BOW Code Development
Modelling Of In-Reactor Bundle Deformation

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1 ABSTRACT

A bundle version of the BOW computer program has been recently developed in AECL to simulate simultaneous elastic and elasto-creep deformations of all fuel elements within a fuel bundle under normal operating conditions. The finite-element method is employed in the BOW code to model lateral deformations of all fuel-bundle components by accounting for temperature variations in the pellet and sheath, neutron flux gradient across the bundle radius, gravity, hydraulic drag force, sheath creep and pellet cracking. A reliable and convergent contact algorithm is implemented into the recent BOW code to handle the contacts among neighboring fuel elements through mating spacers, and to handle contacts of outer fuel elements with the pressure tube through bearing pads while accounting for the creep-induced sagging of the pressure tube. Simultaneous considerations of in-reactor load effects and contacts introduce nonlinear factors to the solution process. This paper presents a methodology to model and solve the complex in-reactor fuel-bundle deformation process. To date applications of the BOW code in nuclear fuels have shown that BOW calculations are accurate and reliable.

2 INTRODUCTION

1.1 Lateral Deformation of a Fuel Bundle

A CANDU® fuel bundle (CANFLEX® 43-element bundle) shown in FIGURE 1, consists of a number of fuel elements whose endcaps are welded to two endplates. On each of these fuel elements, there are spacers (and bearing pads on outer-ring fuel elements) brazed onto the sheath to prevent direct sheath-to-sheath (or sheath-to-pressure-tube) contact. After the fuel bundle is loaded into a reactor fuel channel, Individual fuel elements and the whole fuel bundle deform laterally under various loads during irradiation. Bow is a lateral deflection of a fuel element

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[References 1, 2, 3]. Under normal operating conditions (NOC), the total bow includes elastic, thermo-elastic, and creep (or permanent) components. Elastic and thermo-elastic deformations are recoverable when forces or thermal loads are removed. However, creep-induced deformation is permanent plastic deformation.

The direct driving forces for lateral deformations of a fuel element in a horizontal fuel channel include the temperature gradients, gravity, and hydraulic drag force. The temperature gradients may be caused by neutron flux gradients, non-uniform coolant temperature, or dry patches formed on sheath outer surfaces. Material creep at elevated temperatures and as-fabricated bow (in magnitude of mm) also contribute to the fuel-element lateral deformation.

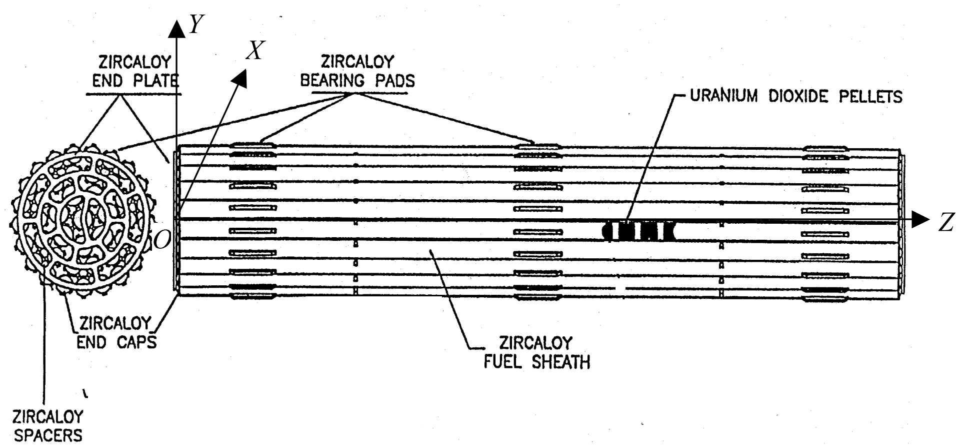

FIGURE 1 A CANFLEX 43-ELEMENT BUNDLE AND ITS GLOBAL COORDINATE SYSTEM

An individual fuel element in a bundle may bend or bow in two lateral directions - radial and tangential, or horizontal and vertical, under in-reactor loads. When a fuel element bends vertically downward, the fuel element is said to sag. When a fuel element bends downward at the two ends, the fuel element is said to droop. Post-irradiation examinations (PIE) of irradiated fuel elements confirm that both sag and droop exist under in-reactor conditions [4]. To determine the degrees of the lateral deformations (bow, sag and droop) of all fuel elements in a bundle, the coupling deformations of fuel elements through the two endplates, and the inter-element contacts need to be considered. The contact conditions for a fuel element depend on the axial location:

- At the midplane of the fuel element, there are spacer pads that maintain space between neighbouring elements.
- At several axial locations, there are bearing pads attached to outer elements that prevent the sheath-to-pressure-tube contact.

Assessment of the fuel-bundle deformations needs to consider (i) all in-reactor loads, namely, from temperature gradients, gravity, hydraulic drag force and creep; (ii) bundle orientations; and (iii) boundary and contact conditions. Because these loads and conditions need to be considered in the modeling, multiple nonlinearities exist in the solution process.

1.2 The BOW Code

BOW, a finite-element (FE) computer code originally developed by M. Tayal [2], is used to assess and quantify on-power and with-load lateral deflections including bow, sag, and droop of nuclear fuel elements in a CANDU reactor. The following factors are addressed in the BOW code [2].

- Circumferential and axial variations in the temperatures of the sheath and of the pellet, caused by neutron flux gradients, dry-patches, non-uniform heat transfer coefficients and non-uniform coolant temperatures;
- Hydraulic drag force, eccentricity of welds between endcaps and endplate, distributed load (e.g., gravity), and lateral concentrated load (e.g., an amount of bundle weight transferred to the endcap of a bottom element via an endplate, lateral contact forces);
- Restraints from endplates, from neighbouring fuel elements and from the pressure-tube;
- Creep-induced additional deflections during bowing, sagging and drooping of fuel elements;
- Interactions between the pellets and the sheath, the effect of the pellet (and pellet cracking) on flexural rigidity, and the contribution of pellet bending to fuel element deflection;
- Other effects such as as-fabricated bow, dependence of material properties on local temperature, effects of cross-sectional geometry (non-circular cross-sections, bearing pads, spacers) and column buckling.

In order to perform complete simulations of on-power and with-load bundle deformation under these conditions, the BOW code was further developed within AECL.

This paper describes the most recent developmental work, which was carried out to enable the finite-element modelling of simultaneous elasto-creep deformations of all fuel elements in a CANDU fuel bundle. The work includes modelling of all bundle components, constraints and contacts, and corresponding development of the BOW code, which are necessary for assessment of on-power and with-load fuel-bundle lateral deformation and fuel-element deflections (e.g., bow, sag and droop).

2. MODELLING

2.1 Modelling of Bundle Components

A typical CANDU fuel bundle consists of two endplates and a number of fuel elements with end caps. Spacer pads and bearing pads are considered attachments to a fuel element.

The two endplates in the fuel bundle are identical. Each endplate consists of several circular rings and a number of radial webs. Endplate rings and the radial webs are modeled using three-dimensional beam finite elements. Circular rings are meshed using beam elements having nodes at (i) all locations where fuel elements are welded, and (ii) all locations where the radial webs are joined to the rings.

To model the bundle lateral deformation, the location of a fuel element in the fuel bundle is defined by both the radial distance from the bundle centre to the fuel element centre, and the angular position with reference to the global coordinate system. In the FE model, fuel elements are numbered from the outer ring to the inner ring, starting from the element located nearest to the reference line or the endplate-fixed *x* axis. FIGURE 2 shows the numbering of the 43 fuel elements in a fuel bundle.

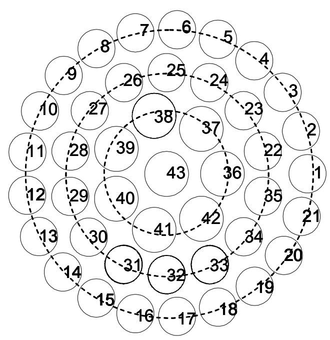

FIGURE 2 NUMBERING OF FUEL ELEMENTS IN A 43-ELEMENT FUEL BUNDLE

To model the lateral deformation of a fuel element in a fuel bundle, the two-node straight beam finite elements are used (same as the BEAM 4 in ANSYS [5]), which are capable of capturing axial, torsion and bending deformations and different element sizes. Because of the relatively large dimensions of the bearing pads used for the outer fuel elements, their strengthening effects on flexural rigidities are included.

2.2 Modelling Fuel Element to Fuel Element and Fuel Element to Pressure Tube Contacts

Because contact conditions are included in the fuel bundle deformation assessment, nonlinear analyses are necessary. The total number of degrees of freedom (DOFs) in the finite-element model is large. However, a significant number of these DOFs do not directly contact other components including the

pressure tube (they are the interior DOFs). These interior DOFs can be eliminated through a so-called substructure method [6], hence, the DOF of the nonlinear problem can be significantly reduced.
The main advantage of the substructure method in fulfilling the requirement for formulation of the linear complimentary equations (LCE) [7] is that the number of DOFs in the final LCEs is significantly reduced. For example, if each fuel element in the bundle is meshed with 50 beam elements, a total of 13,000 (50x43x6) DOFs will exist in the resulting LCEs. However, after the restructuring procedure, only 170 DOFs remain in the final LCEs.
Contact represents a special class of nonlinear problems in the branch of solid mechanics. Contact algorithms can be divided into three groups: the iterative method, the penalty method and the direct method. The main disadvantages associated with the iteration and penalty methods include the trial and error strategy itself, unreliable convergence and potentially long CPU time. These disadvantages become problematic for multi-body contact in a nuclear fuel bundle involving multi-bodies. Therefore, use of either the iteration method or the penalty method in dealing with the kind of contact that occurs in a fuel bundle is not desirable. In this paper, a direct algorithm, based on a point-to-point scheme and the linear complementary equations method (LCEM), is implemented. The main advantages of the implemented method are that (i) no initial guess of contact stiffness is required; (ii) existence and uniqueness of a solution are guaranteed if the contact matrix in the LCEs satisfies the positive-definite or semi-positive-definite conditions; and (iii) contact conditions are simultaneously satisfied at all contact nodes.

A standard set of LCEs is written as [7]

$$\begin{gathered} P - Mg = Q \\ P_j \geq 0, g_j \geq 0, g_j P_j = 0 \quad (j = 1,2,...,n) \end{gathered} \tag{1}$$

where M is a $n \times n$ matrix; P , g and Q are $n \times 1$ vectors. In contact calculation, M is a flexibility matrix, g is the contact force vector, P is the gap vector and Q is load caused relative displacement vector.

To obtain the LCEs for the contact problems in a fuel bundle, a new approach is implemented in the BOW code. Once the LCEs are obtained, the robust Lemeke's algorithm [3, 7, 8] can be used to solve the equations with guaranteed convergence and computational speed.

FIGURE 3 shows a portion of the midplane cross-section of a fuel bundle. For node *i*, the contact conditions considered are the four inter-element contact pairs *i* to *j, i* to *k, i* to *l,* and an element-to-pressure-tube contact pair *i* to pressure tube. For other nodes, similar contact conditions need to be considered.

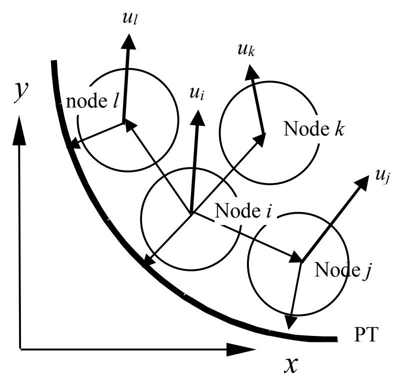

FIGURE 3 CONTACT DIRECTIONS FOR A NODE

2.3. Modelling of Creep

In the BOW code, the sheath creep deformation is considered. To determine the fuel element lateral deformations caused by the creep under NOC, the creep bending moment is calculated by integrating the creep effects over the sheath cross-section and length [9].

Three types of creep laws are used in the BOW code:

- The first creep law is for an irradiated CANDU Zircaloy sheath. This law is applicable to a fuel element in a reactor during normal operation.
- The second creep law is for an unirradiated CANDU Zircaloy sheath at high temperature (in α phase).
- The third creep law is a time-hardening power law.

2.4. Modelling of Initial (As-Fabricated) Bow

Initial bow, or as-fabricated bow, can have a significant effect on fuel element bowing. From the measurement of some bundles, it was found that outer-elements are typically not straight and they can be C-, S-, W-, or M-shaped. The bow of a fuel element can be inward, outward, or a combination.

In the BOW code, the initial bow induced deformations were calculated. The BOW code accounts for the effects of single-wave (or C-shape) initial bow, and multi-wave shapes, such as *S*-, *M*-, or *W*- shapes, which are also observed in many outer elements of an irradiated bundle.

The single-wave initial bow is modelled by polynomials or single wave sinusoidal function. The multi-wave shaped initial bow can be modeled by a set of sinusoidal functions with multi-wave over the fuel element length.

2.5. Modelling of In-Reactor Thermal and Mechanical Loads

The in-reactor thermal loads include circumferential and axial temperature variations in the sheath and pellet, caused by neutron flux gradients, dry-patches, non-uniform heat transfer coefficients and non-uniform coolant temperatures. The mechanical loads include gravity and hydraulic drag. These loads were modeled in previous versions of the BOW code and are also considered in the current version. The details are described in Reference [1, 2].

3. COMPARATIVE STUDY

In a comparative study, calculations of the BOW code were compared with analytical solutions, an out-reactor test and results from PIE.

3.1 Bundle Deformation Considering Neither Contact Nor Creep

In the first test case, the deflections of the 43 fuel elements in a fuel bundle under weight were obtained from BOW and ANSYS without modelling contacts anywhere and without considering the pellet strengthening effects. However, the deformations of individual fuel elements are coupled through the two endplates. To remove the rigid body displacements, the vertical displacement freedoms of the two bottom elements (at 6 o'clock positions) at the outboard bearing pad nodes are constrained. The constrained degrees of freedom are so chosen that they simulate a fuel bundle simply supported on the pressure tube. The loads, fuel element length and material properties are given in TABLE 1.

TABLE 1 COMPARISON OF MAXIMUM DEFLECTIONS OF CANFLEX BUNDLE FUEL ELEMENTS UNDER WEIGHT (FULLY CLAMPED ENDPLATES, NO CONTACTS)

	Element Properties
Outer and Intermediate Elements	Linear weight density = 9.66 N/m, Flexural rigidity = 14.72 Nm^2
Inner and Centre Elements	Linear weight density =12.86 N/m, Flexural rigidity = 26.64 Nm^2

Results for the maximum deflections of the 43 fuel elements were obtained using BOW and ANSYS. For clarity, the midplane horizontal and vertical deflections obtained from BOW and ANSYS [5] are plotted together for each fuel element in FIGURE 4 and FIGURE 5, respectively. It can be concluded that there is an excellent agreement between the two independent solutions. In this calculation, element 15 to 18 are assumed be supported by pressure tube through bearing pads, that is why these element have relative larger deflections at the midplane.

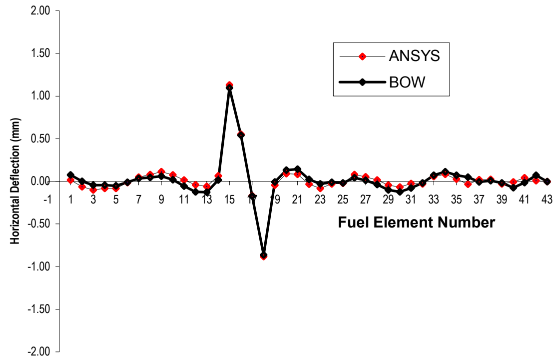

FIGURE 4 MAXIMUM HORIZONTAL DEFLECTIONS: BOW VS. ANSYS

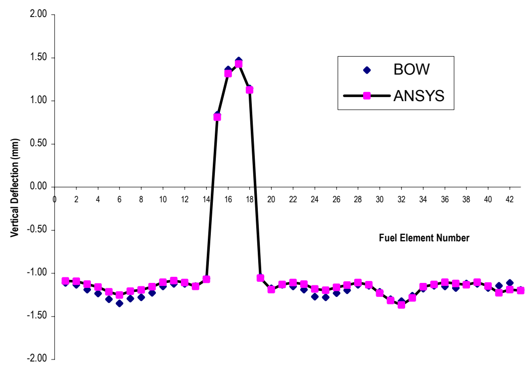

FIGURE 5 MAXIMUM VERTICAL DEFLECTIONS: BOW VS. ANSYS

3.2 Bundle Deformation Considering Contact

The authors developed a simple test case involving only a single fuel element and potential contact with the pressure tube, as shown in FIGURE 6. As the purpose of this test was to check the contact, the single fuel element chosen for this case was constrained in all DOFs except the axial ones at the two ends and was free from any other contact constraints except with the pressure tube at the midpoint. In this case, bending of the chosen element is isolated from the rest of the fuel elements in the bundle. An equivalent model, or a clamped beam subjected to distributed load of variable magnitudes, axial contact force and initial gap, is shown in FIGURE 7. According to Housner and T. Vreeland [10], the deflection of the beam at the midpoint, u_1, without contacting the pressure tube, under uniform distributed load, *q,* and compressive axial load, P_a, may be written as (for $P_a \neq P_{cr,0}$ cases)

$$u_1 = \frac{qL^4}{384EI} \times (1 - \frac{P_a}{P_{cr,0}})^{-1} \tag{2}$$

where L is the length of the beam; EI is the flexural rigidity of the beam; P_a is the axial load simulating hydraulic drag; and $P_{cr,0}$ is the first critical buckling load of the fully clamped beam.

For a clamped beam subject to a concentrated load, *P*, at the midpoint and the axial force, P_a, the deflection at the midpoint is (for $P_a \neq P_{cr,0}$ cases)

$$u_2 = \frac{PL^3}{192EI} \times (1 - \frac{P_a}{P_{cr,0}})^{-1} \tag{3}$$

When contact does not occur, the beam bends freely under its own weight. However, when contact occurs at the midpoint, the deformation of the beam at the midpoint is constrained. The contact force, *P*, may be determined from the following equation

$$P = \begin{cases} 0 & if \quad (u_1 - \delta_0) < 0 \\ (u_1 - \delta_0)\dfrac{192EI}{L^3}(1 - \dfrac{P_a}{P_{cr,0}}) & if \quad (u_1 - \delta_0) \geq 0 \end{cases} . \tag{4}$$

where δ_0 is the initial gap.

Similarly, the gap may be obtained as

$$\delta = \begin{cases} \delta_0 - u_1 & if \quad (u_1 - \delta_0) < 0 \\ 0 & if \quad (u_1 - \delta_0) \geq 0 \end{cases} \tag{5}$$

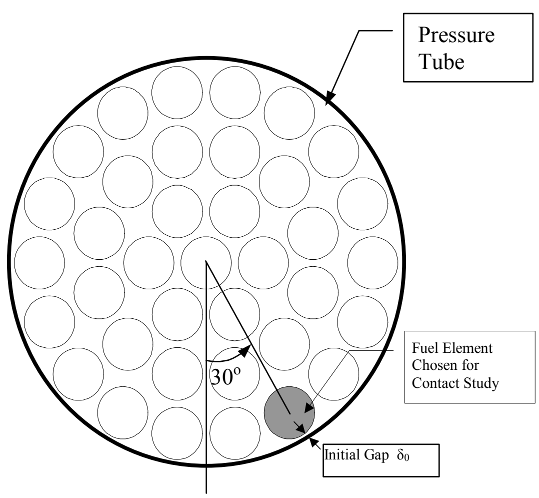

FIGURE 6 FUEL ELEMENT (SHADED) IN A FUEL BUNDLE CHOSEN FOR CONTACT STUDY

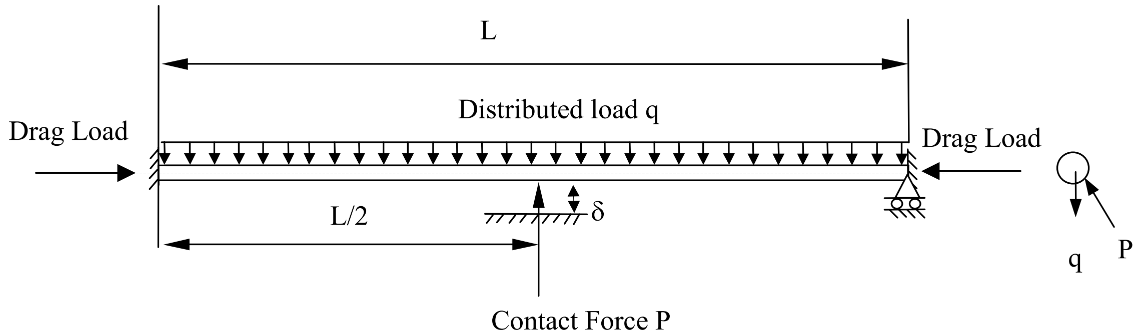

FIGURE 7 A CLAMPED BEAM SUBJECTED TO CONTACT CONSTRAINT AT MIDPOINT

Two scenarios were analyzed in this test case. One did not involve axial load while the other involved an axial compressive load of 800 N. Results for contact force and gap were obtained using BOW and the given analytical solutions, i.e. Eq. (2) to (5). They are plotted in FIGURE 8 and FIGURE 9. It can be seen that the BOW results perfectly match those of the analytical solution for a wide range of distributed load concerning both contact and non-contact situations.

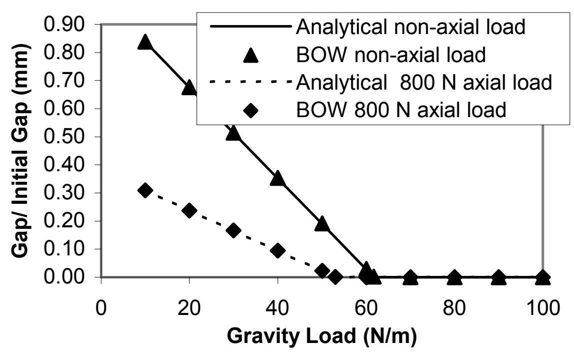

FIGURE 8 GAP VERSUS DISTRIBUTED LOAD

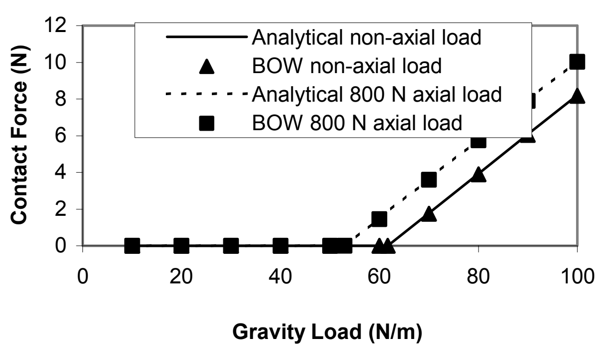

FIGURE 9 CONTACT FORCES VERSUS DISTRIBUTED LOAD

3.3 Bundle Deformation Considering Contact Comparing with Out-Reactor Tests

In the third case BOW was tested against unirradiated bundle droop measurements during an out-reactor test. The only load on the bundle during the outer-reactor test was gravity. The fuel bundle was supported by a cradle, which was cut from a real pressure tube. Testing was performed for the following two configurations for a 43-element CANFLEX bundle having staggered bearing pads:

- bundle fully supported - all bottom bearing pads were in contact with the cradle, and
- bundle overhung - the bundle was supported by the inboard and outboard bearing pads near one bundle end and by the inboard bearing pads only near the other bundle end (the outboard bearing pads were left unsupported).

In the test, the bundle was measured at 21 positions for each of the 21 outer fuel element located at the 6 o'clock position. A total of 14 outer fuel-elements had outboard pads and the remaining did not. For a fuel element with an outboard bearing pad, the deflection datum is the sheath end droop, while for a fuel element without an outboard bearing pad, the deflection datum is only the vertical deflection difference.

FIGURE 10 and FIGURE 11 show the comparisons between BOW calculations and measurements. In both of the two figures, the data are normalized by dividing the maximum values. It can be seen from the figures that the BOW calculations and the measurements have the same trends for both cases and are in good agreement. In the results from the BOW calculations, all of the contact results are reasonable. One set of typical contact results is shown in TABLE 2. In this case, about 100 bearing pad to pressure tube and about 100 inter-spacer contact conditions were checked. In the spacer-to-spacer contact node pairs, 8 pairs are contacted and the rest are kept apart.

The running time for the BOW code analysis for one of those cases is about 30 seconds on a HP UNIX computer.

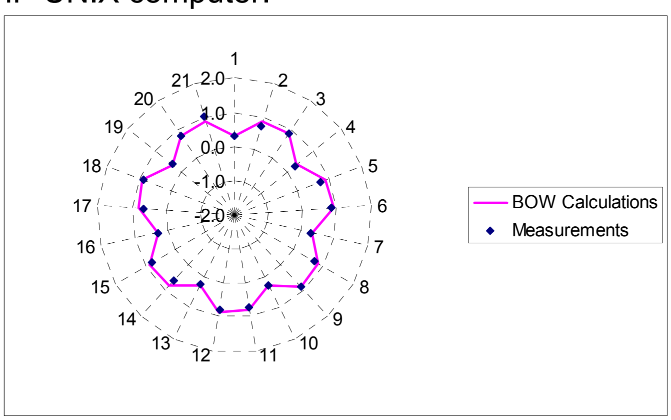

FIGURE 10 COMPARISON BETWEEN BOW CALCULATION AND DROOP MEASUREMENTS (FULLY SUPPORT CASE, NORMALIZED)

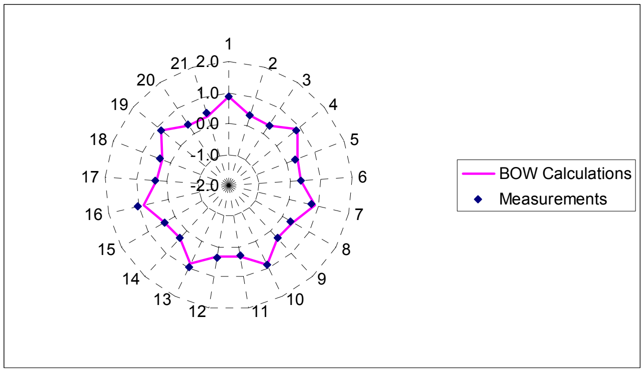

FIGURE 11 COMPARISON BETWEEN BOW CALCULATION AND DROOP MEASUREMENTS (OVERHANG SUPPORT, NORMALIZED)

TABLE 2 BUNDLE MIDPLANE CONTACTED FUEL ELEMENT AND CONTACT FORCE (FOR ELEMENT 17 AT 6 O'CLOCK POSITION CASE)

Contacted Element Pairs	Contact Force/ Bundle Weight (%)
17-18	0.227
16-31	1.260
16-32	0.621
17-32	2.331
18-33	1.551
32-41	1.152
33-42	0.369
41-43	0.142

3.4 Comparison with WL Out-of-Reactor Bow Test

Thermal induced bow, caused by a circumferential temperature distribution around a fuel element, was investigated in AECL's Whiteshell Laboratory (WL) [11]. In the WL out-reactor bowing test, a fuel element simulator (FES) was used in each test. Each FES was made of Pickering size pellets and sheath, representing a fuel element. No end plate was attached. An off-set hole was drilled through the pellets to house the electric heater. With the heater located away from the centre of the pellets, asymmetric heat is generated, creating a circumferential temperature gradient. During the tests, transient bow increases with the top-to-bottom sheath temperature difference and interaction between the pellet and the sheath. The permanent bow was caused solely by creep.

The results of the BOW calculation are shown in FIGURE 12 and are compared to the measurements from a specimen in the WL Test. The measured with-load deflection at the midplane of the fuel element simulator under the sole thermal load (temperature difference between top and bottom of the simulator) is shown as a function of time. It can be seen that the BOW calculated deflection is able to follow the measurement.

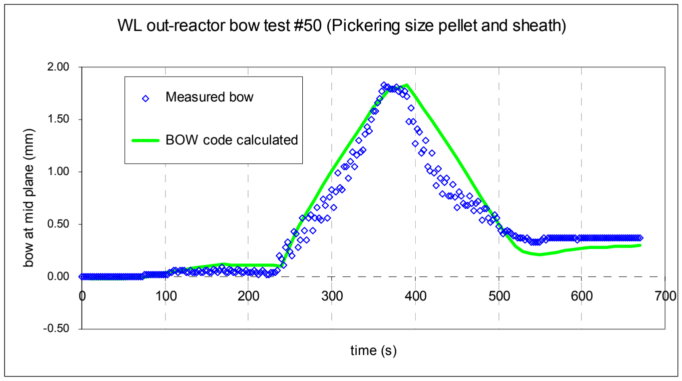

FIGURE 12 DEFLECTION OF FUEL-ELEMENT SIMULATOR AT THE MIDPLANE: WL TEST #50

3.5 Comparison with PIE Results

To check creep calculation capabilities of the BOW code, BOW predictions for permanent bow were compared with PIE results. A CANFLEX-DI bundle, FLX007Z, was irradiated in the Point Lepreau Generating Station (operated in PLGS Channel S08 Position 8) and was also examined via PIE.

The power histories and as-fabricated average bow of the archive bundle were used for the BOW code simulation. FIGURE 13 shows the BOW code prediction for the permanent deflections at the midpoint and the PIE results. The results show that BOW-predicted permanent bow matches the PIE results well for majority of the fuel elements; and the shape of the BOW prediction also matches the PIE results. The resident time for this case is about 250 days and the running time for the BOW code analysis for this case is about 2 minutes on a HP UNIX computer.

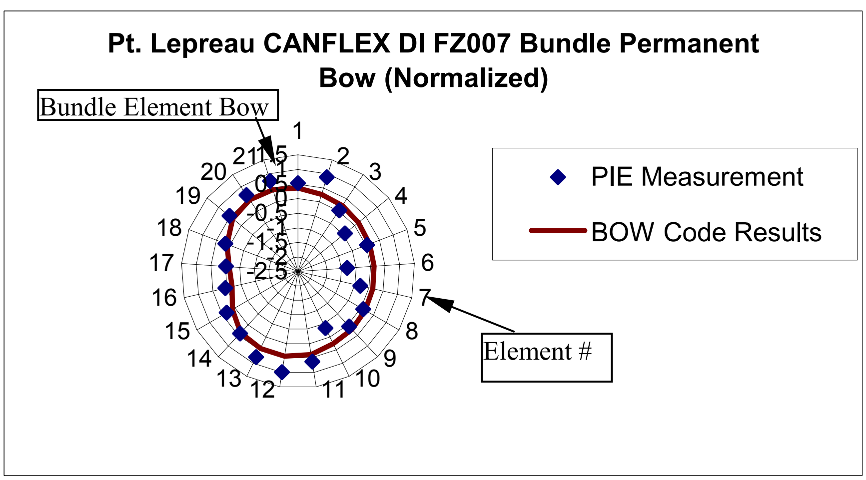

FIGURE 13 BUNDLE-ELEMENT BOW PROFILE AT MID-PLANE (WITH AVERAGE AS-FABRICATED BOW)

4. CONCLUSIONS

A bundle version of the BOW computer program has been recently developed to simulate simultaneous elastic and elasto-creep deformations of all fuel elements within a fuel bundle under normal operating conditions. The BOW code models the deformations of a fuel bundle by accounting for temperature gradients, neutron flux gradients, gravity, hydraulic drag force, sheath creep, pellet cracking, and multiple body contacts.

This paper presents a methodology, developed for and used in the BOW code, and some validation results. The BOW calculations were found convergent and in good agreement with independent calculations and measurements. In addition, BOW runs quickly. A typical run, without the creep option, takes about 30 seconds on a HP UNIX computer. With the creep option, the running time is about 2 minutes to simulate about 250 days of creep.

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