cases}$	$10^4 < Re < 5.0x10^5$	Formulations based on various correlations.

Notter and Sleicher [9] critically examined the correlations available for liquid metal heat transfer to identify the reasons behind the discrepancy among them. The correlation developed by them is based on exhaustive numerical analysis on eddy diffusivity and minimum error of cloud of experimental data. Recently Cheng and Tak [10] carried out a review of various empirical heat transfer correlations. They studied various correlations applicable for different Peclet number ranges and a new correlation is developed which is applicable for a wide range of Peclet numbers in molten lead bismuth eutectic.

An assessment of various correlations given in Table 1 is carried out with the experimental results from Seban [11], Johnson et al. [12] and Lubarski [13]. A discussion on the assessment is given in later section of the paper.

1.2 Heat transfer in entry region

The heat transfer in the entry region of the pipe is of interest, because Nusselt number varies appreciably in this region. For simultaneously hydro-dynamically and thermally developing liquid metal flow in pipe the experimental results are scarce. The correlation presented by Chen and Chiou [14] for the local Nusselt number in developing flow with constant wall heat flux is given below,

$$\frac{Nu_x}{Nu_\alpha} = 0.88 + \frac{2.4}{x/d_h} - \frac{1.25}{\left(x/d_h\right)^2} \qquad (1)$$

The range of applicability of the above correlation is $\Pr \le 0.03$, $Pe > 500$ and $2 \le x/d_h \le 35$.

Where Nu_α is the Nusselt number for fully developed flow and can be found using one of the correlations given in Table 1.

2. CFD application for liquid metal heat transfer

Recently use of CFD in liquid metal thermal hydraulic studies is gaining increasing acceptance for design of liquid metal cooled nuclear reactor systems [4]. But assessment of CFD codes for liquid metal application is essential before using it for the design of system level studies. Due to the much higher surface tension and the drastically lower Prandtl number of liquid metals compared to water, the CFD modelling of liquid metal flows cannot rely on turbulence models developed and tested for water flows. Thus, for physical reasons, a separate benchmarking of the CFD modelling for liquid metal flows is inevitable.

The most widely used turbulence models in CFD are based on eddy diffusivity concepts (e.g., *k*–*ε* model), which use turbulent Prandtl number to evaluate the turbulent heat transport. IAEA-TECDOC-1520 [15] provides a comprehensive report on the different CFD studies and highlights major hurdles for its application in liquid metal systems. It was emphasized that improvement in turbulence modelling and turbulent Prandtl number formulation is essential for precision results. LBE handbook [4] also points out that careful validation of CFD codes with experimental results is necessary for liquid metal flow related analysis. There are some recent works on validation of CFD codes for liquid metal heat transfer prediction (Cheng & Tak [10] and Chandra et al. [16]).

The application of CFD for three-dimensional LBE flow simulation is essential for liquid metal systems. The reliability and accuracy of the CFD simulation essentially depends on the physical model used especially turbulence models and parameters like mesh structures and turbulent Prandtl numbers.

2.1 Turbulent Prandtl number

In turbulent flow, the thermal eddy diffusivity ε_H is correlated to momentum diffusivity as below,

$$\varepsilon_H = \frac{\varepsilon_M}{\Pr_t} \qquad (2)$$

Where $\Pr_t$ is called turbulent Prandtl number. Reynolds analogy assumes that $\varepsilon_H = \varepsilon_M$, i.e. $\Pr_t$=1.0. Experimental evidence indicates that $\Pr_t$=0.9 is correct for most of the conventional fluids, i.e. air, water, etc. (Kays [17]). During 1950s it was found that the experimentally determined heat transfer coefficients for liquid metal flows falls below what would be predicted using Reynolds analogy. The possible reason was found to be the role of $\Pr_t$ in heat transfer prediction. A lot of experimental and theoretical studies were carried out to evaluate $\Pr_t$. Reynolds [18] assessed more than 30 methods to calculate Pr_t and found that the value of Pr_t in a tube depends on molecular Prandtl number, radial distance from the wall and eddy diffusivity. It was emphasized that semi-empirical methods are more promising for the prediction of Pr_t. Gori et al. [19], carried out numerical analysis for heat transfer in low Prandtl number fluids flowing through pipes. Based on the analysis the correlations for Pr_t are specified for different Reynolds number ranges. Kays [17] assessed the present status of the Pr_t for wide range of Prandtl number. It is found that for liquid metal turbulent Prandtl number is higher than unity. In tube flow with heat transfer from the wall, the value of Pr_t varies in radial direction. Recently, Cheng and Tak [10] derived a correlation for Pr_t based on the heat transfer correlations available in literature and CFD analysis. The correlation is reported to be valid for a wide range of Peclet number. Fuchs [20] carried out experimental studies on sodium and Pr_t values are found at different radial locations. The different methods for prediction of Pr_t for liquid metal heat transfer are briefed in LBE handbook [4]. It is mentioned that semi-empirical methods are the best approach to derive turbulent Prandtl number of fluids.

However for practical application, the empirical correlations are preferred for simplicity. But care should be taken while applying those correlations, because they are developed for particular range of parameters and fluids. Some of the empirical correlations developed for Pr_t for liquid metal applications are discussed below,

Aoki [21] proposed the following correlation,

$$\mathrm{Pr}_t = X\left[1-\exp\left(\frac{-1}{X}\right)\right] \tag{3}$$

Where, $X = 0.014\,\mathrm{Re}^{0.45}\mathrm{Pr}^{0.2}$, Reynolds [18] proposed following correlation for liquid metal flows,

$$\mathrm{Pr}_t = \left(1+100Pe^{-0.5}\right)\left[\frac{1}{1+120\,\mathrm{Re}^{-0.5}} - 0.15\right] \tag{4}$$

Gori et al. [19] carried out numerical analysis for low Prandtl number fluids by using one equation (k) turbulence model near wall region and a two-equation (k-ε) turbulence model in the core region. From the analysis, they recommended that the Eq. (3) and Eq. (4) can be used for 40,000<Re<170,000. For 170,000<Re<260,000, the following correlation is proposed,

$$\mathrm{Pr}_t = 0.85 + \frac{0.005}{\mathrm{Pr}} \tag{5}$$

The above correlation is derived by Jischa and Rieke [22] based on the modelling of transport equations for turbulent kinetic energy, turbulent heat flux and turbulent mass flux. For 260,000<Re< 400,000, Pr_t=0.85 is recommended.

Cheng and Tak [10] developed a correlation for Pr_t, based on the evaluation of various heat transfer correlations for liquid metal flow and CFD analysis, which is given as,

$$\mathrm{Pr}_t = \begin{cases} 4.12 & \text{for Pe} \le 1000 \\ \dfrac{0.01Pe}{\left[0.018Pe^{0.8} - (7-A)\right]^{1.25}} & \text{for } 1000 \le \text{Pe} \le 6000 \end{cases} \tag{6}$$

Where, A is given in Table 1.

2.2 Turbulence models and mesh distribution

As it can be seen from Eq. (2) that ε_H is mainly calculated from ε_M. So it is important to analyse whether the turbulent momentum diffusivity, ε_M is calculated by turbulence models are correct or not. There are several models available in PHOENICS, standard k-ε and its variants, k-ω, RSM model, LES model etc. But the k-ε turbulence model remains the work-horse of industrial computation. Four turbulence models which are available in PHOENICS-3.6 have been taken for the evaluation: a) standard k-ε model, which is the most widely used for turbulent CFD analysis. b) RNG (ReNormalised Group) k-ε model, which takes into account of the small scale turbulence by expressing them in terms of larger scale motions and modified viscosity. c) Chen and Kim modified k-ε turbulence model which improves dynamic response of ε-equation by introducing an additional time scale in the equation and d) LVEL (Length-VELocity) model, which is a zero equation turbulence model and a unique feature of PHOENICS. Equilibrium log-law wall functions are used to provide near-wall boundary conditions for the mean-flow and turbulence transport equations.

In the LVEL turbulence model ν_t is calculated via Spalding's law of the wall, which covers the entire laminar and turbulent regimes, the value of ν_t is calculated from the following equation,

$$\nu_t = \frac{k}{E}\left[e^{X^+} - \sum_{i=0}^{4} \left(X^+ \right)^{i/i!} \right] \tag{7}$$

Where k=0.41, E=8.6, and $X^+=ku^+$. The local parameter X^+ is determined iteratively from a non-linear expression involving k, E and the local Reynolds number Re, which is based on a typical local velocity and the normal distance to the nearest wall [6].

The RNG k-ε turbulence model in PHOENICS is based on "Renormalized Group" theory and is proposed by Yakhot and Orszag [23]. In this model an inverse effective Prandtl number is used to calculate the effective thermal conductivity. The inverse effective Prandtl number varies with μ_{mol}/μ_{eff}, which is consistent with experimental evidence and takes care of turbulent heat transfer. There is no need to modify the turbulent Prandtl number because the model itself takes care of the turbulent heat transfer. It is reported that this model works well across a broad range of molecular Prandtl numbers, from liquid metals to paraffin oils. There are few literatures (Wolters et al. [24] and Tarantino et al. [25]) where this model has been applied to LBE heat transfer studies. So an assessment of this model is also carried out in this paper.

Mesh distribution is another factor to be considered carefully during CFD analysis. Here, 2-dimensional structured grid is used in cylindrical coordinate. The mesh density near the tube wall is important for the turbulent model applications. As the wall function used is equilibrium log law function, the mesh density near the wall should be so that $30 < y_1^+ < 130$ (PHOENICS manual [6]). This requirement is met by adjusting the mesh numbers near the wall at different flow rate.

2.3 Evaluation of Nusselt number from PHOENICS results

To compare the CFD results with empirical heat transfer correlations, Nusselt number is calculated from the temperature distribution obtained by the CFD analysis. The Nusselt number for fully developed region is calculated from the PHOENICS results as described below,

$$Nu = \frac{\bar{h} d}{k} \tag{8}$$

Where $\bar{h}$ is the average heat transfer coefficient, and

$$\bar{h} = \frac{q''}{T_w - T_m} \tag{9}$$

Where q" is the heat flux at the inner surface of the wall. T_W is the inner wall temperature of the tube and T_m is the mean fluid temperature found from the CFD analysis.

2.4 Thermo-physical properties of lead bismuth eutectic

The thermo-physical properties for the analysis are calculated from the following equations as shown in Table 2. Care has been taken to maintain the fluid temperature within the range of application of the properties.

Table 2: Thermo-physical properties of LBE used in the thermal-hydraulic analysis

Properties	**Unit**	**Correlation**	**Reference**	**Temperature range (K)**
Density	kg/m^3	ρ=11096-1.3236T	LBE Handbook [4]	403-1100
Dynamic viscosity	Pa.s	$\mu=4.94\times10^{-4}.\exp(754.1/T)$		400-1100
Thermal conductivity	W/mK	$k=3.61+1.517\times10^{-2}.T-1.741\times10^{-6}.T^2$		403-1100
Specific heat	J/kgK	$C_p=159-2.302\times10^{-2}\,T$	Morita et al. [26]	400-1100

3. Results and Discussions

For the analysis a circular tube with the flow conditions described in Table 3 is considered. The inlet temperature is kept at 773K. Figure 1 shows the 2D axi-symmetric geometry considered for the CFD analysis. A wall with thickness of 1.0mm is considered. The boundary condition of constant heat flux is imposed on the outer surface of the tube wall. The velocity and thermal boundary layers are developed as the flow proceeds towards downstream.

Table 3: Parameters for the turbulent flow analysis

Parameters	**Values**
Coolant	LBE
Tube diameter/length (m)	0.02/2.0
Inlet Temperature (K)	773
Reynolds Number	5.0×10^4 to 4.5×10^5
Wall heat flux (W/m^2)	2.0×10^5
y_1^+	30-130

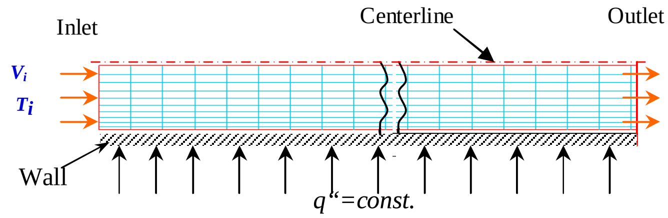

Figure 1. 2D axi-symmetry section of tube with mesh distribution

3.1 Assessment of heat transfer correlations

As discussed in Section 1, there are a number of heat transfer correlations available for liquid metals. Figure 2 depicts variation of Nusselt number as a function of Peclet Number using various heat transfer correlations given in Table 1. The correlations are assessed by comparing experimental results [11, 12, 13] available in literature. The correlation given by Lyon [7] over predicts the experimental data, with Pr_t=1.0, while with Pr_t=3.0 as shown Figure 2, it under predicts. So the influence of Pr_t on heat transfer coefficients can be observed. The correlation from Kirrilov and Ushakov [5] also over predicts the experimental results. It is to be noted that the correlation was developed from the results of experimental studies with pure sodium, whose thermo-physical properties are different from lead bismuth eutectic. The correlation of Notter & Sleicher [9] best fit the experimental data as shown in Figure 2. This correlation has been used in LBE handbook [4] for assessment of other correlations for liquid metal applications. The correlation proposed by Cheng & Tak [10] recently also conforms well to experimental results. This correlation is based on the careful assessment of different correlations for liquid metal applications. The correlation suggested by Lubarski and Kaufman [2] under predicts the experimental results. It should be noted that this correlation was developed from the results from twenty experimental studies. The working fluids of these studies include sodium, lead-bismuth and mercury. Though this can be considered as a general correlation for all liquid metal coolants, it is found that this correlation under predicts in case of lead bismuth coolant. The correlation recommended by Ibragimov [8] also under predicts the experimental results. As mentioned in the literature the possible reason is that the correlation developed is for without control of impurity and oxygen in the liquid metal during the experiments. His objective was to study the heat transfer performance of liquid metal in industrial environment.

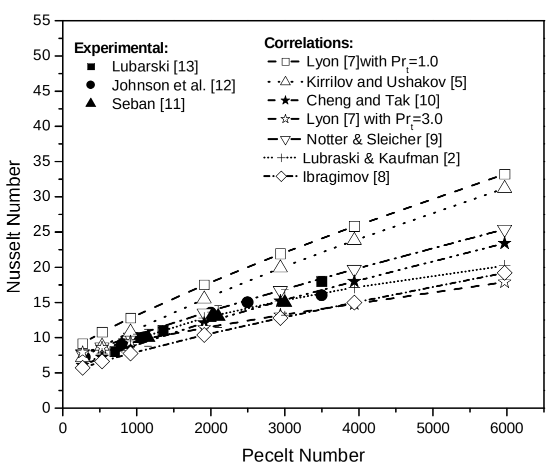

Figure 2. Comparison of liquid metal heat transfer correlations with experimental results

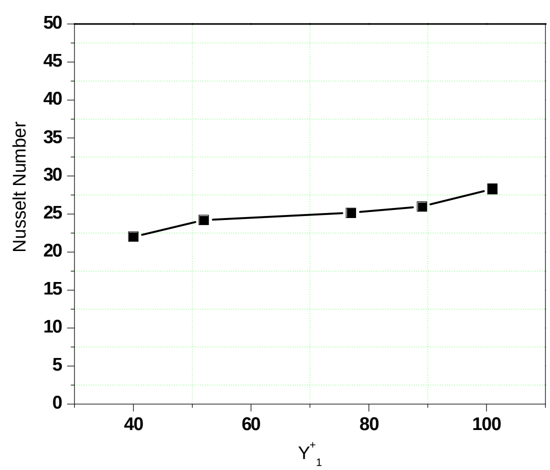

Figure 3. Variation of fully developed Nusselt Number with y_1^+ with k-ε turbulence model

From the above assessment it is found that the correlations of Notter & Sleicher [9] and Cheng & Tak [10] conform better with the experimental results and hence selected for the assessment of CFD predictions in the following sections.

3.2 Effect of mesh size

The geometry is discretised with structured mesh as shown in Figure 1. For discretising the domain, the guide lines suggested by NEA report [27] are followed. As per PHOENICS user manual [6] for k-ε based turbulence model application the value of y_1^+ should be maintained between 30 and 500. However since log law wall function is used for turbulence modelling y_1^+ is maintained between 30 and 120. Figure 3 shows the variation of Nusselt number with y_1^+ for inlet velocity of 2m/s. Standard k-ε model is used for the analysis. It is seen that prediction of Nu in the range of $40< y_1^+ <80$ is unaffected by variation of y_1^+. For further analysis the range of y_1^+ is kept in between 40 and 80 for all the flow velocities by changing the mesh size near the wall.

3.3 Turbulent Prandtl number effect

The turbulent Prandtl number varies with flow rate of the liquid metal. The various correlations for Pr_t are given by Eq. (3) to Eq. (6). Figure 4 shows the variation of turbulent Prandtl number with Peclet number by the above mentioned correlations. The correlation proposed by Cheng &Tak [10] covers entire range of Peclet number shown in the graph, while Reynolds [18] and Aoki [21] correlations are valid up to the Peclet number 2000. Beyond Pe=2000 the correlation from Jischa and Rieke [22] is valid. The Cheng and Tak [10] correlation for Pr_t was derived from empirical heat

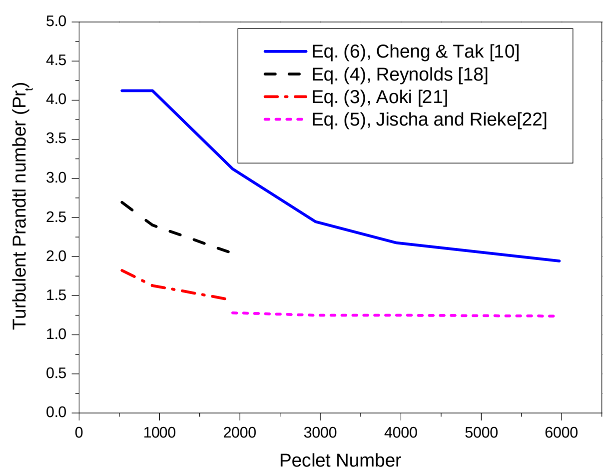

Figure 4. Variation of Turbulent Prandtl Number with Peclet Number for liquid metal

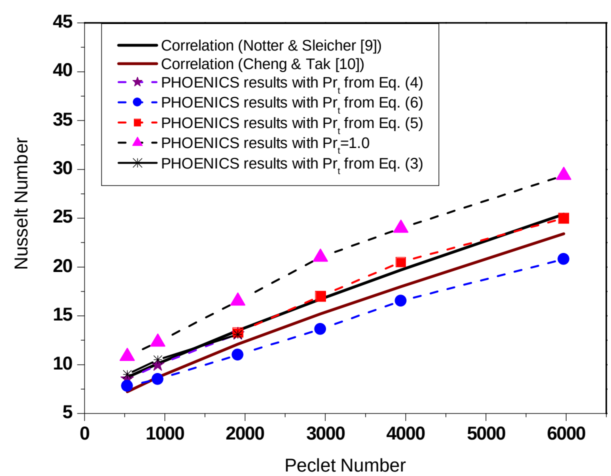

Figure 5. Comparison of CFD prediction using standard k-ε model for different Pr_t with empirical correlation for Nusselt number.

transfer correlations and results of CFD analysis. The value of A in Eq. (6) which is defined in Table 1 is used to fit the various regime of Peclet number. The non-linear values of A makes the Pr_t variation not so smooth, as in the case of other two correlations. It can be seen from Figure 4 that the Pr_t calculated by Eq. (3) is higher than Eq. (4), but trend of Pr_t variation with Pe, by both equations are similar. The prediction by correlation from Jischa and Rieke [22] does not vary significantly with Pe. This correlation is suggested by Gori et al. [19] for high Pe range because at high Pe, turbulence plays dominant role in heat transfer as compared to molecular conduction. So the variation of Pr_t with Pe is not significant. To study the effect of turbulent Prandtl number on the heat transfer prediction using CFD, the Nusselt number for fully developed flow has been calculated for different flow rates. The turbulent Prandtl numbers estimated by Eqs. (3), (4), (5) and (6) are used in the standard k-ε turbulence model in PHOENICS code. Figure 5 shows the variation of Nusselt number with Peclet number. For comparison, the Nusselt number estimated by correlations of Notter & Sleicher [9] and Cheng & Tak [10] is also given in Figure 5. It can be seen that with Pr_t=1.0, the CFD analysis over predicts Nusselt number. In case of liquid metal, thermal conductivity is much higher than that for air or water. The molecular conduction is higher than convective heat transfer near the wall. The turbulent thermal diffusivity plays a lesser role here. So as seen from Eq. (2) when Pr_t = 1.0, $\varepsilon_H=\varepsilon_M$, which means ε_H is overestimated in case of liquid metals. This results in higher prediction of heat transfer as seen in Figure 5. But, as per Eqs. (3), (4) and (6), which are developed especially for liquid metals, Pr_t is always greater than one and can be seen in Figure 4. This makes $\varepsilon_H<\varepsilon_M$ and different values of Pr_t at various values of Pe regulate ε_H. From Figure 5, it can be seen that Pr_t calculated from Eqs. (3), (4) and (5) give best conformation with correlations. The analysis with Pr_t from Eq. (6) under predicts the Cheng & Tak correlation by 15% at Pe=6000, which may be due to the value of Pr_t calculated by the equation is high which suppresses ε_H as per Eq. (2).

3.4 Effect of Turbulence models

The selection of turbulence model is also important for prediction of heat transfer. As discussed in Section 2, four turbulence models are selected for the analysis. Figure 6 shows the prediction of the Nusselt number using various turbulence models. The Pr_t values for this analysis are calculated from Eqs. (4) and (5). The values of Pr_t are incorporated in all the turbulence models except RNG k-ε model. In RNG k-ε model, the turbulent heat transfer is accounted for by calculating effective Prandtl number, which in turn varies with turbulent viscosity and molecular Prandtl number [4]. The analysis using RNG k-ε turbulence model over predicts the Notter & Sleicher [9] correlation by 10% at Peclet number 6000.

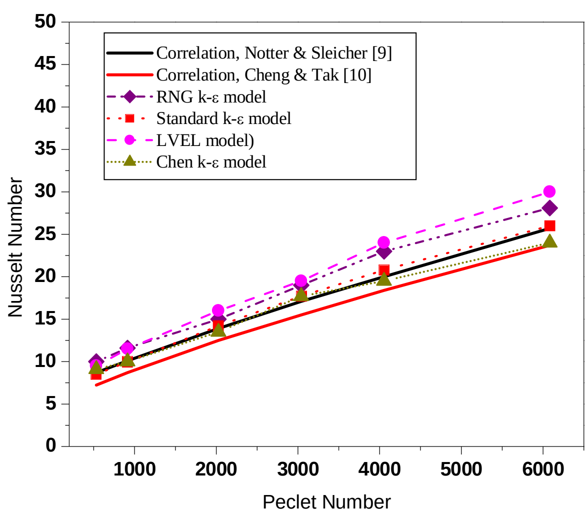

Figure 6. Comparison of CFD prediction using different turbulence models with empirical correlations for Nusselt number

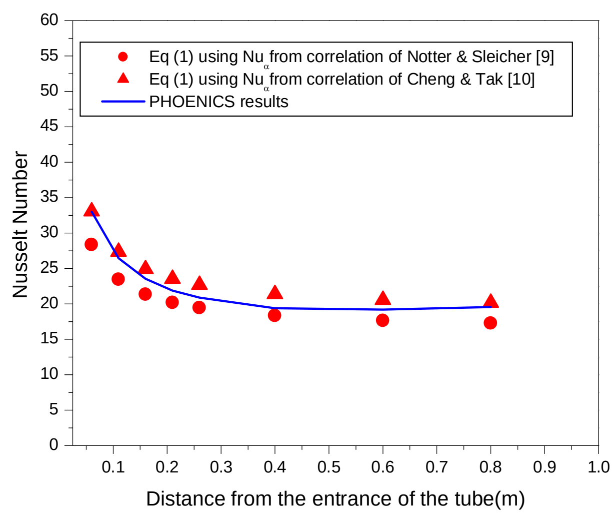

Figure 7. Variation of local Nusselt number along the axial distance.

3.5 Developing length

It is well known that the local Nusselt number varies in the entry length due to the developing boundary layer. As shown in Figure 7 up to 0.5m from the entrance the Nusselt number variation is very clear. Beyond this length the Nusselt number becomes steady, which means the flow is fully developed. It can be seen that at 0.1m length the local Nusselt number is 1.7 times higher than the developed flow. It can further be seen that the CFD prediction is in good agreement with the Eq. (1) for the whole length of the tube. The calculation is carried out with the inlet velocity v=2m/s and with k-ε model. The Pr_t was calculated from Eqs. (4) and (5).

4. Conclusions

The design of LBE based nuclear reactor systems requires reliable physical models for heat transfer and CFD studies. A review of the correlations for liquid metal heat transfer is carried out and a comparison with experimental results is performed. CFD analysis, using PHOENICS-3.6, is carried out to assess the turbulence models. It is found that the prediction of heat transfer by CFD analysis with k-ε model, k-ε RNG model and k-ε Chen model conform well to the correlations. Turbulent Prandtl number, Pr_t is required to be modified in the turbulence models in CFD code for better prediction of liquid metal heat transfer. The variation of Nusselt number in the entry length of the tube is also predicted. It is found that the CFD prediction of variation local Nusselt number along the axial direction follows empirical correlation of Chen and Chiou [14].

5. Nomenclature

A constant

C_p specific heat *(J/kgK)*

d tube inner diameter (m)

d_h hydraulic diameter (m)

Nu Nusselt number, $\left(\frac{\bar{h}D}{k}\right)$

$\bar{h}$ average heat transfer coefficient, *(W/m² K)*

k thermal conductivity, *(W/mK)*

Pe Peclet number, *(Re Pr)*

Pr Prandtl number, $\left(\frac{\mu C_p}{k}\right)$

Pr_t turbulent Prandtl number, $\left(\frac{\varepsilon_M}{\varepsilon_H}\right)$

Re Reynolds number, $\left(\frac{\rho u D}{\mu}\right)$

u velocity (m/s)

u^+ dimensionaless velocity (u/u_τ)

x axial length, m.

y Radial distance from the tube wall, m/s.

y^+ dimensionless distance to wall $y^+ = \frac{y}{\nu}\frac{u}{\left(\tau_s/\rho\right)^{1/2}}$

y^+_1 dimensionless distance to wall at the first mesh

Greek

ε_H eddy diffusivity of heat transfer (m^2/s),

ε_M eddy diffusivity of momentum transfer (m^2/s),

μ dynamic viscosity, Ns/m^2

ν kinematic viscosity, m^2/s

τ_s shear stress, (N/m^2)

ν_t turbulent kinematic viscosity (m^2/s)

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