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# Efficient Compliance with Prescribed Bounds on Operational Parameters by Means of Hypothesis Testing using Reactor Data

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**Abstract** - A common problem in any reactor operations is to comply with a requirement that certain operational parameters are constrained to lie within some prescribed bounds. The fundamental issue which is to be addressed in any compliance description can be stated as follows: *The compliance definition, compliance procedures and allowances for uncertainties in data and accompanying methodologies, should be well defined and justifiable*. To this end, a mathematical framework for compliance, in which the computed or measured estimates of process parameters are considered random variables, is described in this paper. This allows a statistical formulation of the definition of compliance with licence or otherwise imposed limits. An important aspect of the proposed methodology is that the derived statistical tests are obtained by a Monte Carlo procedure using actual reactor operational data.

The implementation of the methodology requires a routine surveillance of the reactor core in order to perform the underlying statistical tests. The additional work required for surveillance is balanced by the fact that the resulting actions on the reactor operations, implemented in station procedures, make the reactor "safer" by increasing the operating margins. Furthermore, increased margins are also achieved by efficient solution techniques which may allow an increase in reactor power.

A rigorous analysis of a compliance problem using statistical hypothesis testing based on extreme value probability distributions and actual reactor operational data leads to effective solutions in the areas of licensing, nuclear safety, reliability and competitiveness of operating nuclear reactors.

# I. INTRODUCTION

The fundamental issue which is to be addressed in any compliance description can be stated as follows: *The compliance definition, compliance procedures and allowances for uncertainties in data and accompanying methodologies, should be well defined and justifiable*. To this end, a mathematical framework for compliance, in which the computed or measured estimates of process parameters are considered random variables, is described in this paper. This allows a statistical formulation of the definition of compliance with licence or otherwise imposed limits. Of particular interest is the area of compliance with limits on reactor channel powers,[1] and other conceptually similar physics parameters, for which the statistical treatment based on extreme value statistics has been shown to be effective.[3]

Generally, inference rules are divided into two categories: Estimation (Confidence Intervals) and Significance (Hypothesis) Testing. While there are affinities between the two categories, in some situations they lead to different solutions. It is shown that in the context of channel power compliance, and other conceptually similar situations, the testing approach is the more appropriate one. Here, probabilities are evaluated for the event that the *true* channel powers do not exceed the licence limits. The compliance criterion is presented using the Type I error probability and naturally gives rise to the concept of *compliance uncertainty.*

The traditional industry approach is based on confidence intervals using the error in the individual channel powers. This approach is shown[2] to be equivalent to a test procedure using a channel power distribution in which exactly one channel is at the licence limit while all other channels have powers zero. Such a channel power distribution is obviously absurd and the related Type I error probability is much smaller than prescribed (such as 2%), for reasonable channel power distributions. Moreover, a complementary result is that the probability of a Type II error (false alarm), which is an incorrect declaration of the violation of the channel power licence limit, is exaggerated.[2]

A significantly more effective approach based on hypothesis testing uses an error in the *maximum* channel power to derive *compliance uncertainties*. A maximum channel power reflects the structure of the power distribution much better than an individual channel power does unless, of course, the channel power distribution is “very peaked”. In an actual reactor the fueling strategy ensures non-peaked power distributions under normal operating conditions. Therefore, the above assertion about the maximum channel power is reasonable for operating reactors under normal conditions and the Type I error probability is closer to the prescribed value (such as 2%) for compliance. It is shown[1] that the corresponding *compliance uncertainty* is much smaller than the one based on the current standard industry practice. (Note that OPG has adopted the proposed methodology.) For reactors with "tight" operating margins, the smaller *compliance uncertainty* leads to substantial economic benefits because a larger *compliance uncertainty* may cause de-rating of the actual reactor power.[1]

Rigorous analysis of the *compliance uncertainty* as well as the error in the *maximum* channel power has been obtained.[1] An analytical solution to the problem for an arbitrary channel power distribution is prohibitive considering the number of channels involved (e.g., 480 at Darlington or Bruce and 380 at Pickering). The problem, *i.e.*, finding the *compliance uncertainty*, is solved numerically by a Monte Carlo procedure (a bootstrap like approach) in which the errors for all reactor channel powers and the reactor power are sampled from known distributions. (These distributions are obtained by comparing the computed values to the existing measurements and/or other available uncertainty analysis means.) The Monte Carlo procedure is used to estimate the error in the maximum channel power and from this an allowance for the *compliance uncertainty* is obtained. The OPG fuel management code SORO[4] is used to obtain the computed channel powers. A fundamental assumption for the Monte Carlo procedure is that these SORO computed powers serve as surrogates for the *true* channel powers. Validity of this assumption is supported by using a subset of channels - the so-called FINCHs (44 at Darlington and 22 at other OPG stations) which are equipped with instrumentation to measure the channel powers directly.[2]

The proposed compliance with channel power limits can readily be adopted in similar situations involving computational or measurement uncertainty. For example, compliance with channel power specific limits can be solved by the above described methodology by considering ratios of actual channel powers to the corresponding limits. In such a situation there is only one "licence limit" having a value 1 (see Section II). Another successful application is the "Enhanced Gap Monitoring" at Bruce plant, which involves statistical testing for a positive minimum margin to fuel constraint under a postulated LOCA. In a traditional deterministic approach, analysis shows that fuel constraint under a postulated LOCA may occur if larger Void Reactivity Error Allowance is assumed. A statistical analysis of actual operational data using measured fuel channels gaps and computed channel powers to determine the expansion of the fuel string under postulated LOCA and a larger Void Reactivity Error Allowance, shows that a fuel constraint does not occur.[3] The analysis was performed using the above methodology with a limiting value of zero and a *compliance uncertainty* based on an error in the computed minimum margin to constraint. This approach has permitted current operation at 90% for the Bruce "B" reactors under larger Void Reactivity Error Allowance.

## II. COMPLIANCE PROBLEM DEFINITION

We will describe a general problem which can be solved within a framework of a managed system in reactor operations. Such a problem is not necessarily in the area of licensing. Nevertheless, we will use the term *compliance* to refer to the fact that a part of the motivation is conformity with a given condition.

The terminology used throughout this paper is summarized in the Section Nomenclature at the end of this paper.

Let $\boldsymbol{Q} = \{Q_1, Q_2, Q_3, \ldots, Q_n\}$ be a set of $n$ parameters. For illustration purposes, throughout this paper, these will often represent channel powers (e.g, $n = 480$ channel powers). But note that the methodology presented applies more generally, e.g., these parameters may represent the margins to fuel constraint in the enhanced gap management problem.[3] In general, values of the parameters $\boldsymbol{Q}$ are not known to us and will be usually referred to as *true* parameters.

Let $C$ be a condition that is imposed on the parameters. Given a number (a limit), say $L$, we will write

$$C = C(\boldsymbol{Q},\ L).$$

We stress that we are considering only one condition imposed on the parameters. Thus, when the $\{Q_1, Q_2, Q_3, \ldots, Q_n\}$ are reactor channel powers and $L$ a license limit, such as 7200 kW, the (licence) condition, $C$, would be

$$max\{Q_1, Q_2, Q_3, \ldots, Q_n\} \ \le\ L.$$

Note that the problem of channel specific limits consists of many conditions, namely,

$$Q_i \ \le\ L_i, \qquad i = 1, 2, 3, \ldots, n.$$

However, it is easy to convert this into a “one condition” problem:

$$max\left\{\frac{Q_1}{L_1}, \frac{Q_2}{L_2}, \frac{Q_3}{L_3}, \ldots, \frac{Q_n}{L_n}\right\} \ \le\ 1.$$

The issue is to ascertain if $C$ is true or false. If $C$ is true we say that we are *complying* with the condition, otherwise we are in *violation* of the condition.

In addition to the problem of the channel power licence compliance, there are many other problems that can be cast in a way described above. For example, a condition imposed on safety analysis stipulates that there be no fuel constraint under a postulated LOCA. The expansion for every fuel string in a reactor is computed from a channel power. The *margin to constraint* is then obtained by subtracting the fuel string expansion from the available gap in the same channel. The condition is that the *minimum margin to fuel constraint* is positive.[3] Another example is a bundle heat-up problem during a reactor outage and a postulated loss of heat sink. For a given cooling time (which determines the bundle decay power), the condition is that a number of fuel failures will not exceed a given limit. Such a limit is obtained from cost-benefit considerations.

Because the parameters $\boldsymbol{Q}$ are not usually known, we must have a computational model or measurements, or both available to estimate $\boldsymbol{Q}$. Let such estimated parameters be denoted by

$$\hat{\boldsymbol{Q}} = \{\hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \ldots, \hat{Q}_n\}.$$

Note that in case of channel powers, the OPG fuel management code SORO[4] based on solving 3-dimensional diffusion equations together with a measured value of a reactor power provide such channel power estimates. In the case of the fuel constraint problem, the margins to fuel constraint are obtained using SORO estimated channel powers, estimated fuel expansion as described in Ref. 5, and measured channel gaps every time a channel is re-fuelled. The bundle heat-up problem would also require decay power calculations in addition to the SORO computed bundle powers.

A crucial assumption in our methodology is that the estimated parameters $\hat{\boldsymbol{Q}}$ are random variables related to the *true* values $\boldsymbol{Q}$ by a statistical model

$$\hat{\boldsymbol{Q}} = \boldsymbol{Q} + \varepsilon,$$

where $\varepsilon$ is the computational (or measurement) error. This error is known and may be obtained from available measurements. Using a suitably chosen number $\eta_\alpha$, referred to as *compliance uncertainty*, depending on a small positive number $\alpha$, we define a new condition $\hat{C}$ :

$$\hat{C} = \hat{C}(\hat{\boldsymbol{Q}}, L, \eta_\alpha).$$

$\hat{C}$ and $C$ are identical if $\hat{\boldsymbol{Q}} = \boldsymbol{Q}$ and $\eta_\alpha = 0$. The way $\eta_\alpha$ and hence $\hat{C}$ are determined is by posing the given problem as hypothesis testing (for details see Ref. 1 and 2). Condition $\hat{C}$ is the test statistic for hypothesis $C$. For a given small positive number $\alpha$, referred to as a *significance level,* $\eta_\alpha$ is calculated in such a way that the *Type I* error[6] satisfies

$$\boldsymbol{Probability}\{\hat{C} \textit{ holds under the assumption that } C \textit{ is false}\} < \alpha.$$

We note that the *compliance uncertainty* $\eta_\alpha$ is defined in terms of $\varepsilon$ and $\boldsymbol{Q}$.

A traditional "industry" approach is to use confidence intervals of $\varepsilon$ to define the *compliance uncertainty* $\eta_\alpha$ rather than the hypothesis testing as described above. Let $u_\alpha$ be an upper 100(1-$\alpha$) confidence interval for $\varepsilon$. That is,

$$\boldsymbol{Probability}\{\varepsilon \le u_\alpha\} < 1-\alpha .$$

In this case compliance is declared as long as $C(\hat{Q}_i + u_\alpha,\ L)$ is true for any $i$. For a given $\alpha$ this is a much less "efficient" procedure than the above described hypothesis testing used in the proposed methodology.[1] Less efficient means that the corresponding Type I error is much smaller than the Type I error associated with the hypothesis testing described above. The implication of this is that

$$\eta_\alpha \le u_\alpha .$$

For reactors with tight operating margins the difference in the magnitudes of the respective *compliance uncertainties* could mean de-rating of the reactor power. (At Bruce or Darlington Nuclear stations the *compliance uncertainty* $\eta_\alpha$ = 3% and $u_\alpha$ = 5%. At $\eta_\alpha$ = 5% the reactors would have to be de-rated by 1% or more.) To understand the less efficient nature of the traditional approach we point out the fact that it is designed for $\boldsymbol{Q} = \{0, 0, 0, \ldots, 0, Q_i, 0, \ldots, 0\}$ while the approach based on the hypothesis testing takes into consideration the full structure of $\boldsymbol{Q} = \{Q_1, Q_2, Q_3, \ldots, Q_n\}$. (For more details see Ref. 1 and 2.)

## III. MANAGED SYSTEM FOR COMPLIANCE

The description of our problem in the previous Section provides only a mathematical formalism (for details see Ref. 1, 2 and 3). The full implementation of the compliance strategy is more involved. The framework for the implementation, or the corresponding managed system embedded in the reactor operation is depicted by a chart in Fig. 1. Such a framework with all its components is needed in order to satisfy the fundamental requirement of any compliance strategy. Namely:

> *The compliance definition, compliance procedures and allowances for uncertainties in data and accompanying methodologies, should be well defined and justifiable.*

We assert that our methodology not only satisfies the above requirement, it does so efficiently as well.

The most important reason that the methodology is efficient is that it is non-deterministic (i.e., probabilistic) and based on *a posteriori* analysis using actual reactor operational data. An *a priori* analysis, by necessity, uses design data and assumptions which either may never occur in an operating reactor or at least may never occur under normal operating conditions. The *a posteriori* method requires a routine surveillance of the reactor core in order to perform the underlying statistical test as well as to look for reactor changes that may necessitate re-evaluating the test itself. But this is no different from any other "quality control" procedure based on a sampling technique.

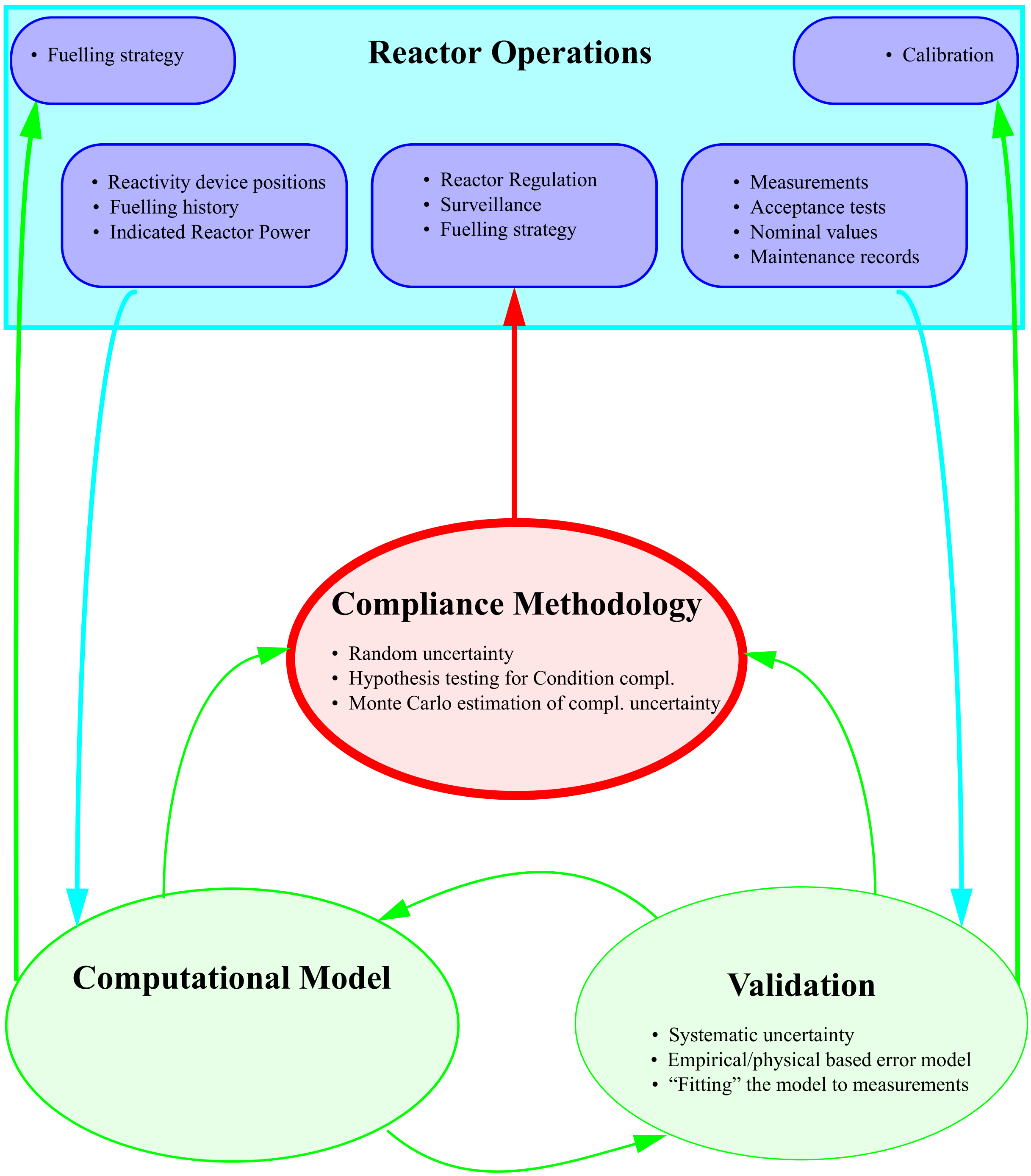


Fig. 1. Interrelationships in the Managed System for Compliance.

The additional work required for surveillance is balanced by the fact that the resulting actions on the reactor operations, implemented in station procedures, allow the reactor to be operated at higher power, and at the same time make the reactor “safer” by increasing the operating margins. A good example is provided by an actual increase in operating margins at OPG Bruce B reactors which occurred as a result of new surveillance procedures[9, 8] following updates to the Compliance Methodology[9, 1], the implementation of an improved SORO model[7] and the implementation of the *enhanced gap management*[3] incorporating larger Void Reactivity Error Allowance.

Another reason for the effectiveness of the proposed methodology is the usage of an extreme value probability distribution in calculating the *compliance uncertainties* $\eta_\alpha$. An extreme value probability distribution naturally arises as a consequence of the “one-condition” property of the given problem with many parameters together with the hypothesis testing. Such a probability distribution is inherently biased with respect to the “parent” distribution (associated with the error $\varepsilon$ in the previous Section) - see Fig. 2. In hypothesis testing this bias is irrelevant since it is embedded in the distribution of the associated test statistic necessarily producing smaller and more accurate *compliance uncertainties*.

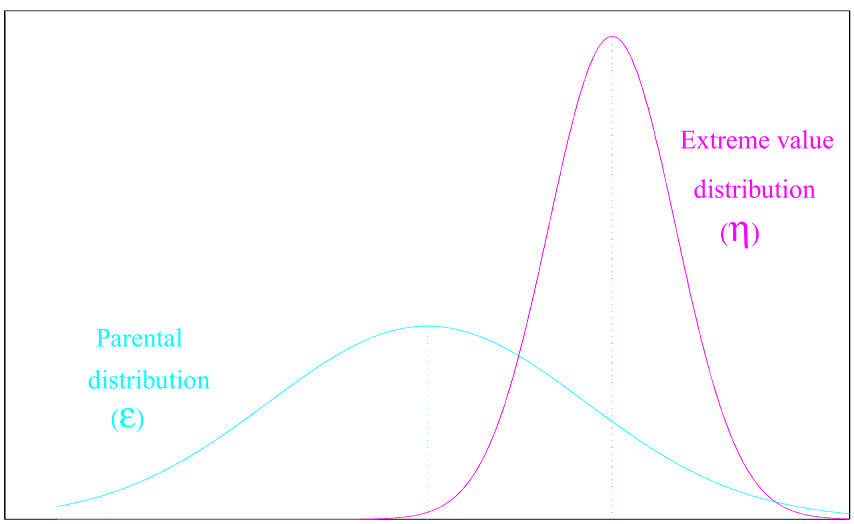


Fig. 2. Parental and the derived extreme value probability distribution.

The most important components of the overall strategy for compliance (see Fig. 1) are:

- Computational Model;
- Reactor Operations;
- Compliance Methodology - formal description;
- Validation;

The rest of the paper contains brief discussion of each of the components.

### III.A. *Computational Model*

For the *true* parameters $\boldsymbol{Q} = \{Q_1, Q_2, Q_3, \ldots, Q_n\}$, we need to provide their estimates $\hat{\boldsymbol{Q}} = \{\hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \ldots, \hat{Q}_n\}$ as described in Section II. Therefore, a computational model which provides such estimates is an integral part of our managed system for compliance. Sometimes, such estimates, either in whole or in part, are obtained by measurements. This is the case in computing margins to fuel constraint, where the available channel gap is a measured estimate.[3] For the sake of simplicity of presentation a "Computational Model" will also include measurements. In order to satisfy the fundamental premise of *well defined and justified uncertainties*, the computational model must be validated (hence the corresponding links in the chart of Fig. 1). Depending on the validation results, the computational tool may be updated or modified (as was the case with the Bruce B SORO model[7]) instead of adjusting the corresponding uncertainty component in $\varepsilon$.

The role of the computational model may sometime be much more involved than just providing $\hat{\boldsymbol{Q}}$ to help estimate $\varepsilon$ in the validation process. For example, a fuel management code such as SORO[4] is an active component of reactor operations. It provides, in particular, bundle and channel powers which are used by fuelling engineers to select channels for re-fuelling and hence affects the actual core configuration. In order to provide an accurate snapshot of a reactor core, SORO, in turn, needs information on the instantaneous reactivity device positions, indicated reactor power as well as recent fuelling history (Fig. 1). Also, SORO provides a number of parameters (such as bundle and channel powers, fuel constraint under postulated LOCA, etc.) which are used in station procedures as test statistics for the various compliance requirements.

The computational model also provides $\hat{\boldsymbol{Q}}$ to the Compliance Methodology (see the chart in Fig. 1) as the surrogates of $\boldsymbol{Q}$ for the Monte Carlo procedure which estimates the *compliance uncertainty* $\eta_\alpha$.

### III.B. *Reactor Operations*

Reactor operation is an active component of the managed compliance system because our methodology is based on *a posteriori* analysis. (In *a priori* based methodologies, reactor operation would only be a passive component.) Therefore, there must exist station procedures which provide for surveillance, or monitoring, of the reactor core. The purpose of such surveillance is to

- execute the actual statistical tests to judge the specific compliances (e.g., see Ref. 8 and 9) which are often done by simply comparing the computed (or measured) parameter to a *reporting limit* which is determined from the *compliance uncertainty*;

- test an identified parameter, other than the one being tested for compliance, to be within specified bounds (e.g., average SORO-FINCH error[9]) in order to confirm assumptions which are made in the Compliance Methodology to compute the *compliance uncertainty.*

If some of the tests mentioned above fail then appropriate actions are taken which include reporting as well as changes to the reactor operations which affect reactor regulations (e.g., de-rating of the reactor power, calibration of Reactor Regulating System and its instruments) or fuelling sequencing to increase the operating margins. Sometimes long term fuelling strategy changes may be required to increase the operating margins and hence the safety of the reactor operations. Also, larger operating margins may be achieved by using more efficient methodologies which enable us to compute smaller *compliance uncertainties.*

### III.C. *Compliance Methodology*

This component of the managed system is the formal mathematical methodology (for details see Ref. 1, 2 and 3). All the theory pertains to uncertainties derived from the random component of $\varepsilon$, the error in $\hat{\boldsymbol{Q}}$ . The systematic component of $\varepsilon$ is estimated in the Validation program (Fig. 1). The probability distribution for the error $\varepsilon = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n\}$ is referred to as the parental distribution. We introduce an error $\eta$ associated with the condition $\hat{C}$ . Since the condition for channel power licence limit compliance is

$$max\{Q_1, Q_2, Q_3, \ldots, Q_n\} \le L,$$

the error $\eta$ is defined as the error in $max\{\hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \ldots, \hat{Q}_n\}$, where $\hat{Q}_i$ is the SORO computed channel power serving as an estimate of $Q_i$. The probability distribution for the error $\eta$ is, naturally, an extreme value probability distribution and is derived from the parental distribution for the error $\varepsilon = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n\}$ - see Fig. 2.

The *compliance uncertainty* $\eta_\alpha$ is some percentage point of $\eta$ depending on the problem. In case of the above mentioned channel power licence limit condition, $\eta_\alpha$ is defined as

$$\boldsymbol{Probability}\{\eta < -\eta_\alpha\} = \alpha.$$

A schematic of the extreme value probability distribution for $\eta$ together with the *compliance uncertainty* $\eta_\alpha$ is depicted in Fig. 3. (Note that in this example $\varepsilon$ and $\eta$ are taken as relative errors, for convenience.) With such defined *compliance uncertainty* the test procedure is

$$\boldsymbol{Accept} \quad max\{Q_i\} \le L \quad \boldsymbol{if\ and\ only\ if} \quad max\{\hat{Q}_i\} \le L(1-\eta_\alpha),$$

where $L$ is the licence limit. It can be shown[1] that the *significance level* for this procedure is less than $\alpha$.

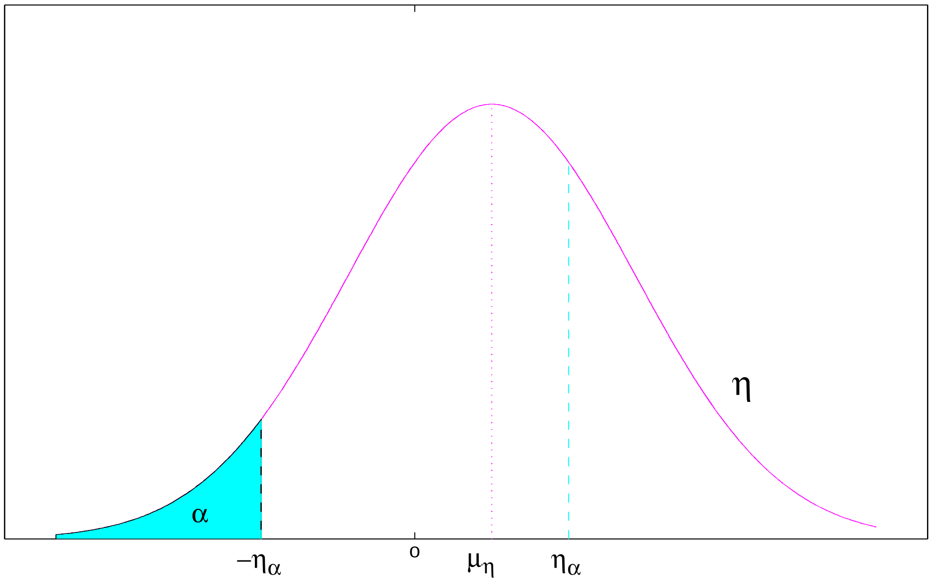


Fig. 3. Probability distribution for $\eta$ and the *compliance uncertainty* $\eta_\alpha$.

Probability distribution for $\eta$ is computed numerically by a Monte Carlo procedure. Again we will demonstrate the approach using channel power licence problem. Generalization to other problems is straightforward. The defining equations in this case are

$$\hat{\boldsymbol{Q}} = \boldsymbol{Q}(1+\varepsilon),$$

$$\eta = \frac{max\{\hat{Q}_i\} - max\{Q_i\}}{max\{Q_i\}}.$$

The estimated channel powers $\hat{\boldsymbol{Q}}$ are computed by SORO in an operating environment. In a Monte Carlo procedure these are, however, computed from the above equation by sampling from a known distribution for $\varepsilon$ (obtained by comparison to the available measured channel powers) and from the given set of *true* channel powers $\boldsymbol{Q}$. Since the *true* channel powers are not known we take the SORO computed channel power estimates $\boldsymbol{S} = \{S_1, S_2, S_3, \ldots, S_n\}$ and assume that these are the *true* channel powers of some fictitious reactor not very different from the real one.[10] Thus, the new equations which are actually used in the Monte Carlo computation are

$$\hat{\boldsymbol{Q}} = \boldsymbol{S}(1+\varepsilon),$$

$$\hat{\eta} = \frac{max\{\hat{Q}_i\} - max\{S_i\}}{max\{S_i\}},$$

where $\hat{\eta}$ is considered a good approximation to $\eta$. Justification for this procedure is provided in.[2]

$\eta$ is a rather complicated function of $\boldsymbol{Q}$ and $\varepsilon$. In order to understand the claimed effectiveness of our procedure we will highlight the following two properties of $\eta$. The probability distribution for $\eta$ is biased (positively in the maximum-type problems - see Fig. 2, or negatively in the minimum-type problems such as in the case of margin to fuel constraint) with respect to the bias of the parental distribution for $\varepsilon$. This means that resulting *compliance uncertainty* $\eta_\alpha$ is necessarily smaller than a *compliance uncertainty* based directly on $\varepsilon$ as is the traditional industry approach (see Figures 2 and 3, or Ref. 1 for details).

Another important property of $\eta_\alpha$ is that it behaves as a convex function of the parameters defining the probability distribution for $\varepsilon$. This allows us to provide guaranteed upper bound estimates on $\eta_\alpha$ without relying on having accurate estimates of $\varepsilon$. Interestingly enough, the maximum values of $\eta_\alpha$ are attained at those values of the parameters of $\varepsilon$ which are very close to our estimates. That means that the obtained upper bound *compliance uncertainties* $\eta_\alpha$ are in fact realistic, contributing to the over all effectiveness of the methodology.

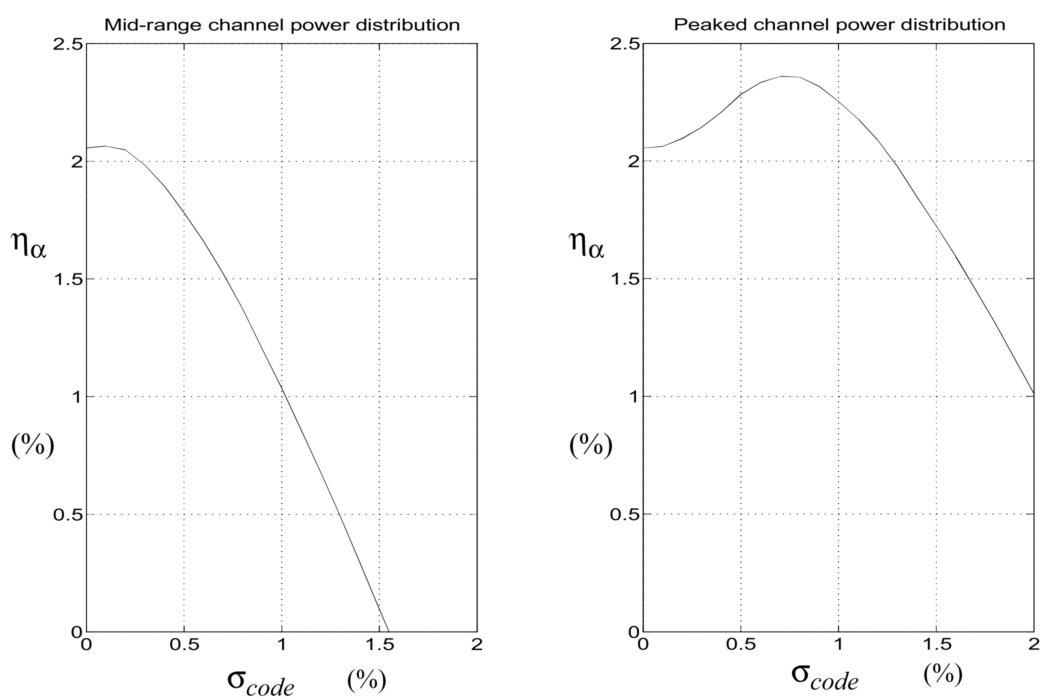


Fig. 4. Relationship between the *compliance uncertainty* and the code error.

In Fig. 4 there are two graphs of $\eta_\alpha$ versus $\sigma_{code}$ for the channel power licence problem, where $\sigma_{code}$ is the standard deviation of the normal probability distribution for the component of $\varepsilon$ describing the error in the SORO code. (The reactor power error component of $\varepsilon$ does not exhibit such a convex property because it is common for all channels. The corresponding relationship would be strictly monotone increasing.) As mentioned earlier $\eta_\alpha$ also depends on $\boldsymbol{Q}$. The two graphs above show the relationship between $\eta_\alpha$ and $\sigma_{code}$ for different channel power profiles $\boldsymbol{Q}$. One is a *peaked* profile, i.e., $\boldsymbol{Q}$ which has only a very few channel powers close to the maximum, while the other is less peaked, or, more *flat*, or, mid-range (the most common channel power distribution that occurs under normal operating conditions) with many channel powers close to the maximum.

The margin to fuel constraint problem is an order of magnitude more complex because the error $\varepsilon$ has two components which randomly vary among channels (again, the reactor power error is common to all channels). One component is the *code* error - the same as above - and the other one is the gap measurement error, $\gamma$. In order to be conservative $\gamma$ is assumed to have uniform distribution described by a parameter $max_\gamma$. The graph, shown in two different azimuthal views, in Fig. 5 depicts the relationship between the compliance uncertainty $\eta_\alpha$, $\sigma_{code}$ and $max_\gamma$. ($\eta_\alpha$ is replaced by $\eta_{98/95}$ in the graph to signify that the *significance level* $\alpha$ is taken to be 0.02 and the "worst" $\boldsymbol{Q}$ was obtained over a 95% of different reactor states.)

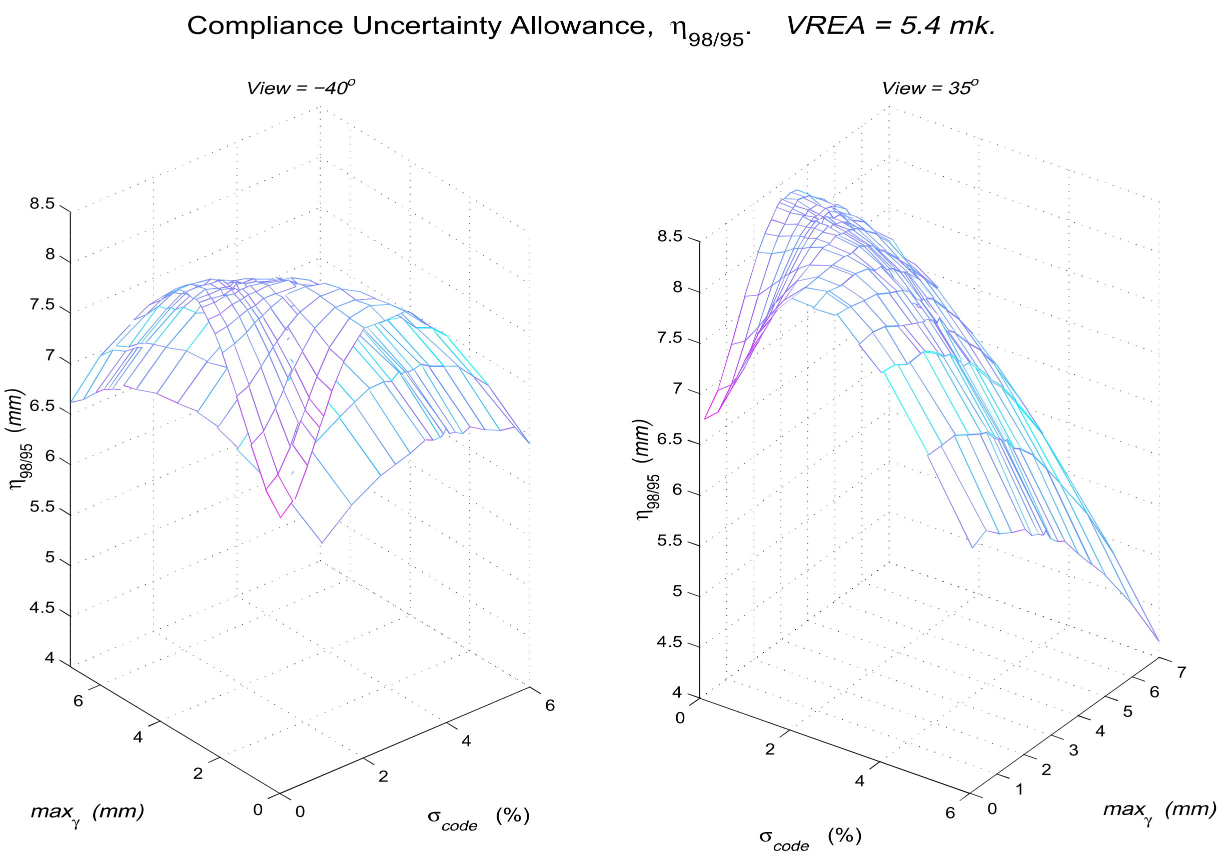


Fig. 5. Relationship between the *compliance uncertainty*, the code error and the gap measurement error.

## III.D. *Validation*

There are two major assumptions that we make in our methodology. First one is that the computational model (including measurements) is sufficiently accurate, i.e., $\hat{\boldsymbol{Q}} \approx \boldsymbol{Q}$. This assumption is made in the Monte Carlo procedure as a necessary condition for $\hat{\eta} \approx \eta$ to hold. In the case of the channel power licence problem we have used the available channel power measurements at selected channels (the so-called FINCHs) and computed $\hat{\eta}_S$ and $\hat{\eta}_M$ using the SORO computed channel powers and measured channels powers, respectively.[2] While we found that the corresponding estimates of the *compliance uncertainties* may be quite at variance for individual reactor states, the agreement was excellent if compared as percentiles taken over many reactor states. This could mean that we might not be able to carry out the statistical testing accurately for individual reactor states, only over many states. Given that $\hat{\eta}_M$ is in itself in error, it is not as yet clear how close $\hat{\eta}$ and $\eta$ are for the individual reactor states. In principle this could be done by estimating sensitivities (first derivatives) of $\eta$ with respect to every $Q_i$ and carry out the first order error analysis for $\hat{\eta}$.

The second important assumption is again made in the Monte Carlo procedure regarding the expected value of $\varepsilon$, or the so-called systematic error component $\mu$. This systematic error component is needed when sampling from $\varepsilon$ to generate the possible random realizations of $\boldsymbol{Q}$. For the SORO code we have assumed that zero is a conservative bound for $\mu$ for the applications we have encountered.

We view the most important role of validation as estimation of the systematic error component of $\varepsilon$. For situations where there are some measurements of $\boldsymbol{Q}$ available we envisage the following approach. First, we identify possible systematic components of $\varepsilon$ and assume a model for these error components which would normally include some free (to-be-determined) parameters. Such models can be either empirical or physically based. An example of an empirically based model is the linear expression involving the channel irradiation as a systematic error component of the SORO *code* error.[11] The physically based model is often obtained by first order sensitivity analysis as can be done for channel power measurement error. In the case of the reactor power error, it is possible to find its fundamental physical components by analysis. Once we build the error models, we compute the free parameters by fitting the model to the existing measurements.

## REFERENCES

1. P. Sermer and C. Olive, "A Probabilistic Approach to Compliance with Channel Power Licence Limits Based on Extreme Value Statistics," OHN Report No. N-REP-06631.04-0575 P, February 15, 1999.

2. F.M. Hoppe, "Ontario Hydro Channel Power Compliance Methodology," McMaster University Report, Pre-print Series 1998/99 No. 2, February 1999.

3. P. Sermer, "Statistical Analysis for Compliance with the Minimum Margin to Fuel Constraint," OPG Nuclear Report No. NK29-REP-03503-00002, April 26, 1999.

4. L. Wilk, "SORO Program Description," Paragon Design Manual, Release 1990.1, Ontario Hydro, April 1990.

5. "Bruce B Large Break LOCA Analysis of Higher Void Reactivity Error Allowance at 90 Percent Full Power," Ontario Power Generation Report No.: NK29-REP-03503-00001, April 26, 1999.

6. E.L. Lehman, Testing Statistical Hypotheses, John Wiley, N.Y. 1959.

7. C. Olive, "Improvements to the Bruce 5-8 SORO Model," OHN Report No. NK29-REP-06631.04-10002 P, February 1999.

8. H.W. Hughes, "Bruce B Minimum Margin to Constraint Compliance Methodology," OHN Report No. NK29-REP-03503-00003, April 26, 1999.

9. G.C. Andognini, Memorandum to J. Blyth, "GAI95G03: Compliance with Bundle and Channel Power Licence Limit," AECB submission, N-00531 P, June 2, 1998.

10. W. H. Press, B. P. Flannery, S. A. Teukolsky and W. T. Vetterling, *Numerical Recipes*, Cambridge University Press, Cambridge 1986, pp. 529 - 532.

11. H.C. Chow and E.V. Carruthers (AECL) with Appendix C by C. Olive (OH), "RFSP Simulations of Darlington FINCH Refuelling Transients," COG-96-579, Rev. 0, November 1977, Appendix C, p. C9.

NOMENCLATURE

| | |
|---|---|
| $\boldsymbol{Q} = \{Q_1, Q_2, Q_3, \ldots, Q_n\}$ | *true* reactor channel powers |
| $\hat{\boldsymbol{Q}} = \{\hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \ldots, \hat{Q}_n\}$ | estimated reactor channel powers |
| $\boldsymbol{S} = \{S_1, S_2, S_3, \ldots, S_n\}$ | SORO computed reactor channel powers |
| $L$, or $L_i$ | (license) limit, or channel specific limits |
| $\alpha$ | level of significance |
| $\varepsilon = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_n\}$ | errors in estimated parameters $\hat{\boldsymbol{Q}}$ |
| $\eta = \dfrac{max\{\hat{Q}_i\} - max\{Q_i\}}{max\{Q_i\}}$ | relative error of maximum channel power |
| $\hat{\eta} = \dfrac{max\{\hat{Q}_i\} - max\{S_i\}}{max\{S_i\}}$ | relative error of maximum channel power using bootstrap |
| $-\eta_\alpha$ | the (lower) $100\alpha$ percentile of the probability distribution of $\eta$ (note the minus sign for convenience as the quantity $\eta_\alpha$ will generally be positive) |
| $\eta_\alpha$ | *compliance uncertainty* |
| $u_\alpha$ | industry standard margin of error (the$100(1-\alpha)$ percentile of the probability distribution of $\varepsilon_i$) |
| $L(1-\eta_\alpha)$ | *reporting limit* (critical value of $max\{Q_i\}$) |
| *peaked channel power profile* | few channels close to maximum power |
| *flat channel power profile* | many channels close to maximum power |
| $\sigma_{code}$ | standard deviation of SORO *code* error |
| $\gamma$ | gap measurement error |
| Bootstrap | statistical methodology using data to generate error distributions by simulation |