# Fuzzy Adaptive Robust Optimal Controller for Wide Range Power Regulation and Increasing Load Follow Capability of Nuclear Reactors

**Mehrdad N. Khajavi**

*Lecturer of The Islamic Azad University of Iran (Takestan Branch)*
*Educational Complex Takestan*
*Ph.D. student of nuclear science and engineering*
*The Amir-Kabir University of Technology*
*Physics Department, Hafez Ave*
*Tehran Iran. e-mail: n6811905@cic.aku.ac.ir*
*Phone : (009821) 4400986 Fax : (009821) 8781181*

**Mohammad B. Menhaj**

Associate Prof. IEEE member.
*The Amir-Kabir University of Technology*
*Electrical Engineering Department, Hafez Ave*
*Tehran Iran. e-mail: menhaj@cic.aku.ac.ir*
*Phone: (009821) 6466009 Fax: (009821) 6407418*

**Amir A. Suratgar**

*Ph.D. student of electrical engineering*
*The Amir-Kabir University of Technology*
*Electrical Engineering Department, Hafez Ave*
*Tehran Iran. e-mail: s7823909@cic.aku.ac.ir*
*Phone: (009821) 6466009 Fax: (009821) 6407418*

Abstract- Nuclear reactors are in nature nonlinear and their parameters vary with time as a function of power, fuel burnup, and control rod worth. Therefore, these characteristics must be considered if large power variations occur in power plant working regimes (for example in load following conditions). In this paper a Fuzzy Adaptive Robust Optimal Controller (FAROC) based on a proposed modified dynamic non-singleton fuzzy logic systems is presented. A Robust Optimal Self-Tuning Regulator (ROSTR) response is used as a reference trajectory to determine the feedback, feedforward and observer gains of the fuzzy controller. The fuzzy controller displays good stability and performance for a wide range of operation as well as considerable reduction in computation time in regard to ROSTR. It also increases the load follow capability of nuclear reactor.

**Keywords:** Modelling an simulation, Nuclear power plant, Control of power systems, Fuzzy systems, Adaptive control

## 1-INTRODUCTION

Many investigations have been done in the field of nuclear reactor control. The design of controller for a nuclear power plant and its robustness to process and measurement noise for 10% variation of reactor power about nominal power (100%) has been reported (Akin and Altin, 1991). Edwards and his colleagues demonstrated improved robustness characteristic of SFAC (State Feedback Assisted Classical Control) to cope with changes of reactor parameters over that of CSFC (Conventional State Feedback Control)[1]. Ramaswamy and his colleagues have designed a fuzzy controller based on a fixed optimal controller [2]. In recent work Khajavi and his colleagues have designed and simulated a Robust Optimal Self-Tuning Regulator (ROSTR) for nuclear reactors [3 ]. In this paper a fuzzy logic controller based on the response of the ROSTR is designed and simulated aiming at improving overall system stability and performance as well as increasing load follow capability. Two cases of a) very large step change of power level from 100% to 10% and b) start-up/shut-down operation of reactor 100% to 10% and back to 100% power have been considered and some test simulations have been carried out. The results show that the response of the fuzzy controller follows very closely the reference response but the computation time is reduced by a factor of three. In section 2 the equations governing reactor are explained. Section 3 gives a brief review of the ROSTR method. In section 4 the design procedure of the proposed fuzzy logic controller is described. Section 5 presents the simulation results. Conclusions are given in section 6.

## 2 -Nuclear Reactor Model

A fifth order nonlinear model, with one delayed neutron group and two thermal feedback mechanism (Edwards et al,1992)[4], is the basis of designing a fuzzy logic controller for controlling power level of a PWR reactor. Point-kinetic equations are assumed for reactor neutronics. The governing equations are as follows

$$\frac{dn}{dt} = \frac{\delta\rho - \beta}{\Lambda} n + \lambda c \qquad (1)$$

and

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c \qquad (2)$$

where,

$n \equiv$ neutron density $(n/cm^3)$ ,

$c \equiv$ (neutron) precursor density $(atom/cm^3)$ ,

$\lambda \equiv$ effective precursor radioactive decay constant $(s^{-1})$ chosen to match the one group reactor transfer function to a six delayed neutron group transfer function as closely as possible (Schultz,1961),

$\Lambda \equiv$ effective prompt neutron lifetime (s),

$\beta \equiv$ fraction of delayed fission neutrons,

$k \equiv k_{eff} \equiv$ effective neutron multiplication factor,

$\delta\rho \equiv \frac{k-1}{k} \equiv$ reactivity ( since $k \approx 1.000$ , $\delta\rho \approx k-1$ ; at steady state $k=1$ , $\delta\rho = 0$ ).

For computational purposes, we will use equivalent normalized versions of Eqs (1) and (2):

$$\frac{dn_r}{dt} = \frac{\delta\rho - \beta}{\Lambda} n_r + \frac{\beta}{\Lambda} c_r \tag{3}$$

and

$$\frac{dc_r}{dt} = \lambda n_r - \lambda c_r \tag{4}$$

where,

$n_0 \equiv$ initial equilibrium(steady-state) neutron density,

$c_0 \equiv$ initial equilibrium(steady-state ) precursor density,

$n_r \equiv n / n_0$ , neutron density relative to equilibrium density,

$c_r \equiv c / c_0$ , precursor density relative to initial equilibrium density.

Reactor temperatures vary as a function of power generation and heat transfer from (or to) the system. Using normalized point-kinetics equations for $n_r$, reactor power can be represented as

$$P_a(t) = P_{0a} n_r(t) \equiv \text{reactor power at time t (MW)} \tag{5}$$

and

$P_{0a} \equiv$ initial equilibrium power level (MW)

The power $P$ and power demand $P_d$ used in the block diagrams are assumed to be relative to the initial equilibrium power (i.e., $P = P_a / P_{0a}$ ) and are therefore equal to $n_r$. The following thermal-hydraulic model represents a two-temperature feedback mechanism for a PWR.

$$P_c(t) = \Omega(T_f - T_c) \tag{6}$$

and

$$P_e(t) = M(T_l - T_e) \tag{7}$$

where,

$P_c \equiv$ power transferred from fuel to coolant (MW),

$P_e \equiv$ power removed from the coolant (MW),

$\Omega \equiv$ heat transfer coefficient between fuel and coolant $(MW/^{\circ}C)$

$M \equiv$ mass flow rate times heat capacity of the water $(MW/^{\circ}C)$

$T_f \equiv$ average reactor temperature $(^{\circ}C)$

$T_l \equiv$ temperature of the water leaving the reactor $(^{\circ}C)$

$T_e \equiv$ temperature of the water entering the reactor $(^{\circ}C)$

$T_c \equiv$ average reactor coolant(water) temperature $(T_l + T_e)/2$.

The differential equations for the lumped fuel and coolant temperature are as follows:

$$f_f P_a(t) = \mu_f \frac{dT_f}{dt} + P_c(t) \tag{8}$$

and

$$(1 - f_f)P_a(t) + P_c(t) = \mu_c(t)\frac{dT_l}{dt} + P_c(t) \tag{9}$$

where

$f_f \equiv$ reactor power fraction deposited in the fuel,

$\mu_f \equiv$ total heat capacity of the fuel and structural material;

$W_f C_f \equiv$ weight of fuel times specific heat $(MW.s/^{\circ}C)$,

$\mu_c \equiv$ total heat capacity of the reactor coolant; $W_c C_c \equiv$ weight of the coolant times specific heat of the coolant $(MW.s/^{\circ}C)$.

Reactivity input and feedback to the point-kinetics equations are represented by

$$\frac{d\delta\rho_r}{dt} = G_r z_r \tag{10}$$

and

$$\delta\rho = \delta\rho_r + \alpha_f (T_f - T_{f0}) + \alpha_c (T_c - T_{c0}) \tag{11}$$

where,

$\delta\rho_r \equiv$ reactivity due to the control rod,

$z_r \equiv$ control input, control rod speed (fraction of core length per second),

$G_r \equiv$ reactivity worth of the rod per unit length(with rod speed in units of fraction of core length per second, $G_r$ is the total reactivity of the rod),

$\alpha_f \equiv$ fuel temperature reactivity coefficient,

$\alpha_c \equiv$ coolant temperature reactivity coefficient,

$T_{f0} \equiv$ initial equilibrium(steady-state) fuel temperature,

$T_{c0} \equiv$ initial equilibrium(steady-state) coolant temperature.

Linearization of equations (3) through (11) about nominal working point $n_r$ results in the following state-space representation of the reactor model (Edwards et al,1992)[4].

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{y} &= \mathbf{Cx} + \mathbf{Du} \end{aligned} \tag{12}$$

where,

$$\mathbf{A} = \begin{bmatrix} -\dfrac{\beta}{\lambda} & \dfrac{\beta}{\lambda} & \dfrac{n_r\alpha_f}{\Lambda} & \dfrac{n_r\alpha_c}{2\Lambda} & \dfrac{n_r}{\Lambda} \\ \lambda & -\lambda & 0 & 0 & 0 \\ \dfrac{f_f P_{0a}}{\mu_f} & 0 & \dfrac{-\Omega}{\mu_f} & \dfrac{\Omega}{2\mu_f} & 0 \\ \dfrac{(1-f_f)P_{0a}}{\mu_c} & 0 & \dfrac{\Omega}{\mu_c} & \dfrac{-(2M+\Omega)}{2\mu_c} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\mathbf{B} = [0 \quad 0 \quad 0 \quad 0 \quad G_r]^T$ ; $\mathbf{C} = [1 \quad 0 \quad 0 \quad 0 \quad 0]$ ; $\mathbf{D} = [0]$

$\mathbf{x} = [\delta n_r \quad \delta C_r \quad \delta T_f \quad \delta T_l \quad \delta \rho_r]$ ; $\mathbf{y} = [\delta n_r]$ ; $\mathbf{u} = [z_r]$

The symbol $\delta$ indicates the deviation of a variable from an equilibrium value; e.g., $\delta n_r(t) = n_r(t) - n_{r0}$ with $n_{r0} \equiv$ the nominal value of $n_r$ at the equilibrium condition.

The values of the constants used for controller design and simulations in this paper are summarized in Table I. These parameters are representative of a Three Mile Island-Type reactor at the middle of the fuel cycle.

TABLE I

Parameters for ROSTR Design at the Middle of the
Fuel Cycle of a TMI-Type PWR

| | | |
|---|---|---|
| $\beta = 0.006019$ | $G_r = 0.0145 \quad \Delta k / k$ | $T_{f0} = 700 \quad {}^\circ C$ |
| $\Lambda = 0.00002 \quad \sec$ | $\lambda = 0.150 \quad s^{-1}$ | $T_{c0} = 305 \quad {}^\circ C$ |
| $P_{0a} = 2500 \quad MW$ | $f_f = 0.92$ | |
| $\mu_f = 26.3 \quad MW.s/{}^\circ C$ | $T_e = 290 \quad {}^\circ C$ | |

Also $\mu_c, \Omega, M, \alpha_f$ and $\alpha_c$ are not constant but rather a function of the power level $n_r$ as follows:

$$\mu_c(n_r) = \left(\frac{160}{9} n_r + 54.022\right) \quad MW.s/{}^\circ C \tag{13a}$$

$$\Omega(n_r) = \left(\frac{5}{3} n_r + 4.933\right) \quad MW/{}^\circ C \tag{13b}$$

$$M(n_r) = (28.0 n_r + 4.9333) \quad MW/{}^\circ C \tag{13c}$$

$$\alpha_f(n_r) = (n_r - 4.24) \times 10^{-5} \quad \frac{\delta k}{k} / {}^\circ C \tag{13d}$$

$$\alpha_c(n_r) = (-4.0 n_r - 17.3) \times 10^{-5} \quad \frac{\delta k}{k} / {}^\circ C \tag{13e}$$

Moreover parameters of matrix **A** [Eq (12)], the linearized state space representation of reactor equation about nominal power level $n_r$, depends on power level $n_r$.

## 3 -Robust Optimal Self-Tuning Regulator

Self-Tuning Regulator (STR) is one of the methods for controlling plants with time varying parameter [5]. The block diagram of a STR is shown in Fig. 1. We call this controller a self-tuning regulator because it can tune it's own parameters. As can be seen in Fig. 1 this controller consists of the following two loops. The main loop includes the process and a linear feedback controller. The second loop, the auxiliary loop, has the task of adjusting the parameters of the main loop's controller. The auxiliary loop consists of a recursive parameter estimator and a control design scheme. The design scheme box in Fig. 1 solves the design problem for the system whose parameters have been estimated by the estimator in real time.

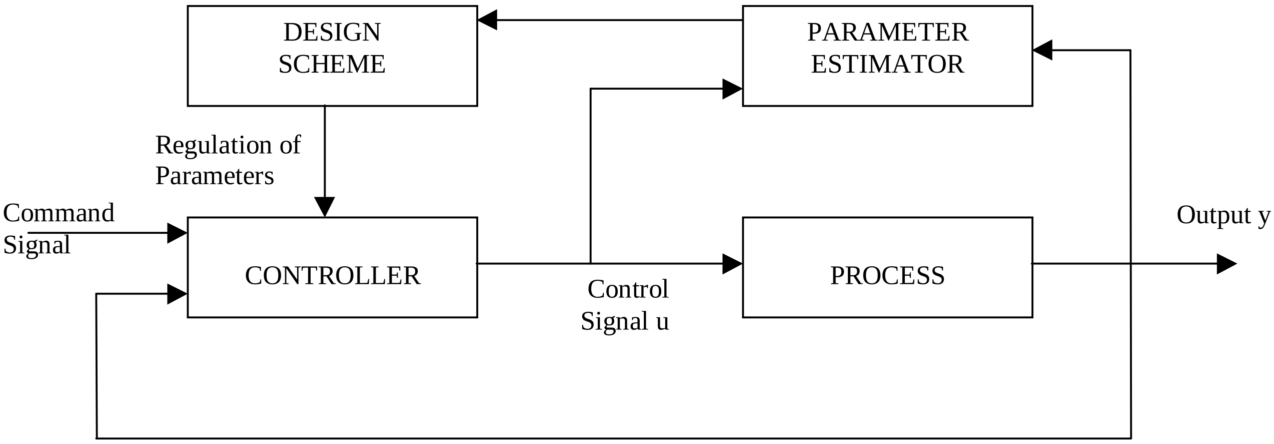



Fig. 1. Block diagram of a self-tuning regulator (STR)

Edwards et al. [1] have shown the improved performance and stability robustness of the State Feedback Assisted Classical Control (SFAC) in controlling primary coolant temperature in a nuclear reactor over that of conventional state feedback control (CSFC). Figure 2 shows the SFAC configuration.

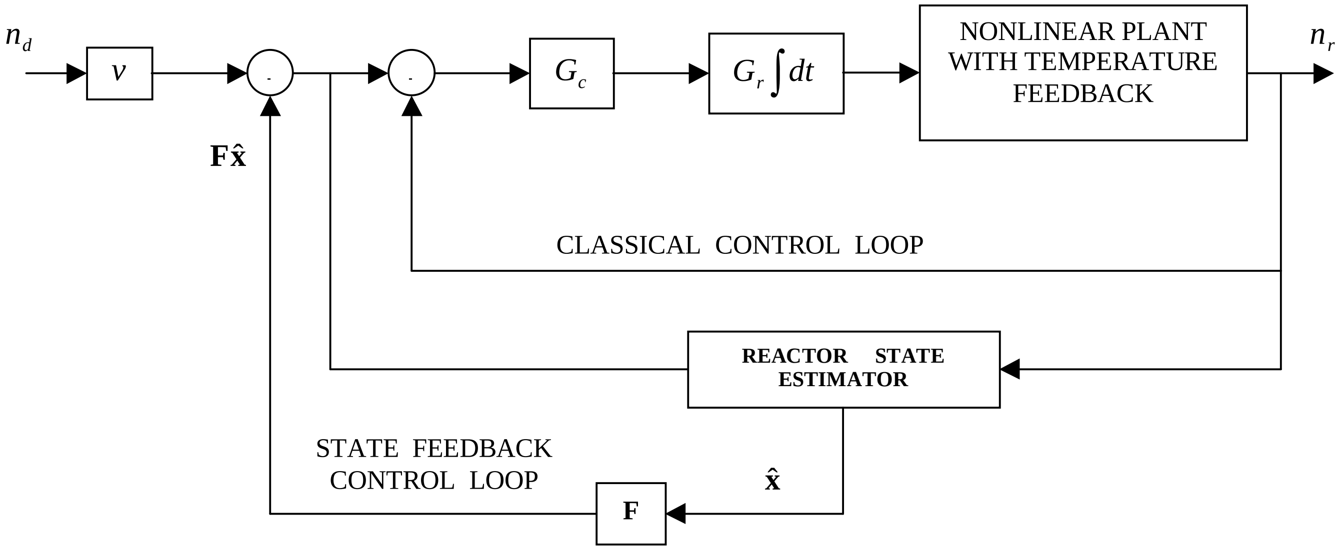



Fig. 2. State Feedback Assisted Classical Control configuration.

Khajavi et al. [3] used a self-tuning regulator based on optimal control theory in a SFAC configuration to obtain Robust Optimal Self-Tuning Regulator (ROSTR) to control reactor power in wide range of operations. Figure 3 shows the ROSTR configuration. . The ROSTR shows good performance over wide range of power level variations. Therefore, we use ROSTR response as a reference trajectory to tune our fuzzy logic controller.

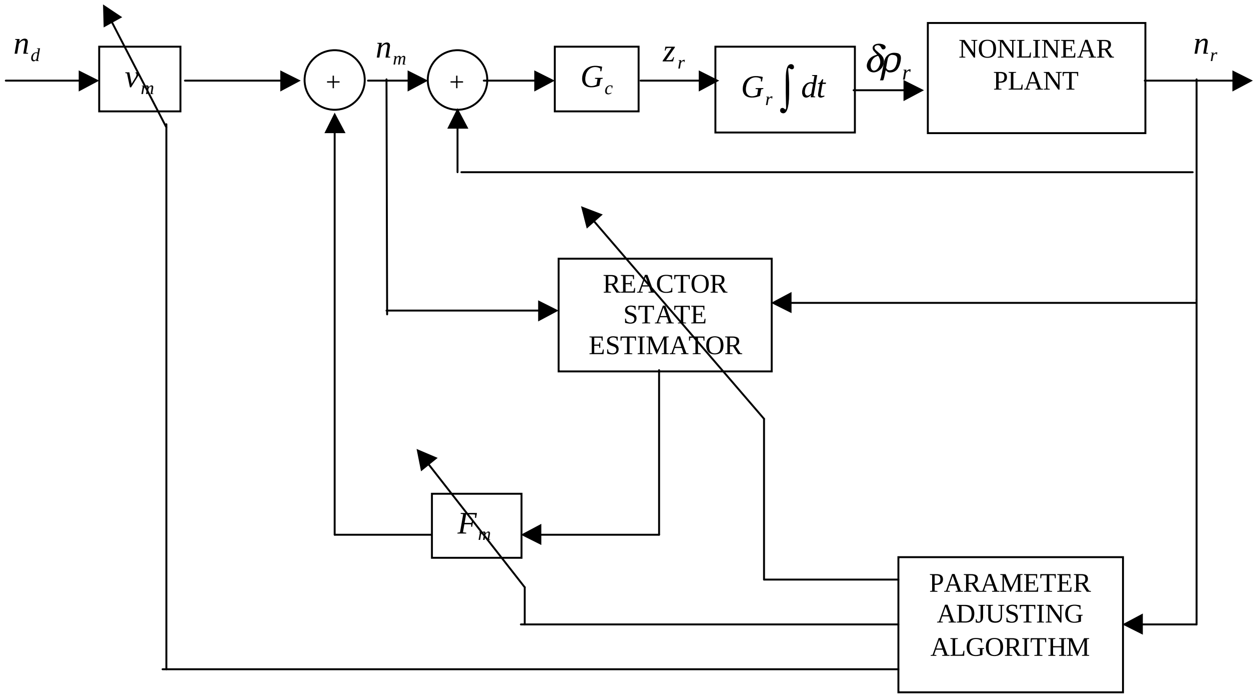



Fig. 3. Robust Optimal Self-Tuning Regulator configuration.

## 4-FUZZY LOGIC CONTROLLER (FLC)

As an alternative to model based controllers, fuzzy logic controllers neither relies on accurate description of plants, nor on the precise measurements. An introduction to the fundamental concepts of fuzzy logic has been given by Zadeh [6]. Figure 4 shows the structure of fuzzy inference system.

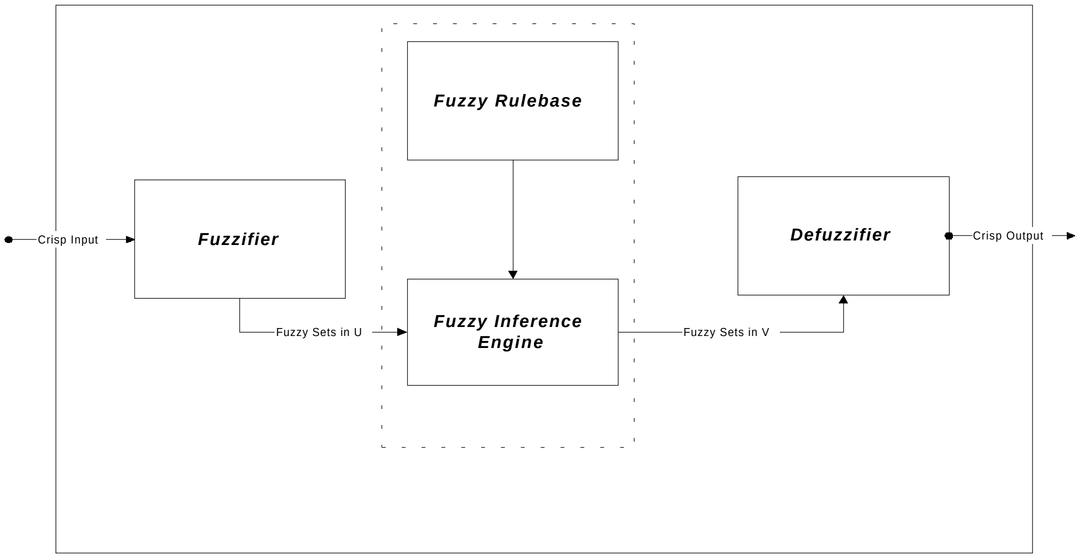



Fig. 4. Structure of fuzzy inference system

Information used for fuzzy logic controller will be placed in two groups:

a) quantitative information from measurements,
b) qualitative information from expert operators.

In neural network analysis only quantitative information will be used, but in fuzzy logic qualitative data can be used as well. Here we use a method for constructing rule base based on available quantitative and qualitative data. The controller used in this paper has 9 inputs and 11 outputs. For implementing this controller, we use 11 parallel controllers each with 9 inputs and 1 output. Figure 5 shows the structure of the fuzzy controller used in this paper. For simplicity, we describe the design method for a controller with 2 inputs and 1 output, extension to 9 inputs and 1 output situation is straightforward.

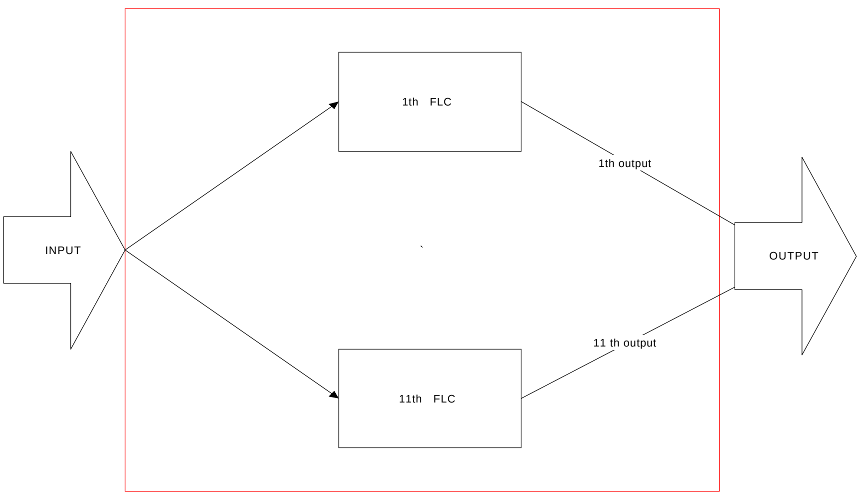



Fig. 5. 11 Fuzzy Logic Controllers in parallel

### 4-1- FUZZY CONTROLLER DESIGN STEPS

In this section the steps of fuzzy controller design is listed. The interested reader may refer to [7]. Here we consider a series of data consisting of two inputs and one output as input-output of the desired controller, $(x_1^{(1)}, x_2^{(1)}, y^{(1)}), \ldots, (x_1^{(i)}, x_2^{(i)}, y^{(i)})$, with $x_1^{(i)}, x_2^{(i)}$ as inputs and $y^{(i)}$ as output.The method can be easily extended for multi-input multi output systems. Our objective is to map the inputs to the desired output(s).

The design procedure is summarized below:

Step 1: Select the maximum fuzzy partitions and weighting coefficients (R, Q) in the cost function as below:

J = (Euclidean norm of error )*R + (Number of partitions)*Q

R, Q measure the amount of fuzziness of controller. These parameters will be selected by an expert familiar with the system. By increasing the error weighting coefficient R, response accuracy of the fuzzy controller will be increased. But this makes the number of linguistic terms grows up, which inturn increases the computation time. On the other hand by increasing the number of the fuzzy partitions the method becomes more crisp and noise rejection ability of the controller will decrease. To limit the amount of unnecessary rule generation, another term with coefficient Q has been added to the cost function. The next step is to construct membership functions. The interval of variables $x_1, x_2, y$ are considered as $[x_1^-, x_1^+]$ , $[x_2^-, x_2^+]$ and $[y^-, y^+]$ in which the operating data exist more probably.

Step 2: Initialize fuzzy partitions. We considered five primary fuzzy partitions with equal length. However in general each partition can have it's particular length.

Step 3: Consider Gausian membership functions with the centers coincident with fuzzy partitions midpoints, and with spread equal to distance between neighborhood centers.

$$\mu(x) = \exp\left(-\frac{(x - \mathrm{m})^2}{\sigma}\right)$$

$\mu$ =Membership function

$\sigma$ =Spread of membership functions

m =Center of membership functions

Different input/output intervals are labeled as follows:

$\mathrm{S_N}, \ldots, \mathrm{S_1}, \mathrm{CE}, \mathrm{L_1}, \ldots, \mathrm{L_N}$

Fig. 6. shows the schematic of membership functions.

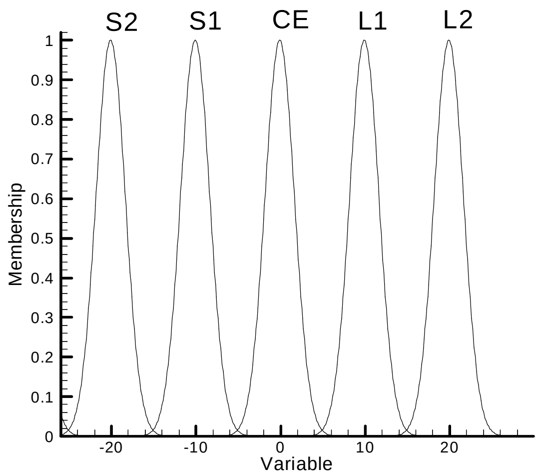



Fig. 6. Schematic of membership functions

Step 4: Membership degree of each pair of given data $(x_1^{(1)}, x_2^{(1)}, y^{(1)})$ in different partitions are specified and then $x_1, x_2, y$ are assigned to the partitions with maximum membership degree. At last one rule from the input-output pair will be produced. For example for the given triple $(x_1^{(1)}, x_2^{(1)}, y^{(1)})$, we have:

If Maximum membership of $x_1^{(1)}$ is in $\mathrm{L}_1$ and

If Maximum membership of $x_2^{(1)}$ is in $\mathrm{S}_1$ and

If Maximum membership of $y$ is in CE

Then we conclude following rule:

IF $x_1^{(1)}$ is $\mathrm{L}_1$ and $x_2^{(1)}$ is $\mathrm{S}_1$ THEN y is CE.

Step 5: Assign a degree of credit to each rule. Since there are many data pairs and each pair makes a rule, there may be overlapping rules (rules with the same if-part and different then-part). To overcome this problem a degree of credit is assigned to every rule, which is equal to the product of membership functions of that rule. Among the overlapping rules, those with greater degree of credit are chosen and will be placed in the rule base.

Step 6: Construct a compound rule base.The rule base is constructed based on rules resulted from numeric data and rules stated by expert operator. Each rule expressed by expert operator will be stated in a conditional form, and a degree of credit is assigned to it. Then this rule will be added to the rule base.

Step 7: Evaluate the I/O mapping. For defuzzification COA is used and we proceed as follows to find a crisp output. First the if parts of the ith rule will be combined to find the degree of firing of the ith rule based on inputs $x_1, x_2$. We use the product method for combination:

$$\mathrm{m}_{\mathrm{O_i}} = \mathrm{m}_{\mathrm{T_{i1}}}(x).\,\mathrm{m}_{\mathrm{T_{i2}}}(x_2)$$

where $O_i$ and $T_{ij}$ denote respectively output and input region.
Then center of area (COA) is applied:

$$y = \frac{\sum_{i=1}^{k}\left(m^{i}_{O_i} * y\right)}{\sum_{i=1}^{k} m^{i}_{O_i}}$$

In the above, $y^i$ indicates center of region $O_i$ and k is the number of rules.
For the purpose of learning, output of fuzzy controller and its error will be computed and the cost function will be determined. The learning algorithm terminates if the cost function increases and the number of fuzzy partitions obtained so far is considered as the optimum number of partitions. Otherwise we will increase the number of fuzzy partitions by one and return to step 3.

## 5-SIMULATION RESULTS

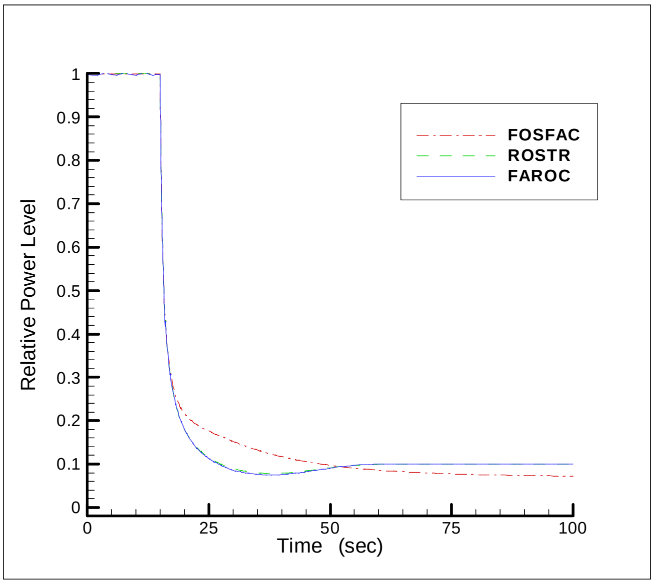
The nuclear reactor system, as well as the three different controllers FOSFAC, ROSTR, and FAROC have been simulated by MATLAB/SIMULINK(ver 5.2)[8]. Simulation results for the most stressed power level change (100% to10%) as well as start-up/shut-down operation $(100\% \rightarrow 10\% \rightarrow 100\%)$ have been shown in figures 7-14. As observed from Fig.7 the response of FOSFAC shows steady-error in the case of a) 100% to 10% power level change. Also from Fig. 12 it is apparent that FOSFAC can not track the desired power level trajectory in the case b) 100% to 10% to 100% (start-up/shut-down operation of reactor). The proposed methods ROSTR and FAROC both perform well. In order to compare better these two methods, Table 2 shows the number of floating point operations (FLOPS) for ROSTR and FAROC methods for (100% to 10%) case. It is apparent from this table that FAROC is more than 3 times faster than ROSTR, and therefore one can consider the proposed FAROC as the superior method.

Table 2: complexity with respect to number of floating point per operations for case a) 100% to 10% power level change.

| METHOD | FLOPS |
|---|---|
| FAROC | 3.442e+6 |
| ROSTR | 10.806e+6 |

## 6-CONCLUSIONS

A Fuzzy Adaptive Robust Optimal Controller (FAROC) based on the response of a Robust Optimal Self-Tuning Regulator (ROSTR) has been designed and simulated. The simulation results shows good performance of this controller for wide range power regulation and good load follow capability. FAROC is also three times faster than ROSTR.



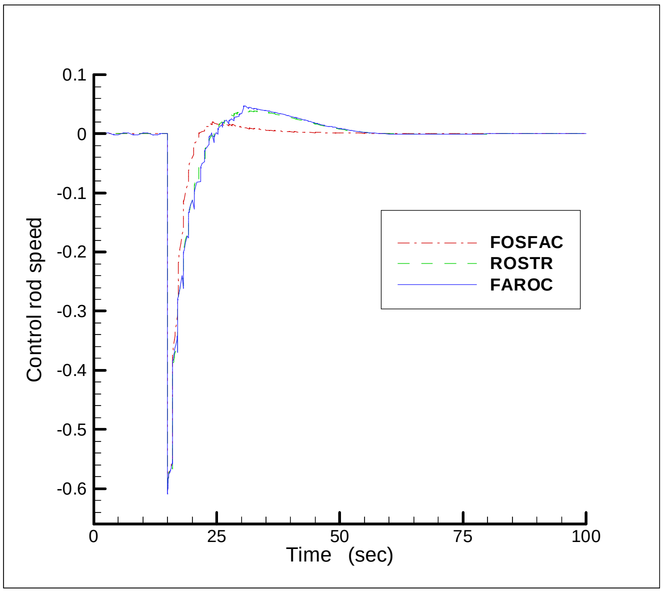
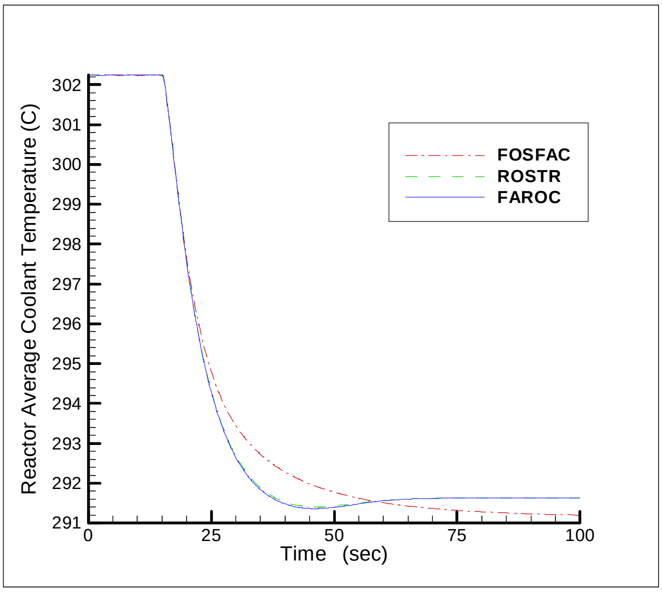




Fig. 7. Very large step change of power level from 100% to 10%.

Fig. 8. Control rod speed for 100% to 10% power level change.



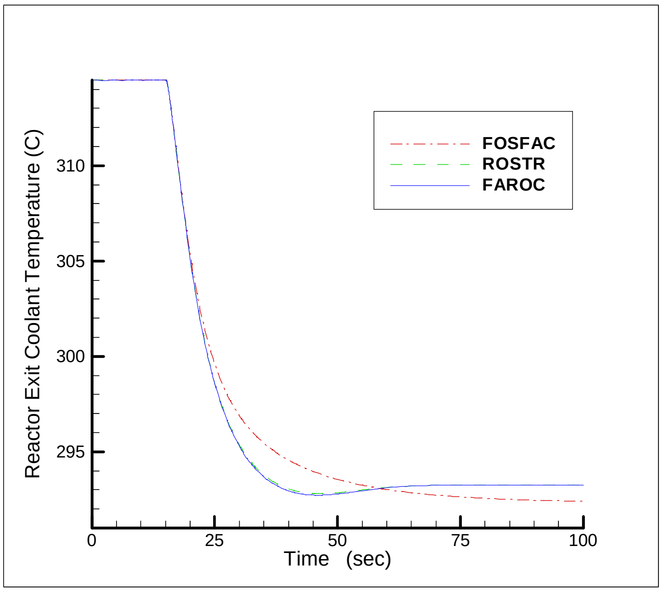




Fig. 9. Reactor average coolant temperature for 100% to 10% power level change

Fig. 10. Reactor exit coolant temperature for 100% to 10% power level change

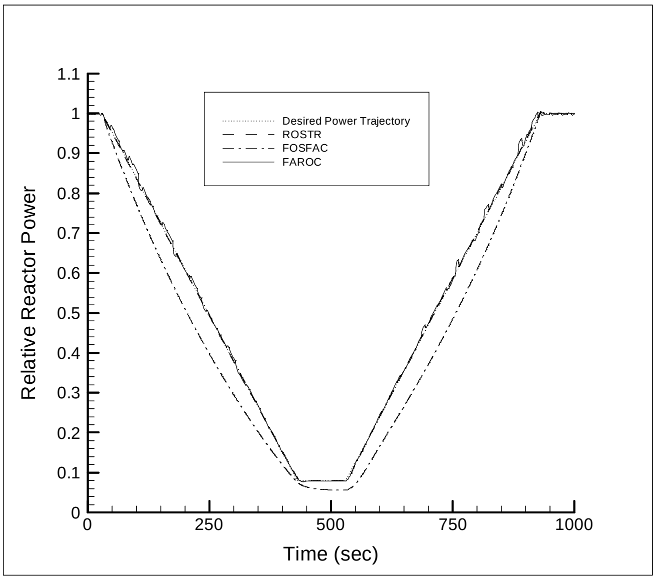



Fig. 11. Relative reactor power for start-up/shut-down operation $100\% \rightarrow 10\% \rightarrow 100\%$.

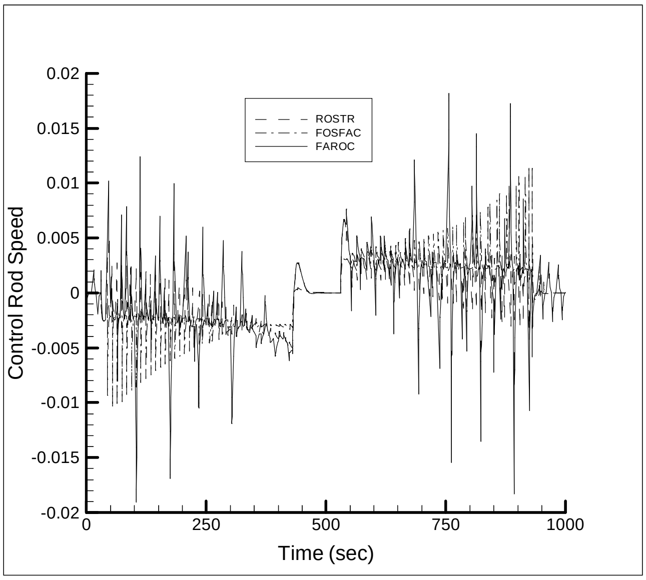



Fig. 12. Control rod speed for start-up/shut-down operation $100\% \rightarrow 10\% \rightarrow 100\%$.

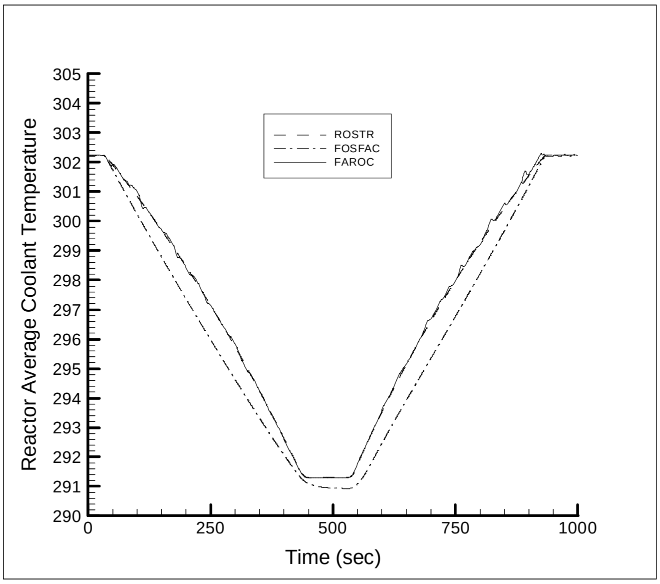



Fig. 13. Reactor average coolant temperature for start-up/shut-down operation$100\% \rightarrow 10\% \rightarrow 100\%$.

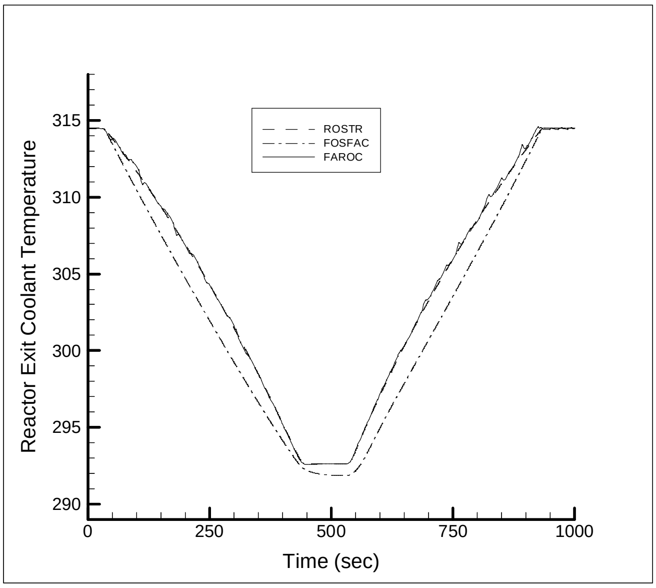



Fig. 14. Reactor exit coolant temperature for start-up/shut-down operation$100\% \rightarrow 10\% \rightarrow 100\%$.

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