FLUIDELASTIC INSTABILITY OF A TUBE BUNDLE PREFERENTIALLY FLEXIBLE IN THE FLOW DIRECTION TO SIMULATE U-BEND IN-PLANE VIBRATION

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1. INTRODUCTION

Fluidelastic instability is the most important vibration excitation mechanism for steam generator tube bundles. It leads to very high vibration amplitude and may cause short term failure by fatigue or fretting-wear. In nuclear power plant, steam generators U-tubes are susceptible to fluidelastic instability because of the high velocity of the two-phase flow in the U-tube region. Prior to 1980, very little work had been done to study flow- induced vibration of tube bundles subjected to two-phase cross flow. Since then, a few studies were conducted in this area. This work was reviewed by Pettigrew and Taylor [1] in 1994. Since 1994, several researchers have contributed relevant results, in particular, Feenstra et al. [2] in Freon 11 two-phase flow, Mann et al. [3] in Freon 12 two-phase flow, and Nakamura et al. [4], Mureithi et al. [5], and Hirota et al. [6] in steam-water cross-flow. Most of the work done so far indicates that fluidelastic instability is more likely to happen in the direction transverse to the flow.

In order to support the U-tubes in the out-of-plane direction, flat bar supports sometimes called "Anti Vibration Bars" or AVBs have been introduced in steam generator designs. The main assumption behind this support design is that fluidelastic instability is not likely to happen in the flow direction. However, in a recent paper, Mureithi et al. [7] demonstrated experimentally that instability can happen in the flow direction of an array of tubes subjected to air flow. Weaver and Schneider [8] have studied the effect of such support on the stability of a U-tube bundle. They found that the presence of AVBs with small clearance was preventing instability for both out-of-plane and in-plane modes. Another experimental study on the vibration response of U-tube bundles supported by AVBs was recently reported by Janzen et al. [9]. They observed fluidelastic instabilities for an in-plane mode in water flow and in low-void fraction (25%) air-water flow.

The logical next step to the above work is to verify if fluidelastic instability can occur for an array of cylinders flexible only in the streamwise direction subjected to high-void- fraction two-phase cross flow. Such an experimental study is presented in this paper.

2. EXPERIMENTAL CONSIDERATIONS

The test section is connected to the air-water test loop shown in Fig. 1. The reader is referred to a paper by Pettigrew et al. [10] for a description of the two-phase flow test loop. The test section, shown in Fig. 2, has a flow area of 0.038 m^2 ($0.2m \times 0.19m$). It includes twelve rigid tubes and seven flexible tubes in a rotated triangular configuration.

Each tube has a diameter of 0.038 m and the pitch to diameter ratio (P/D) is 1.5. Half tubes have been added to the lateral walls of the test section to better simulate a real tube bundle. The test section has been designed so that it is possible to switch the position between flexible tubes and rigid tubes. This offers the possibility to experiment with different configurations of flexible and rigid cylinders. So far, four configurations have been tested: a single flexible tube inside a rigid array (Fig. 2a), a cluster of flexible tubes that are all placed in the middle of the test section

referred to as the central cluster case (Fig. 2b), a single flexible column (Fig. 2c) and two partially flexible columns (Fig. 2d).

A typical instrumented tube assembly is shown in Fig. 3. Each assembly consists of a rigid cylinder attached to a flexible cantilever beam. Two sets of flexible beams were used. The first set has a rectangular cross section which makes the flexible tube assembly much more flexible in one direction. The second set has a circular cross section (0.011 m diameter) which allows the tube to be axisymetrically flexible. For the latter, the natural frequency in air is 30 Hz. Two different sets of rectangular beams were tested: one of 0.00415m x 0.025m cross section and one of 0.00675m x 0.025m. The flexible beam was oriented so that the tube assembly would be much more flexible inflow resulting in tube streamwise frequency in air of 14 Hz (81 Hz for the transverse direction) and 28 Hz (103 Hz in the transverse direction) for the thinner and the thicker beam respectively. Strain gages were mounted on the beam near the clamped end for vibration measurement.

3. FLUIDELASTIC INSTABILITY RESULTS

For each void fraction tested, tube vibration signals were recorded for every test flow conditions for about eight minutes. Tube rms vibration amplitudes were evaluated from the averaged spectra. The homogeneous two-phase flow model was used in order to determine two-phase flow parameters, i.e. void fraction (ε), two-phase mixture density (ρ) and pitch flow velocity ($U_p = U_{\infty}P/(P-D)$). Further description of the two-phase flow parameters used may be found in Pettigrew and Taylor [1].

3.1 Single flexible tube configuration

Fig. 4 shows the behavior of the rms response of the tube flexible only infow (14 Hz) for every void fraction tested (i.e. 65, 80, 90 and 95%). Clearly, no instability was developed up to the maximum velocity tested. Moreover, it can be seen for the three lowest void fractions that the amplitude curves start decreasing beyond a certain velocity. This is due to impacting between the extremity of the flexible tube and the test section wall. The very high flow velocities cause a large deflexion of the flexible tube. From this position a little vibration amplitude makes the tube impact with the test section wall and limits its movement. Those results indicate that fluidelastic instability does not occur for a single tube flexible in the flow direction up to the maximum flow velocity tested. Instability would have normally been expected for a single axisymetrically flexible tube at lower velocities.

Results obtained for the axisymetrically flexible tube are illustrated on Fig. 5. As expected, the response of the tube inflow (drag direction) is much less important than in the lift direction at instability. In fact, no instability was observed whatsoever in the drag direction. Thus Fig. 5 shows only the response of the tube in the lift direction. Fluidelastic instability does occur for all void fraction tested, i.e. 0%, 20%, 40%, 50%, 60%, 80% and 90%, respectively at 2 m/s, 2.4 m/s, 3.75 m/s, 4.5 m/s, 5.5 m/s, 9.5 m/s and 12 m/s. The critical flow velocity appears to be less well defined for the high void fractions (80% and 90%).

It may be seen from Fig. 4 and 5 that the response to two-phase flow turbulence at flow velocity below instability does not increase proportionally with flow velocity. This somewhat unexpected trend has been observed before in two-phase flows. It is explained in terms of changes in the structure of the two-phase flow. Sometimes low flow velocities result in larger characteristic flow structures and somewhat intermittent flows. This causes larger vibration excitation forces. With increasing flow velocity the turbulence scale diminishes resulting in lower excitation forces in

spite of the larger flow velocity. Smaller turbulence scale also reduces the spatial correlation of the excitation forces decreasing further the vibration response. Tests done at 50% and 80% void fraction with an axisymetrically flexible tube (shown on Fig. 5) confirm that the vibration amplitude decreases sharply to zero when reducing considerably the two-phase flow velocity. This puzzling result needs further investigation and understanding.

3.2 Central bundle configuration

Tests with the central cluster configuration were done with both the 14 Hz and the 28 Hz assemblies of the tubes flexible inflow. Fluidelastic instability was observed with both assemblies and the centrally located tube in the flexible cluster (see Fig. 1b) had the highest response in both cases. For the tests with the 14 Hz tubes, it can be seen from Fig. 6 that fluidelastic instability did occur for 65% void fraction at 6 m/s, for 80% void fraction at 7.75 m/s, for 90% void fraction at 9.5 m/s and for 95% void fraction at 13.5 m/s. The instability point is clearly defined for all void fractions except for 95% where the response curve appears more like an exponential curve. For the 28 Hz assembly, it can be seen from Fig. 7 that the tube bundle did go unstable for 90% void fraction at 14.5 m/s. The onset of fluidelastic instability was abserved for 85% void fraction at 13 m/s, however, the tests were limited by the water pump flow rate.

Fig. 8 shows the rms response of Tube 7 versus pitch flow velocity for the case where axisymetrically flexible tubes were used for all void fractions except 95%. For the latter, the response of Tube 4 is shown on Fig. 8 because instability was much more developed for this particular tube for that void fraction. The tube vibration response was considerably higher in the lift direction than in the drag direction for Tube 7. Thus only the response in the lift direction appears on Fig. 8 for Tube 7 and for Tube 4 at 95% void fraction.

For all void fractions below 80% (i.e. 0%, 20%, 40%, 50% and 60%), Fig. 8 shows that instability occurs at similar pitch velocities as those for the single axisymetrically flexible tube. However, the amplitude of vibration at instability is much higher for the central cluster case. For 80% and 90% void fraction, the critical flow pitch velocities, which are respectively 7.75 m/s and 9.5 m/s, are much better defined and have a lower value than in the case of a single flexible tube in a rigid array. The critical flow pitch velocity for 95% void fraction is about 10.5 m/s.

Unlike for the single axisymetrically flexible tube, the response at instability occurred also in the drag direction for most of the tubes resulting in an orbital motion. Fig. 9 illustrates the orbital motion of the tubes at instability for 20%, 40%, 80% and 95% void fraction. Fig. 9 also shows that for low void fractions (20% and 40%), the instability was well developed throughout the flexible bundle. This was also the case for 0%, 50% and 60% void fraction. For the two highest void fraction displayed on Fig. 9 (80% and 95%) the instability is, however, much more developed for the downstream tubes. The physical phenomenon behind this observation is not yet understood.

3.3 Single flexible column

The single flexible column configuration simulates the hypothetical case where there could be a clearance between one column of tubes and the support bars. As a result, the entire column could be ineffectively supported in the in-plane direction. Also, the U-tubes would be free to go unstable in the out-of-plane direction within the available gap. The numbering of the flexible tubes is shown in Fig. 1c. For the case were the tubes are flexible only inflow, the tests were done only with the 14 Hz tube assembly. In this case, Tube 7 had the highest vibration response. Its response is plotted as a function of pitch flow velocity in Fig. 10. It can be seen that the tube

bundle did not go unstable for 95% void fraction. For the case of 80% and 90% void fraction, Tube 7 seemed to experience an increase in vibration amplitude up to a relatively high level followed by a decrease. For these void fractions, the flow limit of the loop was reached before the level of vibration for Tube 7 diminished.

The results obtained with an axisymetrically flexible single column are shown in Fig. 11. It shows the rms response in the lift direction of Tube 4 for all void fractions below 80% and of Tube 7 for void fraction of 80% and above. The response of both tubes in the drag direction does not appear on Fig. 11 because they vibrate exclusively in the lift direction. In other words, their response in the drag direction is negligible compared to the lift direction. This is also the case for all the other tubes inside the column. This is indeed a significant difference between the central cluster and the single flexible column configuration in the case of tubes that are axisymetrically flexible. However, the tendency of the instability to develop only for the downstream tubes at high void fraction (80%, 90% and 95%) that was observed in the case of the central cluster is emphasized in the case of the single flexible column. Fig. 11 shows that for all void fractions tested (i.e. 0%, 20%, 40%, 50%, 60%, 80%, 90% and 95%) the critical flow pitch velocity is similar to those obtained for the central cluster configuration with the same tubes.

3.4 Two-partially flexible columns

As observed earlier, there are huge differences between the results obtained with the tubes flexible inflow for the central cluster and for the single flexible column configuration. Test were done with the flexible tubes placed over two adjacent columns to get a configuration that is situated somewhere in between the latter two. The results obtained for this configuration appear in Fig. 12 which shows the rms response of Tube 7 with pitch flow velocity. Tube 7 had the highest response. As shown in Fig. 12 instability did occur at slightly higher velocities than in the case of the central cluster configuration, i.e. 8m/s for 80%, 10.5 m/s for 90% and 14 m/s for 95% void fraction.

3.5 Flow regimes

Not much work has been done to predict flow regimes for tube bundles subjected to cross-flow. Grant's [11] map (shown in Fig. 13) has been used to evaluate flow regime here. It uses two dimensionless parameters: the Martinelli parameter X and the dimensionless gas velocity U_g . Definition of these two parameters can be found in Pettigrew and Taylor [1]. Three flow regimes appear on Fig. 13: bubbly, spray and intermittent. Bubbly and spray flow are considered to be continuous flow regimes while intermittent flow is considered to be highly non-stationary. Points corresponding to critical velocities observed in the present study are plotted on Fig. 13. All the test points fall well inside the continuous bubbly flow regime.

4. DISCUSSION

4.1 Fluidelastic instability result comparison

It is now confirmed that fluidelastic instability can occur in a rotated triangular tube bundle flexible only in the flow direction when subjected to high-void-fraction two-phase cross flow. The fluidelastic test results reported in the previous section are now compared to existing data for the rotated triangular configuration (Pettigrew et al. [12]) on the instability map of Fig. 14. On this map, the abscissa is the mass-damping parameter and the ordinate is the reduced velocity. The factor K is the proportionality constant defined in the relation:

$$\frac{U_{p,crit}}{fD} = K \left(\frac{2\pi\zeta m}{\rho D^2}\right)^{0.5}$$
(1)

In Eq. (1), f is the tube frequency at instability, m is the tube total linear mass including the hydrodynamic mass and ζ is the tube total damping ratio. The added mass is determined using the relation given in Pettigrew et al. [13]. For the total damping ratio, values measured at approximately one-half of the critical flow velocity with the single flexible tube configuration were used for the case of axisymetrically flexible tube. For the case of 95% void fraction, damping measurement was taken with a single flexible tube inside a rigid array at approximately two-third of the critical flow velocity determined for the central cluster configuration. This was done in order to avoid intermittent flow regime. A similar approach was used for the tubes flexible inflow. As frequency variation is found to have little effect on the value of two-phase damping (Pettigrew et al. [14]), two-phase damping values obtained for the 14 Hz flexible tube assembly were also applied to the 28 Hz assembly.

As shown in Fig. 14, the experimental points reported in this work for tubes that are flexible inflow collapse reasonably well above the K=6 line. Also appearing on this figure is the results obtained by Mureithi et al. [7] in a wind tunnel for a tube bundle similar to the one used in this study for two flexible bundle configurations: the central cluster and the single flexible column. Both configurations use tubes that are flexible only in the flow direction. As it is shown on Fig. 14, those results are in good agreement with those obtained in the present study for the tubes flexible inflow. However, they showed that fluidelastic instability does occur for the single flexible column configuration in the wind tunnel which was not observed in this study in two-phase flow.

Results obtained with axisymetrically flexible tubes with a mass-damping parameter value smaller than about 1.5 appear to be in good agreement with those of Pettigrew et al. [12]. They collapse very well slightly above the K=3 line. However, beyond this value for the mass-damping parameter, there is a jump in the reduced velocity value. This jump is predicted by the Lever and Weaver [15] and by the Païdoussis and Price [16] theoretical models. The transition value they predict for the mass-damping parameter is of the same order of magnitude as the one found in this study, i.e. around one. They explain this jump by a sudden change in the phase angle between the tube motion and the resulting forces.

Since variation of the mass-damping parameter in the present study is caused only by void fraction variation, flow regime changes could also be the cause of the jump observed in the data for the tubes flexible in all directions. As seen in Fig. 13, all flow conditions tested for those tubes are within the bubbly flow regime. However, bubbly flow regime in the Grant's map accounts for more than one continuous flow regimes. Thus, there could be a transition from one continuous flow regime to another that could be missed in the flow regime map presented in Fig. 13.

4.2 On fluidelastic instability mechanism

It is well accepted that there is two mechanisms that can cause instability (Chen [17], Païdoussis and Price [16], Yetisir and Weaver [18]). The first, the damping controlled mechanism, manifests itself when the forces acting on the tube are in phase with its velocity. This mechanism needs only one degree of freedom to exist. The second, the stiffness controlled mechanism, needs at least two degrees of freedom to materialize.

From the results obtained for the tubes flexible only in the flow direction, it can be concluded that there needs to be several flexible tubes for fluidelastic instability to occur (multiple degrees of freedom system). Since the damping controlled mechanism needs only one degree of freedom to cause instability, it can be deduced that it is only the stiffness controlled mechanism that produced the instability for the tubes flexible only in the flow direction.

It was mentioned in this paper that impacts between the flexible tube and the test section caused by very high static deformation was limiting the movement of the tube. One can wonder if instability could occur for a single flexible tube if, in a hypothetical case, the tube was not limited in its movement. Païdoussis and Price [16] demonstrated with their theoretical model that the derivative of the lift coefficient (C_i) to displacement in the lift direction (y) must be negative and large $(\partial C_L / \partial y \le 0)$ for an instability by damping controlled mechanism to occur for a tube that is free to vibrate only in the lift direction. By the same reasoning, the derivative of the drag coefficient (C_D) vs. displacement in the drag direction (x) must be negative and large ($\partial C_D / \partial x < <$ 0) for an instability by the damping controlled mechanism to occur for a tube that is free to vibrate only in the drag direction. Païdoussis et al. demonstrated experimentally in a wind tunnel with a tube bundle similar to the one in question in the present study that the derivative of the drag coefficient to the displacement in the drag direction is close to zero $(\partial C_D/\partial x \approx 0)$. Despite the fact that those results were obtained in a wind tunnel, they seem to be applicable in the present case. Therefore, no instability is possible for a single tube flexible only inflow inside a rigid array. The instabilities observed with tubes that are flexible only inflow are then caused only by the stiffness controlled mechanism.

Further discussion on fluidelastic instability mechanisms and, in particular, on the effects of tube bundle configurations may be found in Violette et al [20].

4.3 Practical significance of the results

Fluidelastic instabilities of a tube bundle preferentially flexible in the flow direction to simulate U-bend in-plane vibration were observed probably for the first time in high-void -fraction two-phase cross flow. This vibration excitation mechanism was not up to now considered to be possible and not included in earlier design guidelines. It may be desirable to consider in-plane fluidelastic instability in future steam generator designs. In this respect, a fluidelastic instability constant K=6.0 would be appropriate based on the results outlined in this paper.

5. CONCLUDING REMARKS

Well-defined fluidelasic instabilities were observed for a rotated triangular tube bundle preferentially flexible in the flow direction and subjected to two-phase cross flow. However, these instabilities occurred at somewhat higher flow velocities than for axisymetrically flexible tube bundle. An instability coefficient K=6.0 would be an appropriate design guideline to avoid U-bend in-plane fluidelastic instabilities.

Instabilities were not observed for a single tube nor for a single column of tube preferentially flexible in the flow direction. However, two adjacent columns became unstable at about the same flow velocity as a tube cluster.

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7. NOMENCLATURE

- C_D , C_L : Drag and lift coefficient
- f: Frequency (Hz)
- m: Linear mass (kg/m)
- P, D: Pitch and tube diameter (m)
- Q_l , Q_g : Liquid and gas volumetric flow (m³/s)
- U_{σ} : Dimensionless gas velocity
- U_{∞} , U_p , $U_{p,crit}$: Free stream, pitch, critical pitch velocity (m/s)
- *X* : Martinelli parameter
- x, y: Tube position in drag and lift direction
- ε : Void fraction
- $\zeta_s, \zeta_v, \zeta_{TP}, \zeta$: Structural, viscous, two-phase and total damping ratio (percent)
- κ_{ii} : Fluid static coupling coefficient (N/m)
- ρ : Two-phase mixture density (kg/m³)

8. REFERENCES

[1] Pettigrew, M. J., Taylor, C. E., 1994, "Two-Phase Flow-Induced Vibration: An Overview", ASME Journal of Pressure Vessel Technology, Vol. 116, pp. 233-253.

[2] Feenstra, P. A., Weaver, D. S. and Judd, R. L., 2002, "Modeling two-phase flow-excited damping and fluidelastic instability in tube arrays", Journal of Fluids and Structures, Vol. 16, No. 6, pp. 811-840.

[3] Mann, W., Mayinger, F., 1995, "Flow Induced Vibration of Tube Bundles Subjected to Single- and Two-Phase Cross-Flow", Advances in Multiphase Flow, Elsevier Science, pp. 603-612.

[4] Nakamura, T., Hirota, K., Watanabe, Y., Mureithi, N. W., Kusakabe, T., and Takamatsu, H., 2002, "Dynamics of an In-Line Tube Array Subjected to Steam-Water Cross-Flow. Part I: Two-Phase Damping and Added Mass", Journal of Fluids and Structures, Vol. 16, No. 2, pp. 123-136.

[5] Mureithi, N. W., Nakamura, T., Hirota, K., Murata, M., Utsumi, S., Kusakabe, T. and Takamatsu, H., 2002, "Dynamics of an In-Line Tube Array Subjected to Steam-Water Cross-Flow. Part II: Unsteady Fluid Forces" Journal of Fluids and Structures, Vol. 16, No. 2, pp. 137-152.

[6] Hirota, K., Nakamura, T., Kasahara, J., Mureithi, N. W., Kusakabe, T., and Takamatsu, H., 2002, "Dynamics of an In-Line Tube Array Subjected to Steam-Water Cross-Flow. Part III:

Fluidelastic Instability Test and Comparison With Theory", Journal of Fluids and Structures, Vol. 16, No. 2, pp. 153-173.

[7] Mureithi, N. W., Zhang, C., Ruël, M. and Pettigrew, M. J. 2005, "Fluidelastic Instability Tests on an Array of Tubes Preferentially Flexible in the Flow Direction", Journal of Fluids and Structures, Vol. 21, No. 1, pp. 75-87.

[8] Weaver, D. S., Schneider, W., 1983, "The Effect of Flat Bar Supports on the Crossflow Induced Response of Heat Exchanger U-Tubes", Journal of Engineering for Power, Vol. 105, pp. 775-781.

[9] Janzen, V. P., Hagberg, E. G., Pettigrew, M. J. and Taylor, C. E., 2005, "Fluidelastic Instability and Work-Rate Measurements of Steam-Generator U-Tubes in Air-Water Cross-Flow", ASME Journal of Pressure Vessel Technology, Vol. 127, No.1, pp. 84-91.

[10] Pettigrew, M. J., Zhang, C., Mureithi, N. W. and Pamfil D. 2005, "Detailed Flow and Force Measurements in a Rotated Triangular Tube Bundle Subjected to Two-Phase Cross Flow", Journal of Fluids and Structures, Vol. 20, No. 4, pp. 567-575.

[11] Grant, I. D. R., 1975, "Flow and Pressure Drop with Single Phase and Two Phase Flow in the Shell-Side of Segmentally Baffled Shell and Tube Heat Exchangers", NEL Report No. 590, National Engineering Laboratory, Glasgow, pp.1-22.

[12] Pettigrew, M. J., Tromp, J. H., Taylor, C. E., Kim, B. S. 1989, "Vibration of Tube Bundles in Two-Phase Cross-Flow: Part 2 – Fluid-Elastic Instability", Journal of Pressure Vessel Technology, Vol. 111, pp. 478-487.

[13] Pettigrew, M. J., Taylor, C. E., Kim, B. S. 1989, "Vibration of Tube Bundles in Two-Phase Cross-Flow: Part 1 – Hydrodynamic Mass and Damping", Journal of Pressure Vessel Technology, Vol. 111, pp. 466-477.

[14] Pettigrew, M. J. and Taylor, C. E., 2004, "Damping of Heat Exchanger Tubes in Two-Phase Flow: Review and Design Guidelines", ASME Journal of Pressure Vessel Technology, Vol. 126, pp. 523-533.

[15] Lever, J. H., Weaver, D. S. 1982, "A Theoretical Model for the Fluid-Elastic Instability in Heat Exchanger Tube Bundles", Journal of Pressure Vessel Technology, Vol. 104, pp. 147-158.

[16] Païdoussis, M. P., Price, S. J. 1988, "The mechanisms underlying flow-induced instabilities of cylinder arrays in crossflow", Journal of Fluid Mechanic, Vol. 187, pp. 45-59.

[17] Chen, S. S. 1987, "Flow-Induced Vibration of Circular Cylindrical Structures", Hemisphere Publishing Corporation.

[18] Yetisir, M. and Weaver, D. S. 1993, "An Unsteady Theory for Fluidelastic Instability in an Array of Flexible Tubes in Cross-Flow. Part II: Results and Comparison with Experiments", Journal of Fluids and Structures, Vol. 7, pp. 767-782.

[19] Païdoussis, M. P., Price, S. J. and Mureithi, N. W. 1996, "On the Virtual Nonexistence of Multiple Instability Regions for Some Heat-Exchanger Arrays in Crossflow", Journal of Fluids Engineering, Vol. 118, pp.103-109.

[20] Violette, R., Pettigrew, M. J. and Mureithi, N.W. 2006, "Fluidelastic Instability of an array of Tubes Preferentially Flexible in the Flow Direction Subjected to Two-Phase Cross Flow", ASME Journal of Pressure Vessel Technology, Vol. 128, No.1, pp. 148-159.



Fig. 2: Configurations of flexible tubes tested within the test section: (a) single flexible tube, (b) central cluster, (c) single flexible column (d) two-partially flexible columns



Fig. 3: Flexible tube assembly



Fig. 4: Rms response vs. flow pitch velocity for a single tube flexible inflow, 14 Hz frequency: $-\frac{1}{2}$ $\varepsilon = 65\%$, $-\frac{1}{2}$ $\varepsilon = 80\%$, $-\frac{1}{2}$ $\varepsilon = 90\%$, $-\frac{1}{2}$ $\varepsilon = 95\%$



Fig. 5 – Rms response in lift direction vs. flow pitch velocity for a single axisymetrically flexible tube, 30 Hz frequency: $+\epsilon = 0\%$, $-\epsilon = 20\%$, $-\epsilon = 40\%$, $-\epsilon = 50\%$, $-\pi = 60\%$, $-\epsilon = 80\%$, $-\epsilon = 90\%$



Fig. 6 – Rms response of Tube 7 vs. flow pitch velocity for the central cluster configuration, 14Hz assembly: $-\frac{1}{2} \epsilon = 65\%$, $-\frac{1}{2} \epsilon = 80\%$, $-\frac{1}{2} \epsilon = 90\%$, $-\frac{1}{2} \epsilon = 95\%$



Fig. 7 – Rms response of Tube 7 vs. flow pitch velocity for the central cluster configuration, 28Hz assembly: $- - \epsilon = 85\%$, $- - \epsilon = 90\%$



Fig. 8 – Rms response in lift direction vs. flow pitch velocity for the central cluster configuration, axisymetrically flexible tubes (30 Hz): $\rightarrow \epsilon = 0\%$ (Tube 7), $\rightarrow \epsilon = 20\%$ (Tube 7), $\neg \Box - \epsilon = 40\%$ (Tube 7), $\neg \Delta - \epsilon = 50\%$ (Tube 7), $\neg \pm \epsilon = 60$ (Tube 7), $\neg \pm \epsilon = 80\%$ (Tube 7), $\neg \pm \epsilon = 90\%$ (Tube 7), $\neg \pm \epsilon = 90\%$ (Tube 7), $\neg \pm \epsilon = 95\%$ (Tube 4)

ε = 20%	ε = 40%
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ε = 80%	ε = 95%
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Fig. 9 - Orbital motion of axisymetrically flexible tubes at instability



Fig. 10 – Rms response of Tube 7 vs. flow pitch velocity for the single flexible column case, tubes (14 Hz) flexible inflow: → ε = 80%, → ε = 90%, → ε = 95%



Fig. 11 – Rms response in lift direction vs. flow pitch velocity for the single flexible column configuration, axisymetrically flexible tubes (30 Hz): $+-\epsilon = 0\%$ (Tube 4), $-\phi = \epsilon = 20\%$ (Tube 4), $-\Box = \epsilon = 40\%$ (Tube 4), $-\Delta = \epsilon = 50\%$ (Tube 4), $-\Xi = 60$ (Tube 4), $-\Phi = \epsilon = 80\%$ (Tube 7), $-\Xi = 90\%$ (Tube 7), $-\Delta = \epsilon = 95\%$ (Tube 7)



Fig. 12 – Rms response of Tube 7 vs. flow pitch velocity for the two-partially flexible columns case, tubes (14 Hz) flexible inflow: → ε = 80%, → ε = 90%, → ε = 95%



Fig. 13 – Flow pattern map for two-phase flow across cylinder arrays (Grant [11]) with flow critical conditions: ● tubes flexible inflow (central cluster), ▲ tubes flexible in all direction (central cluster)



Fig. 14 – Instability map: • axisymetrically flexible tube bundle in air-water two-phase flow (Pettigrew et al. [12]), □ single tube flexible in all direction (present study), △ central cluster with tubes flexible in all direction (present study), △ central cluster with tubes flexible inflow (present study), △ central cluster with tubes flexible inflow (present study), + wind tunnel result for the central cluster with tubes flexible inflow (Mureithi et al. [7]), × wind tunnel results for a single flexible column configuration with tubes flexible inflow (Mureithi et al. [7])