Fluid-to-Fluid Scaling of Heat Transfer in Supercritical Fluids

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Abstract

Model fluid technique has been widely applied in the thermal-hydraulic studies of nuclear engineering. In spite of growing activities of heat transfer at supercritical conditions using model fluids, there does still not exist any reliable fluid-to-fluid scaling methods, to transfer the test data in model fluids directly to the conditions of prototype fluid. This paper presents a fluid-to-fluid scaling method for heat transfer in circular tubes cooled with supercritical fluids. Based on conservation equations and boundary conditions, on set of dimensionless numbers and the requirements of a complete scaling are determined. Scaling of pressure and temperature ensures the similarity of thermo-physical properties of various fluids. A new dimensionless number, presenting the product of the so-called pseudo Boiling number, Reynolds number and Prandtl number, is applied to scale heat flux. The distortion approach is used to scale mass flux. The preliminary validation results show good feasibility and reasonable accuracy of the proposed scaling method.

1. Introduction

One of the most challenging tasks in the SCWR fuel assembly design of supercritical water-cooled reactors (SCWR) is to keep the maximum cladding temperature well below the design upper limit, to guarantee the integrity of the fuel rods. Thus, an accurate prediction of heat transfer behavior plays an important role and attracts extensive investigations. Due to the strong variation of thermal-physical properties in the vicinity of the pseudo-critical point, heat transfer of supercritical fluids shows abnormal behavior compared to that of conventional fluids (Cheng and Schulenberg, 2001). One of the main features of heat transfer of supercritical fluids is its strong dependence on heat flux, especially as bulk temperature close to the pseudo-critical value.

In spite of extensive studies in the past five decades and a large number of prediction models, prediction of heat transfer of supercritical (SC) fluids uses mainly empirical approaches. In the open literature there exist a large number of empirical correlations, which were derived based on experimental data with limited parameter ranges, as reviewed and summarized by Pioro and Duffey (2005), and Cheng and Yang (2009). In the frame of the development of SCWR, heat transfer in SC fluids becomes a focusing topic in the research of nuclear thermal-hydraulics. A literature survey (Cheng and Schulenberg 2001) emphasizes big deficiency in experimental data in the SCWR typical parameter range, and consequently, big deficiency in an accurate description and prediction of heat transfer behavior at SCWR conditions.

Experimental studies using supercritical water require high pressure, high temperature and high heat power. To reduce both technical difficulties and economic expense, heat transfer experiments have often been performed in a scaled model system. Two different modeling techniques are available, i.e. geometric modeling and fluid modeling. By the geometric modeling simplified flow channels, e.g. circular tubes or small rod bundles, instead of prototypical rod bundles are used. By using such simple flow channels it is possible to study systematically the effect of different parameters on heat transfer and to gain detailed knowledge of heat transfer for a wide range of test parameters. By the fluid modeling a substitute fluid is used instead of the original fluid (water). By a proper selection of model fluids the operating pressure, operating temperature, and the heat power required would be reduced significantly. As it was successfully exercised in the nuclear thermal-hydraulics, experimental technique using model fluids is a feasible and effective measure to achieve scientific and engineering purposes and at the same time to overcome technical and economic problems associated with the experiments using the prototypical fluid.

The key issue concerning the fluid modeling is the transfer of the test data obtained in the model fluid to the prototypical fluid (water), the so called fluid-to-fluid modeling. The success in the application of model fluids depends on the reliability of the scaling methods, which transfer the experimental data from the model fluids directly to conditions of prototypical fluid. Unfortunately, there are still very limited studies on fluid-to-fluid modeling of heat transfer at supercritical conditions.

This paper describes some important requirements on scaling methods for heat transfer of SC fluids. Starting from the governing equations (continuity, momentum, energy, surface heat transfer), which are rearranged in dimensionless form, a set of dimensionless parameters is derived. Based on phenomenological analysis and the distortion approach of Ahmad (1973), a fluid-to-fluid scaling law is proposed, which is then validated on existing test data from various fluids combined with existing heat transfer correlations.

2 Fluid-to-fluid scaling method

2.1 Dimensionless parameters

The purpose of this paragraph is to derive dimensionless parameters which are important to heat transfer and have to be taken into consideration in the scaling approach. To achieve this, we start the procedure from the conservation equations for developed flow at steady state conditions:

Continuity:

$$\frac{\partial \rho u_i}{\partial x_j} = 0 \tag{1}$$

Momentum conservation:

$$u_i \frac{\partial \rho u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_j}\right) + \rho g_i \tag{2}$$

Energy conservation:

$$\rho C_P u_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right) \tag{3}$$

Boundary conditions on heat transfer surface:

$$u_{i,w} = 0 \tag{4}$$

$$q_w = \left(\lambda \frac{\partial T}{\partial r}\right)_w \tag{5}$$

Integral condition of continuity

$$\oint_F (u_k \rho)\, dF = m \tag{6}$$

Here *'F'* is the cross section surface at arbitrary elevation.

The final goal of the scaling model is to determine the scaling factor for heat transfer coefficient, which is defined as below:

$$\alpha = \frac{q_W}{T_W - T_B} \tag{7}$$

We introduce the following characteristic values for various parameters:

Pressure difference:

$$\Delta P_0 = \rho_0 U_0^2 \tag{8}$$

Velocity

$$U_0 = \frac{G}{\rho_0} = \frac{m}{\rho_0 F} \tag{9}$$

Length

$$L_0 = d \tag{10}$$

Properties

$$\rho_0 = \rho_B,\ \mu_0 = \mu_B,\ C_{P,0} = C_{P,B},\ \lambda_0 = \lambda_B \tag{11}$$

Heat transfer coefficient

$$\alpha_0 = \frac{\lambda_0}{L_0} \tag{12}$$

Using the characteristic variables, the parameters in the governing equations are replaced by the following expressions:

$$P = P_0 + \Delta P_0 p \tag{13}$$

$$T = T_0 + \Delta T_0 \theta \tag{14}$$

$$u = U_0 u' \tag{15}$$

$$\rho = \rho_0 \rho' \tag{16}$$

$$\mu = \mu_0 \mu' \tag{17}$$

$$C_P = C_{P,0} C_P^{'} \tag{18}$$

$$\lambda = \lambda_0 \lambda' \tag{19}$$

$$x = L_0 x' \tag{20}$$

Inserting equations (13) to (20) into equations (1) to (7) yields dimensionless equations:

Continuity:

$$\frac{\partial \rho' u_i^{'}}{\partial x_j^{'}} = 0 \tag{21}$$

Momentum conservation equation:

$$u_j^{'} \frac{\partial \rho' u_i^{'}}{\partial x_j^{'}} = -\frac{\partial p}{\partial x_i^{'}} + \frac{1}{\mathrm{Re}} \frac{\partial}{\partial x_j^{'}} \left(\mu' \frac{\partial u_i^{'}}{\partial x_j^{'}} \right) + \frac{1}{Fr} \rho' \frac{g_i}{g} \tag{22}$$

Energy conservation equation:

$$\rho' C_P^{'} u_j^{'} \frac{\partial \theta}{\partial x_j^{'}} = \frac{1}{\mathrm{Re} \cdot \mathrm{Pr}} \frac{\partial}{\partial x_j^{'}} \left(\lambda' \frac{\partial \theta}{\partial x_j^{'}} \right) \quad (23$$

Wall boundary conditions:

$$u_w^{'} = 0 \tag{24}$$

$$\left(\lambda' \frac{\partial \theta}{\partial r'} \right)_w = \frac{q_w L_0}{\lambda_0 \Delta T_0} \tag{25}$$

Integral condition of continuity:

$$\oint_{F'} \left(u_k^{'} \rho' \right) dF' = \frac{m}{U_0 \rho_0 L_0^2} \frac{4}{\pi} \tag{26}$$

Heat transfer coefficient:

$$\frac{\alpha}{\alpha_0} = \frac{\alpha L_0}{\lambda_0} = Nu = \frac{q_W L_0}{\lambda_0 \Delta T_0 \left(\theta_W - \theta_B \right)} \tag{27}$$

We get four dimensionless numbers:

Reynolds number:

$$\mathrm{Re} = \frac{L_0 U_0 \rho_0}{\mu_0} \tag{28}$$

Froude number

$$Fr = \frac{U_0^2}{L_0 g} \tag{29}$$

Prandtl number

$$\mathrm{Pr} = \frac{C_{P,0} \mu_0}{\lambda_0} \tag{30}$$

and

$$\pi_q = \frac{q_w L_0}{\lambda_0 \Delta T_0} \tag{31}$$

The dimensionless governing equations have the same solution for different fluids, which have identical values of the above four dimensionless numbers and the same dependence of thermo-physical properties on the dimensionless temperature θ.

2.2 Scaling of parameters

A scaling method has to take the following issues into consideration:

(a) Scaling of parameters has to guarantee that the property variation of different fluids is similar.

(b) Four dimensionless parameters, i.e. Re, Pr, Fr and π_q, dominate the flow and heat transfer behaviour and need to be taken into account in the scaling method.

(c) There are totally six parameters which need to be scaled, five of which can be adjusted independently during the experiment. They are tube diameter, pressure, bulk temperature, heat flux and mass flux. The sixth parameter, i.e. heat transfer coefficient, is a dependent parameter.

2.2.1 Scaling of tube diameter

For simplicity we assume in our model that identical tube diameter is used for both model fluid and prototype fluid, i.e.:

$$(D)_M = (D)_P \tag{32}$$

2.2.2 Scaling of pressure and bulk temperature

The target of the pressure and bulk temperature scaling is to achieve similar property variation of both fluids. For fluid-to-fluid scaling of heat transfer at sub-critical conditions, pressure is often scaled either using their critical values or using density ratios. At sub-critical conditions, there are two characteristic density values at a fixed pressure, i.e. density of saturated liquid phase and vapour phase. However, supercritical fluids do not undergo phase change. Examination of using various density values as characteristic values shows that the approach using density ratio is not feasible for supercritical conditions. Therefore, the present model scales pressure by taking the critical pressure into consideration, such as

$$\left(\frac{P}{P_C}\right)_M = \left(\frac{P}{P_C}\right)_P \tag{33}$$

In the open literature there are different options for scaling bulk temperature. Two most widely applied parameters are normalized with the critical temperature and pseudo-critical temperature, respectively

$$\left(\frac{T}{T_C}\right)_M = \left(\frac{T}{T_C}\right)_P \tag{34}$$

$$\left(\frac{T}{T_{PC}}\right)_M = \left(\frac{T}{T_{PC}}\right)_P \tag{35}$$

Here all the temperatures are in Kelvin.

The feasibility of the above scaling approaches is assessed by the variation of properties with dimensionless temperature. Figure 1 shows the dependence of dimensionless heat capacity versus dimensionless temperatures at a normalized pressure of 1.13 for three different fluids, i.e. water, CO_2 and Freon R134a.

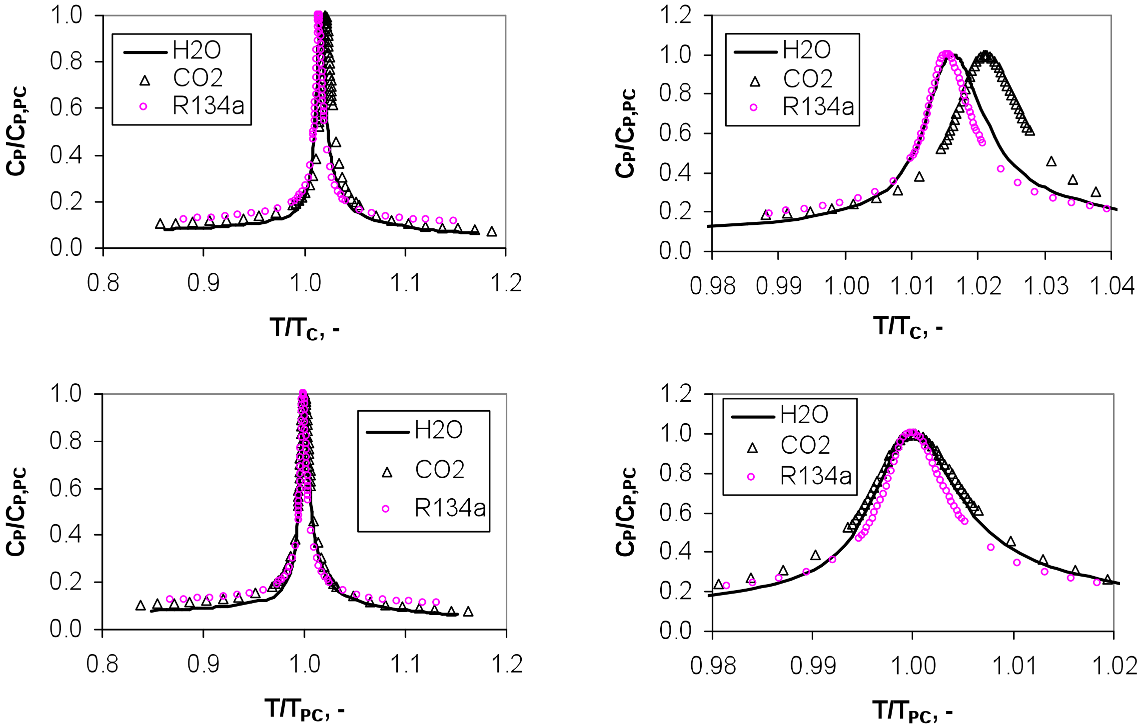


Figure 1: Dependence of dimensionless heat capacity versus dimensionless temperatures at a normalized pressure of 1.13 for three different fluids

Over a wide range of temperature, all three fluids show very similar behaviour with respect to the variation of specific heat. Large deviation occurs between the curve for CO_2 and the other curves, in case the relative specific heat is presented versus the relative temperature based on the critical value. The agreement is significantly improved if the relative temperature based on the pseudo-critical value is applied.

In this paper we propose another dimensionless temperature, which is based on the ratio of temperature difference and expressed as below:

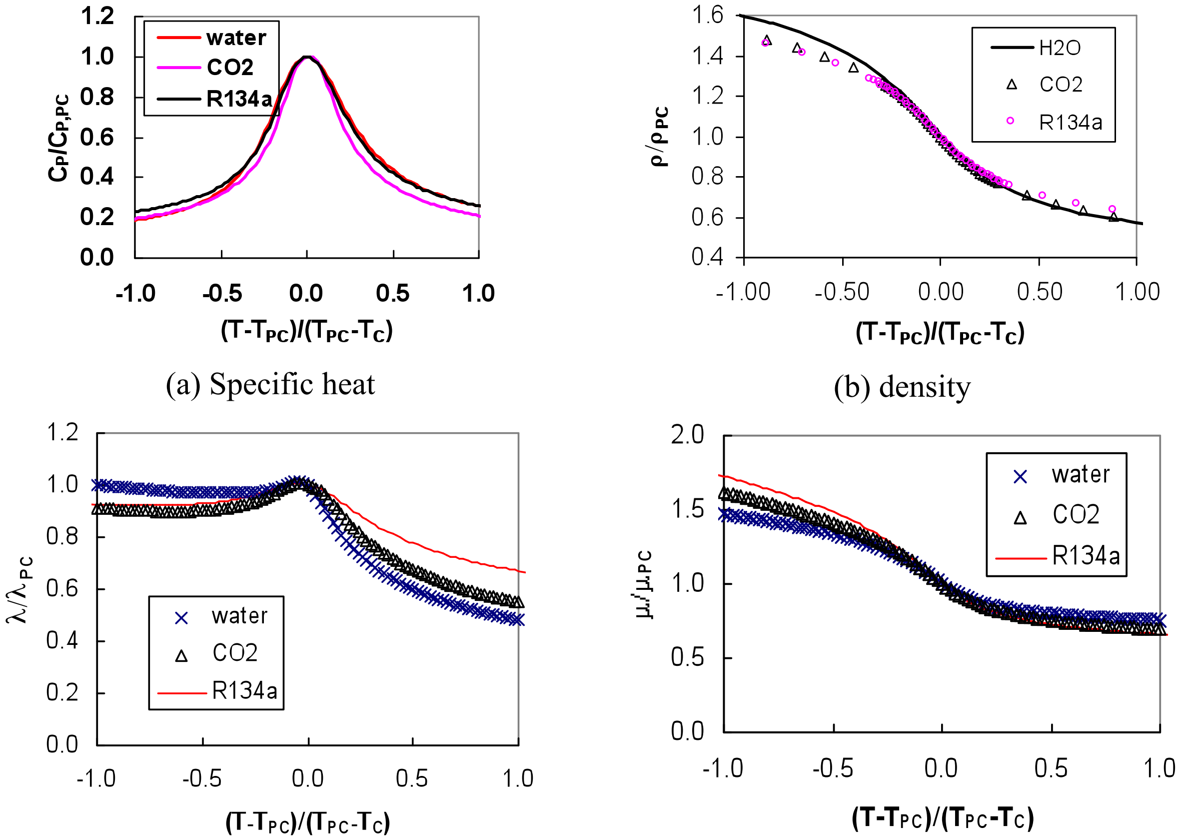
$$\theta = \frac{T - T_{PC}}{T_{PC} - T_C} \tag{36}$$

The parameter θ has negative values, when fluid temperature is below the pseudo-critical point, whereas it gives positive values, if the fluid temperature exceeds the pseudo-critical value. Therefore, this parameter is characterized as 'pseudo steam quality' in this paper.

Figure 2 shows the relative specific heat and the relative density versus pseudo steam quality in the region close to the pseudo-critical value. Compared with Figures 1, better agreement is achieved with the new dimensionless temperature for both relative specific heat and relative density. Thus, the present paper proposes the temperature scaling as below:

(a) $$\left(\frac{T-T_{PC}}{T_{PC}-T_C}\right)_M = \left(\frac{T-T_{PC}}{T_{PC}-T_C}\right)_P \tag{37}$$

(b)



(a) Specific heat (b) density

(c) Thermal conductivity (d) dynamic viscosity

Figure 2: Dependence of dimensionless properties versus pseudo steam quality at a normalized pressure of 1.13 for three different fluids

2.2.3 Scaling of heat flux

The purpose of heat flux scaling is to achieve similarity in the effect of heat flux on heat transfer. Among the four dimensionless numbers, only the Acceleration number contains heat flux. Rearrange this dimensionless number, we get

$$\frac{q_w L_0}{\lambda_0 (T_{PC}-T_C)} = \frac{q_w L_0}{\lambda_0 (T_W - T_B)} \frac{(T_W - T_B)}{(T_{PC}-T_C)} = Nu(\theta_W - \theta_B) \tag{38}$$

This parameter contains two terms, i.e. Nusselt number and the dimensionless temperature difference between the heated wall and the bulk. In case that we require the similarity in Nusselt number to scale heat transfer coefficient, the parameter π_q corresponds directly to the temperature variation across the tube cross-section. As it is pointed out in the previous papers (Cheng et al. , 2009) that the effect of heat flux is mainly resulted by the strong property variation across the flow channel cross section, especially in the area close to the heated wall.

The temperature variation over the cross-section does play the key role in the heat transfer. Therefore, for the scaling of heat flux we propose:

$$\left(\frac{q_w L_0}{\lambda_0\left(T_{PC}-T_C\right)}\right)_M = \left(\frac{q_w L_0}{\lambda_0\left(T_{PC}-T_C\right)}\right)_P \tag{39}$$

2.2.4 Scaling of mass flux

There are still three dimensionless parameters remaining available for mass flux scaling. Referring to the distortion approach of Ahmad (1973), these three remaining dimensionless numbers are combined into one single dimensionless parameter:

$$\pi_C = \mathrm{Re}^{n1}\,\mathrm{Pr}^{n2}\,Fr^{n3} \tag{40}$$

The task now is to select values for the three exponents n_1, n_2 and n_3. Heat transfer of supercritical fluids can be divided into two regions. In the first region with low heat flux or bulk temperature far away from the pseudo-critical point, heat transfer can be well predicted with the conventional correlations, e.g. Dittus-Boelter correlation. This region is characterized as "normal" heat transfer region. In the region of high heat fluxes and bulk temperatures close to the pseudo-critical value, heat transfer behavior deviates from the conventional correlations. In the present paper this region is called "heat flux affected region". In the present fluid-to-fluid model, the similarity of heat transfer in the heat flux affected region is achieved with equation (39). The target of the mass flux scaling is to achieve the similarity of heat transfer in the normal heat transfer region. Assuming that in the normal heat transfer region, heat transfer coefficient can be expressed by the following equation:

$$Nu = c\cdot \mathrm{Re}^m\,\mathrm{Pr}^n \tag{41}$$

The mass flux scaling is thus selected to give

$$\left(\mathrm{Re}^m\,\mathrm{Pr}^n\right)_M = \left(\mathrm{Re}^m\,\mathrm{Pr}^n\right)_P \tag{42}$$

For the first assessment, we assume the conventional heat transfer correlation of Dittus-Boelter for the normal heat transfer region and get:

$$\left(\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\right)_M = \left(\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\right)_P \tag{43}$$

This corresponds to equation (41) with

$$n_1 = 0.8$$
$$n_2 = 1/3$$
$$n_3 = 0$$

2.2.5 Scaling of heat transfer coefficient

The equality of Nusselt number is required, to scale heat transfer coefficient, i.e.:

$$\left(Nu\right)_M = \left(Nu\right)_P \tag{44}$$

2.2.6 Complete set of scaling factors

From equations (32), (33), (36), (39), (43) and (44), a complete set of scaling factors is established:

$$f_D = \frac{D_M}{D_P} = 1.0 \tag{45}$$

$$f_P = \frac{P_M}{P_P} = \frac{P_{C,M}}{P_{C,P}} \tag{46}$$

$$f_\theta = \frac{\theta_{B,M}}{\theta_{B,P}} = 1.0 \tag{47}$$

$$f_G = \frac{G_M}{G_P} = \frac{\mathrm{Pr}_{B,P}^{5/12}}{\mathrm{Pr}_{B,M}^{5/12}} \cdot \frac{\mu_{B,M}}{\mu_{B,P}} \tag{48}$$

$$f_q = \frac{q_M}{q_P} = \frac{\lambda_{B,M}\left(T_{PC} - T_C\right)_M}{\lambda_{B,P}\left(T_{PC} - T_C\right)_P} \tag{49}$$

$$f_\alpha = \frac{\alpha_M}{\alpha_P} = \frac{\lambda_{B,M}}{\lambda_{B,P}} \tag{50}$$

3 Validation of fluid-to-fluid scaling model

For validation, test data from one fluid will be transferred to the equivalent conditions of the other fluid using the new developed scaling method, i.e. equations (45) – (50). With the equivalent parameters in model fluid new values of heat transfer coefficients are calculated using the selected heat transfer correlations. The calculated heat transfer coefficient will be then compared with the equivalent measured heat transfer coefficient. The deviation between test results and correlation for each test point is defined as:

$$e_i = 2\frac{\left(\alpha_C - \alpha_M\right)_i}{\left(\alpha_C + \alpha_M\right)_i} \tag{51}$$

Two statistic parameters, i.e. the average value and the standard deviation, of the deviation parameter are defined as below, to evaluate the accuracy of the scaling method:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} e_i \tag{52}$$

$$\sigma = \left[\frac{1}{N-1}\sum_{i=1}^{N}\left(\mu - e_i\right)^2\right]^{1/2} \tag{53}$$

This paper applied two test data sets, as summarized in Table 1. One data set is from water and the other from CO_2. Two heat transfer correlations are sued. The correlation of Bishop is for water and the correlation of Krasnoshchekov for both water and CO_2.

Table 1: Test data selected for validation

ID-No.	Authors	Ranges of test parameters					
		Fluid	Diameter mm	Pressure MPa	Mass flux Kg/m2s	Heat flux MW/m2	Data points
2	Herkanrath	H_2O	10.0, 20.0	22.5 – 25.0	700 – 3500	0.30 – 2.0	4599
5	Kim	CO_2	4.4	7.75 – 8.85	400 – 1200	0.01 – 0.15	2662

Table 2 summarizes the comparison results. The test data transferred from CO_2 to water conditions agrees on average well with both correlations. For 2661 test data points, the average value of the deviation is about 1%. The correlation of Krasnoshchekov over-predicts the CO_2 data transferred from water experiments of Herkanrath.

Table 2: Comparison of various correlations with different test data

ID-No.	Data Source	Correlations					
		Bishop			Krasnoshchekov		
		N	μ	σ	N	μ	σ
2	Herkanrath	---	---	---	4599	0.2838	0.3117
5	Kim	2661	-0.0039	0.3813	2661	0.0099	0.3822

4. Summary

Experimental investigations of heat transfer at prototypical conditions of supercritical water cooled reactors (SCWR) are strongly limited due to their huge technical and financial efforts required. One of the possible solutions is the application of model fluid technique, which has been widely applied in nuclear thermal-hydraulics. The key issue for the success of the model fluid technique is accurate fluid-to-fluid scaling, which is still missing nowadays.

This paper presents a fluid-to-fluid scaling method for heat transfer in circular tubes cooled with supercritical fluids. Based on conservation equations and boundary equations, important dimensionless numbers governing flow and heat transfer behaviour and the requirements on a complete similarity are derived. A thorough evaluation of the similarity of thermo-physical properties of various fluids, scaling criteria for pressure and temperature are determined. The introduction of the so-called 'pseudo steam quality' gives reasonable similarity of thermo-physical properties variation. A new dimensionless number, presenting the product of the so-called pseudo Boiling number, Reynolds number and Prandtl number, is applied to heat flux scaling. The distortion approach is used for mass flux scaling.

The derived scaling method is validated based on test data and selected heat transfer correlations, and its feasibility is well proven.

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