MATHEMATICAL MODELING OF CANDU-PHWR

BY

FATMA. A. GABER, R.A.ALY and AZZA O. EL-SHAL

Atomic Energy Authority, Cairo, Egypt

ABSTRACT

The paper deals with the transient studies of CANDU 600 pressurized Heavy Water Reactor (PHWR) system. This study involved mathematical modeling of CANDU–PHWR major system components and the developments of software to study the thermodynamic performances. Modeling of CANDU-PHWR was based on lumped parameter technique.The integrated CANDU-PHWR model includes the neutronic, reactivity, fuel channel heat transfer, piping and the preheater type U-tube steam generator (PUTSG). The nuclear reactor power was modelled using the point kinetics equations with six groups of delayed neutrons and reactivity feed back due to the changes in fuel temperature and coolant temperature.

The complex operation of the preheater type U- tube steam generator (PUTSG) is represented by a non- linear dynamic model using a state variable, moving boundary and lumped parameter techniques. The secondary side of the PUTSG model has six separate lumps including a preheater region, a lower boiling section, a mixing region, a riser, a chimmeny section, and a down-comer. The tube side of PUTSG has three main thermal zones. The PUTSG model is based on conservation of mass, energy and momentum relation-ships. The CANDU-PHWR integrated model are coded in FORTRAN language and solved by using a standard numerical technique. The adequacy of the model was tested by assessing the physical plausibility of the obtained results.

1- INTRODUCTION:

The CANDU 600 pressurized heavy water reactor (PHWR) is a large, horizontally-oriented cylindrical tank, called the calandria, which contains the cool, low pressure heavy water moderator. This tank is penetrated by a number of horizontal tubes, called fuel channels which contain the natural Uranium fuel and the pressurized high temperature heavy water coolant. This coolant is pumped through the fuel channels, removing heat from the fuel and then through a preheater type U- tube steam generator (PUTSG) where this heat is given up to produce steam which is fed to the turbine. The steam generators and coolant pumps are located at each end of the reactor so that flow is in one direction through one half of the fuel channels and in the opposite direction through the other half. A pressurizer maintains the coolant circuit pressure at a relatively high value. High circuit pressure permits high coolant temperatures which in turn permits the generation of steam at a high enough pressure to achieve reasonable turbine cycle efficiencies. The CANDU reactor simplified flow diagram is shown in figure 1. The reactor consists of a cylindrical stainless steel calandria structure which contains the heavy water moderator, reactivity control mechanisms and 380 fuel channel assemblies. The fuel channel assemblies contain the fuel and heavy water coolant and pass through the calandria tubes. The gap between each fuel channel and calandria tube is gas – filled to provide thermal insulation . A cooling system is provided, to dissipate heat transferred to the moderator from the high temperature fuel channels and heat generated in the heavy water itself through interaction with fission neutrons and gamma radiation. The reactor is fuelled with natural Uranium in the form of pellets, stacked end – to end, are sealed in zirconium alloy sheath to form a fuel element. Thirty – seven of these elements are assembled into a fuel bundle (Figure 2) . Each fuel channel accommodates twelve fuel bundles .

2- MATHEMATICAL MODELING:

2.1 Reactor Model :

The CANDU–PHWR reactor power was modelled using the point kinetics equations with six groups of delayed neutrons and reactivity feedback due to changes in fuel temperature and coolant temperature as follows:

$$\frac{dn}{dt} = (R - B_T)\frac{n}{L} + \sum_{i=1}^{6}\lambda_i C_i$$

$$\frac{dC_i}{dt} = \frac{B_i \cdot n}{L} - \lambda_i C_i$$

$$R = R_c + R_{fb}$$

$$R_{fb} = \alpha_F \cdot \Delta T_F + \alpha_c \cdot \Delta T_c$$

where:

n=Neutron density,

B_T=total delayed neutron fraction,

B_i=delayed neutron fraction for the i^{th} group,

L=neutron generation time,

C_i ,λ_i =precursor concentration and decay constant respectively,

R,R_{fb},R_c=total, feedback and control rod reactivity respectively,

α_F,α_c=Doppler coefficient of reactivity and coolant temperature coefficient of reactivity respectively,

ΔT_F ,ΔT_c=change in fuel and coolant temperature respectively.

The reactor heat transfer model was based on lumped parameter technique, which assumes average system parameters over defined lumps. This approach gives moderate internal information of the system depending on the number of lumps used with reasonable accuracy at moderate computer time and cost .The fuel channel heat transfer model is simulated by an average channel compossed of horizontal central fuel region surrounded by coolant . Two coolant lumps were used for the fuel lump to obtain good approximation of the heat transfer driving force between fuel and coolant which is taken as the difference between the average fuel temperature and the average temperatures of each coolant lump respectively. The model equations are derived from the basic principles of thermohydroulics by applying the laws of conservation of mass and energy on each model lump, and the equations of state for the heavy water coolant as follows:

For the fuel-lump:

$$M_F \cdot c_{PF} \frac{d}{dt}(\overline{T}_F) = P_\circ \cdot N - U_{Fc} \cdot A_{Fc} \cdot (\overline{T}_F - \overline{T}_c)$$

For coolant lumps:

$$M_{c1} \cdot c_P \cdot \frac{d}{dt}(T_{c1}) = \frac{1}{2} U_{Fc} \cdot A_{Fc} \cdot (\overline{T}_F - \overline{T}_{c1}) + w_P \cdot c_P (T_L - T_{c1})$$

$$M_{c2} \cdot c_P \cdot \frac{d}{dt}(T_{c2}) = \frac{1}{2} U_{Fc} \cdot A_{Fc} \cdot (\overline{T}_F - \overline{T}_{c2}) + w_P \cdot c_P (T_{c1} - T_{c2})$$

where:

A_{Fc}=Heat transfer area between fuel and coolant,

c_{PF} ,c_P=specific heat of fuel and coolant respectively,

M_F=fuel mass,

M_{c1} , M_{c2}=coolant mass in the first and second coolant lumps respectively,

$N=n/n_o$ =normalized neutron density,

P_o=reactor power at normal operation,

$\overline{T}_F$=average fuel temperature,

T_L=temperature of coolant entering the fuel bundle in the first coolant lump,

T_{c1} ,T_{c2}=output coolant temperature of the first and second coolant lumps respectively,

U_{Fc}=overall heat trasfer coefficient between fuel and coolant,

w_P=primary heavy water coolant flow rate,

$$\overline{T}_{cl} = (T_L + T_{cl})/2$$

$$\overline{T}_{c2} = (T_{cl} + T_{c2})/2$$

$$\overline{T}_c = (\overline{T}_{c1} + \overline{T}_{c2})/2$$

For the application of the mathematical model, it is necessary to obtain the effective heat transfer coefficient between the fuel with an average temperature $\overline{T}_F$ and the heavy water coolant with an average temperature $\overline{T}_C$. This heat transfer coefficient can be derived by equating the heat generated in the fuel with the heat transferred by conduction through the cladding material and the heat transferred by convection through the coolant . It is assumed that no heat generated in the cladding and the resistance to heat at the fuel cladding interface is neglected.

2.2 PUTSG Model :

2.2.1 Introduction

Four indentical steam generators with integral preheaters transfer heat from the reactor coolant to raise the temperature of, and boil, the feedwater. The steam generators consist of an inverted vertical U- tube bundle installed in a shell . Steam separating equipment is housed in the upper end of the shell . Incoming feedwater is pumped into the baffled preheater section and flows over the reactor coolant outlet portion of the U – tube bundle. The feedwater emerges from the preheater section at saturation temperature and mixes with the recirculated saturated water flowing over the other portion of the U – tube bundle . Saturated water flows to the tube bundle from the steam separating equipment through an annulus between the tube bundle shroud and the inside surrface of the shell . The water passes through holes in the bottom of the shroud and penetrates and flows over the reactor coolant inlet portion of the U – tube bundle .The steam water mixture rising from the upper end of the U- tube bundle passes through the steam separators. The separated water recirculates to the tube bundle. The steam with less than 25 percent moisture by weight leaves the boiler through the outlet nozzle.The water level at the separators is controlled by a combination of level measuement and feedwater flow measurement .

2.2.2 The Heat Transfer Model :

Starting from the first principles and employing a moving boundary non- linear lumped parameter state variable formulation a mathematical model for the PUTSG has been developed. The model used to simulate the PUTSG is shown in figure 3 . The secondary side of the PUTSG model has six separate lumps including a preheater region (PS), a lower boilling section (LBS), a mixing region (ML), a riser (RL), a chimmeny section (CL), and a downcomer (DC). Th PUTSG model is based on conservation of mass, energy and momentum relationships. The tube side of PUTSG has three main thermal zones.

A-Secondary Side:

By applying the mass, energy and momentum conservation equations on each of the following secondary side lumps, a set of secondary side equations are obtained :

1-Preheater Sectin (Ps)

The fedwater flow enters this section with an inlet temperature T_{F1}. Heat is transferred from the metal (at temperature T_{WP}) to the liquid which leaving the PS lumps with temperature T_{F2} . The change in the liquid mass in PS is neglected .

2- The Downcomer (DC) :

The downcomer is assumed to have a fixed volume and the change in water density is negligible. Therefore the DC secondary fluid lump in this model acts as a time delay section.

3- The Lower Boiling Section (LBS) :

The fluid (water) enters this lump in saturated liquid state. It leaves the lump at the same level as the top of the preheater section and in the saturated steam water state .

4- The Mixing Lump (ML) :

The moving boundary ML is needed to combine the mixture coming from the LBS lump and the saturated liquid coming from PS lump.

5- The Riser Lump (RL):

This lump is a moving boundary lump, with a variable length and density .

6- The Chimney Lump(CL) .

7- Steam Drum Lump (SDL) :

The Steam drum is regards as two distinct regions, one contains saturated water and the other contains saturated steam.

B- Primary Side :

By aplying the mass, energy and momentum conservation equations on each of the following primary side lumps, a set of primary side equations are obtained :

1-Primary Inlet (Pi) and Primary outlet (P_o), Plenum Lumps:

where:

M_{Pi} ,M_{Po}=mass of primary coolant (heavy water coolant) in the inlet and outlet plenum respectively,

w_{Pi} ,w_{Po} =mass flow rates of the primary coolant intering and leaving the PUTSG,
H_{Pi} ,H_P=enthalpy of primary fluid entering and leaving the inlet plenum respectively,

T_{PP} ,T_{Po}=temperature of primary fluid entering and leaving the outlet plenum respectively,

$$\overline{H}_{Pi} = (H_P + H_{Pi})/2$$

$$\overline{T}_{Po} = (T_{PP} + T_{Po})/2$$

2-Evaporative Section Primary Lumps.

In the evaporative section the primary coolant flow path is divided into the following lumps as shown in Figure 3 :

-The PL, PMI and PR1 lumps, in which the primary flow passes through the tubes parallel to the secondary flow in the LBS, ML and RL lumps respectively .

-The PR2 and PM2 lumps, in which the primary flow passes through the tube counter to the secondary flow in the RL and ML lumps respectively .

3-Preheating Section Primary Lump (PPL) :

In this lump, the primary flow passes through the tubes counter to the secondary subcooled flow in the preheater section. The preheating zone is in effect a liquid to liquid counter-flow heat exchanger.

C- Tube Metal Lumb (ML) :

The tube metal is divided into six lumps adjacent to the six primary flow lumps in the tube bundle region. For the tube wall lumps, there is no mass flow with respect to fixed coordinates. However, the change of boundaries results in the mass exchange. The mass exchange will be considered as a mass flow coming or leaving the lumps. Then by applying the mass and energy conservation equations on each of these six metal lumps, a set of metal equations are obtained.

3.CANDU - PHWR PROGRAM DESCRIPTION :

For the state variables chosen, the integrated model (reactor, piping and steam generator) equations are solved using standard numerical method called "Merson" for solving a set of coupled nonlinear differential equations.

The equations form is then given by:

$$\frac{d}{dt}(\bar{Y}) = \bar{F}$$

where:

$\bar{Y}$= State variable vector,

$\bar{F}$= Vector of non linear and forcing terms (including differential terms) .

The elements of vector F will be listed in subroutine "DIFF", which is a part of the "Merson" program used. The Merson program concerned with the calculation of the transient behaviour of the state variables using the forcing and steady state values as an input. This program consists of three sections, the first is the main program that includes the forcing values, the basic model data, the geometrical parmeters and the steady state values as an input to the system. The second section is the subroutine "DIFF" which includes the state equations, the heat transfer correlations, algebraic variables and all physical properties of heavy water coolant and steam-water mixure as time dependent for the integrated CANDU- PHWR model . The third section is the subroutine "Merson" which includes the algorithm for solving the system equations .

4.TRANSIENT RESPONSE OF INTEGRATED CANDU- PHWR MODEL :

The transient responses of the integrated CANDU- PHWR mathematical model for various step reactivity changes is shown in Figure 4.

The first effect of the reactivity decrease is shown as a rapid decrease in power generated due to the external reactivity step decrease. Decreasing the fuel temperature is followed by a decrease in the coolant temperatures. The rapid increase of the nuclear power generated following its initial rapid reduction is due to the negative feed back reactivity. This rapid increase is followed by relatively slower increase of power due to the decrease in the negative feed back reactivity till the power reaches a steady state value .

The effect of the reactivity increase is shown as a rapid increase in the nuclear power generated. Increasing the fuel temperature is followed by increasing coolant temperature inside the core. The rapid decrease of the nuclear power following its initial increase is due to the negative feed back reactivity increase. This rapid decrease is followed by slower power increase (for small external reactivity increase) or rapid power increase (for large external reactivity increases).

Figure 5 illustrates the CANDU reactor transients for various step reduction in the primary coolant flow. The simulation of this perturbation is done through the change in the primary flow valve coefficient .

The decrease in the primary flow increase the fuel temperature. The rapid increase in the reactor power is followed by a rapid decrease due to the negative feed back effect till the power reaches a steady state value.

5.CONCLUSIONS:

Starting from the first principles and employing a moving boundary non- linear lumped parameter state variable formulation, an integrated mathematical model for the CANDU-PHWR major components has been developed. This model has been coded in a general purpose language (FORTRAN) and solved using a standard numerical technique. The adequacy of the model was tested by assessing the physical plausibility of the obtained results.

The results showed that for both the external reactivity increase and primary flow decrease perturbations the CANDU-PHWR reactor is inherently stable due to its high negative temperature coefficient of reactivity.

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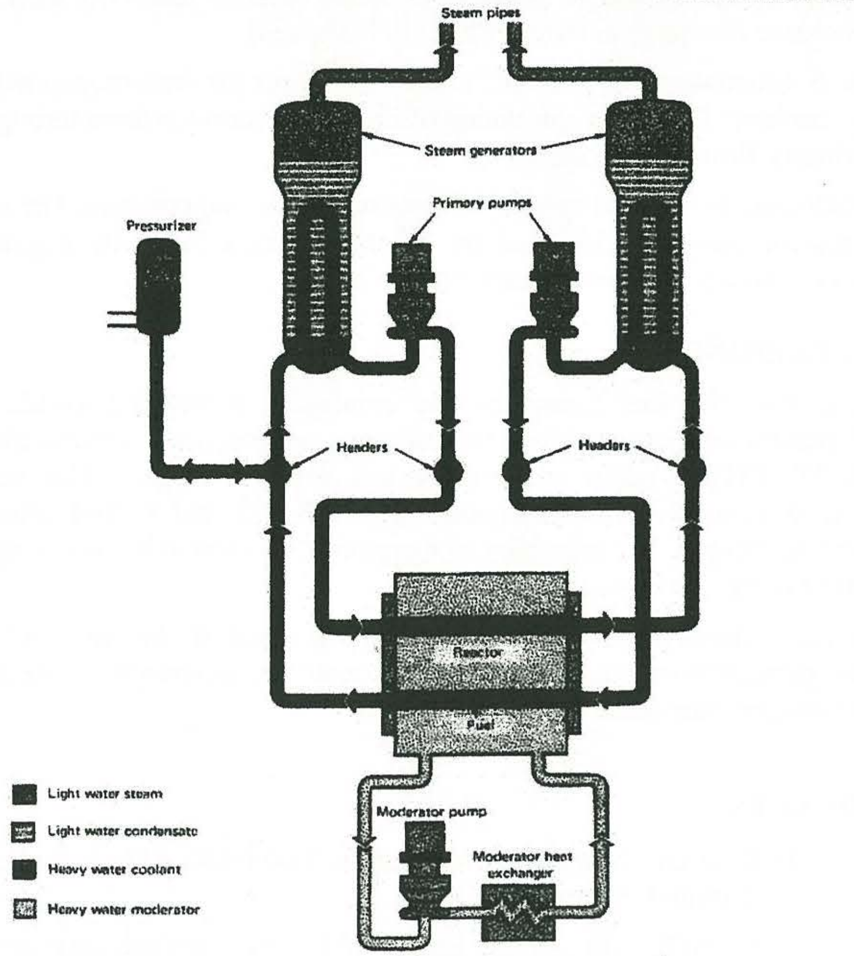
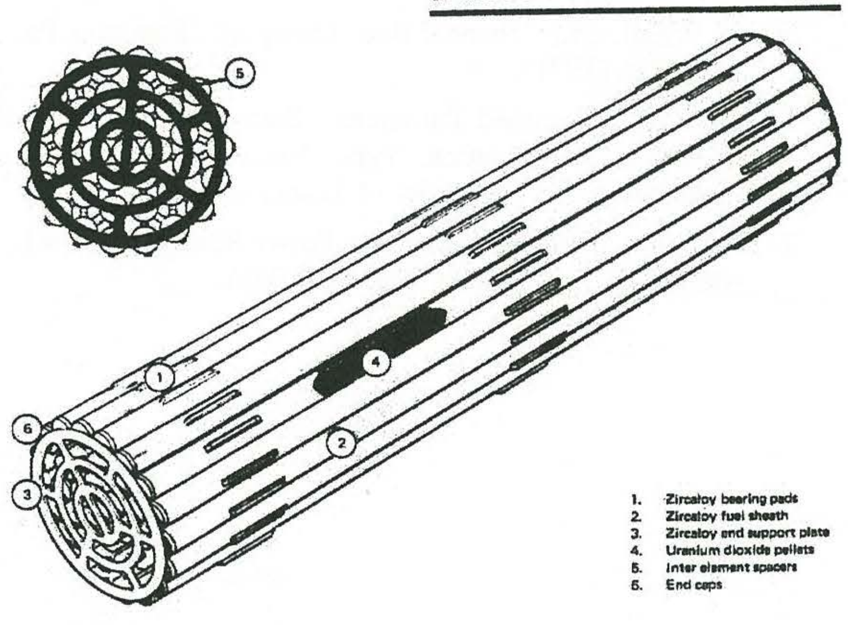
CANDU Reactor Simplified Flow Diagram Figure 1



37 Element Fuel Bundle Figure 2



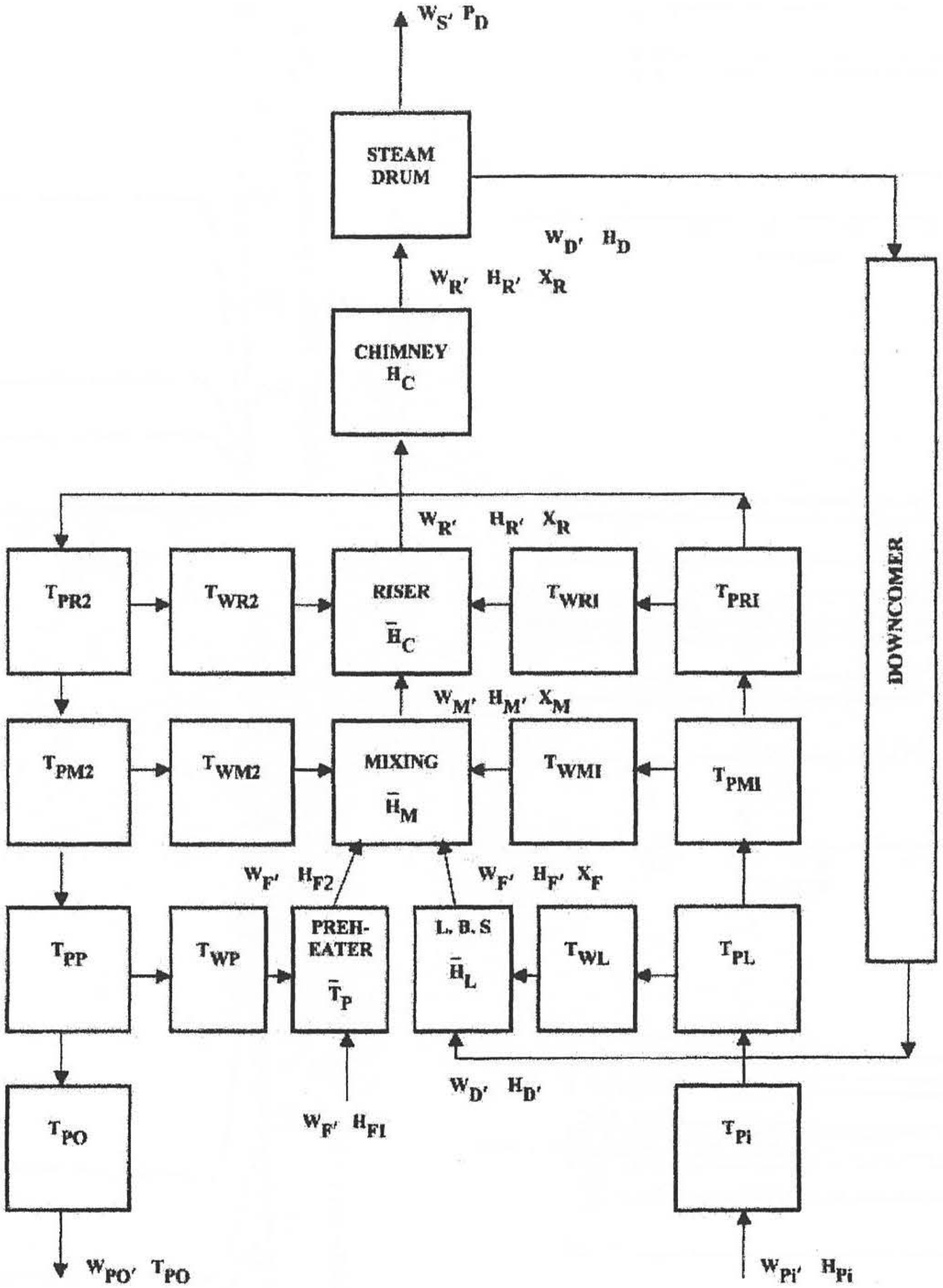


Figure (3) Schematic diagram for PUTSG model

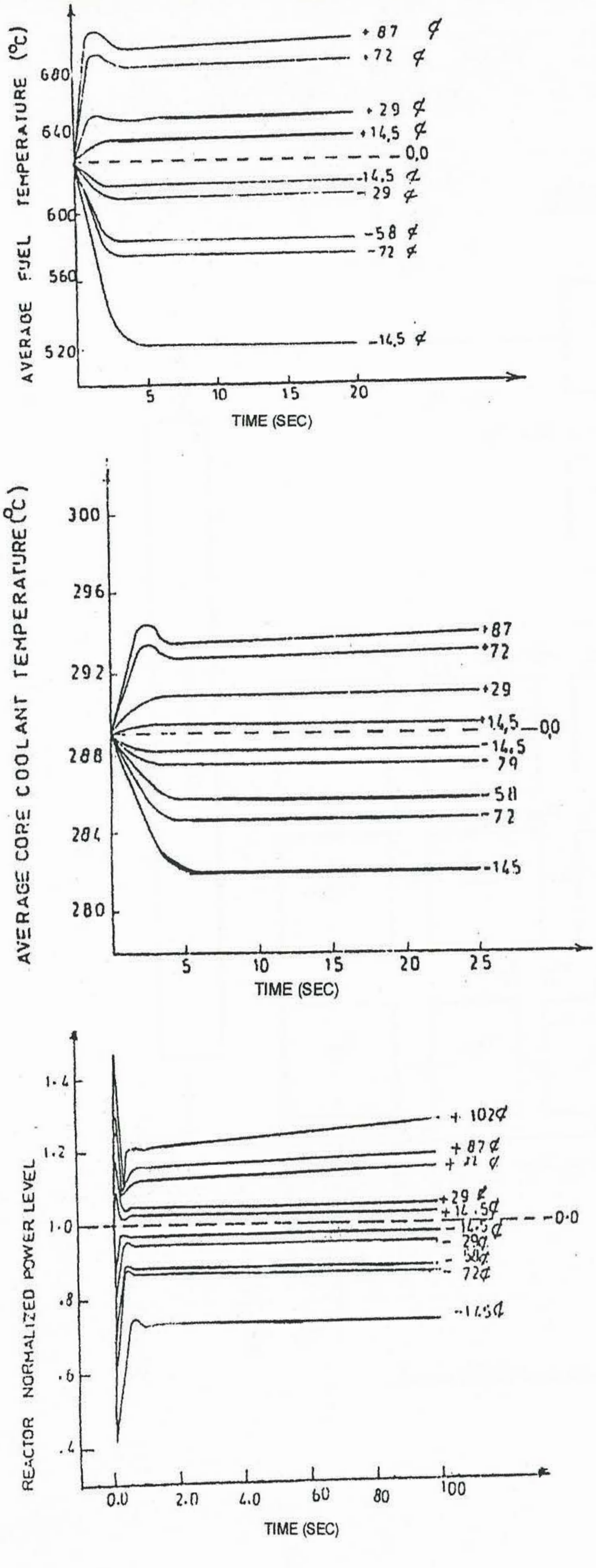


FIGURE (4) THE CANDU REACTOR TRANSIENT FOR VARIOUS STEP REACTIVITY CHANGES

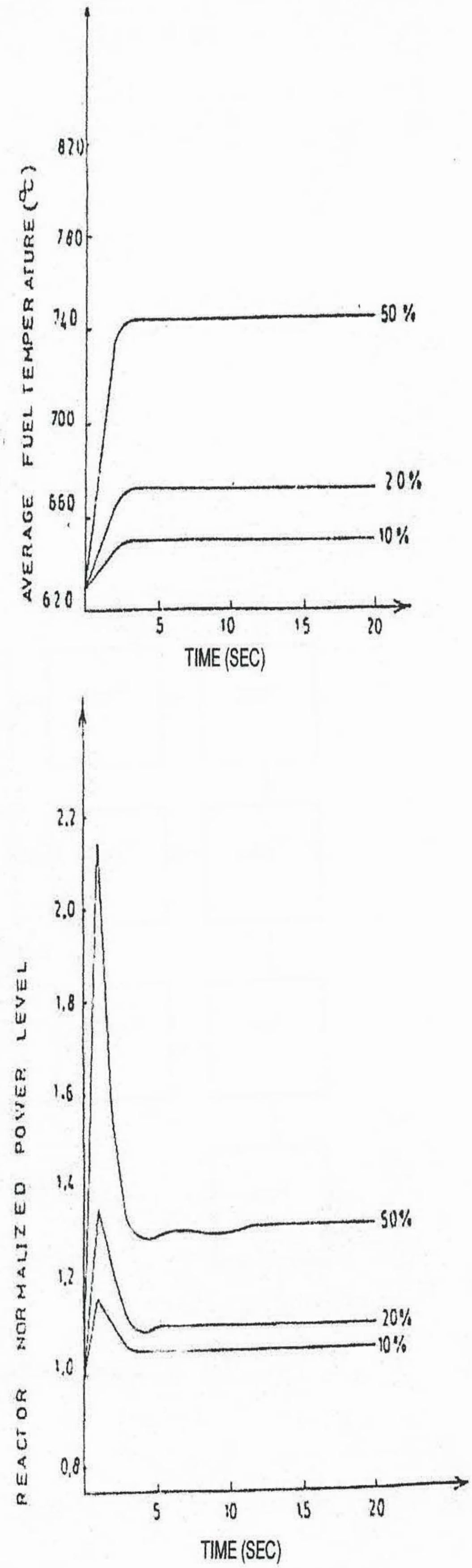


Figure (5) THE CANDU REACTOR TRANSIENT FOR VARIOUS STEP CHANGES IN THE COOLANT FLOW RATES